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MODELLING CLAIM COUNTS OF HOMOGENEOUS INSURANCE RISK GROUPS USING COPULAS

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1. INTRODUCTION

We study an application of copula modelling to insurance using count data of the automobile line of business. We consider the following homogeneous risk groups: third party liability property damages, third party liability bodily injury and material own damages. First, we model the marginal behaviour of each group, then we estimate a tri-variate copula. Data suggests similar correlations between groups. We did a continuous and a discrete approach. Due to the limitations of a direct discrete approach we perform a continuous approximation. In the discrete case we fit a negative binomial to each risk group and in the continuous one we try the gamma and normal approximations.

As our application relies on the assumption of parametric univariate marginals we perform the goodness-of-fit tests by proposing a parametric extension of the test presented by Genest et al. (2009). The test is based on the empirical copula,

$$\hat{C}(u) = \frac{1}{t + 1} \sum_{j=1}^{t} I\{Z_{j1} \leq u_1, \ldots, Z_{jn} \leq u_n\}$$

where $u = (u_1, \ldots, u_n) \in [0, 1]^n$ and $I$ is the indicator function. In the semiparametric approach by Genest et al. (2009) the sample of the vector $Z$ is given by

$$z_j = (z_{j1}, \ldots, z_{jn}) = \left(\frac{\text{Rank}(x_{j1})}{n + 1}, \ldots, \frac{\text{Rank}(x_{jn})}{n + 1}\right) \quad j = 1, \ldots, t$$

---

†The author thanks Seguros LOGO S.A. for financial support.
§The author gratefully acknowledges financial support from FCT-Fundação para a Ciência e a Tecnologia (Programme FEDER/POCI 2010)
where \( \text{Rank}(x_{ji}) \) is the rank of \( x_{ji} \) amongst the sample of the claim counts \( (x_1, ..., x_m) \). To evaluate the goodness-of-fit of a parametric copula model we propose that the sample of the vector \( Z \) is given by

\[
z_j = (z_{j1}, ..., z_{jn}) = (F_1(x_{j1}), ..., F_n(x_{jn})) \quad j = 1, ..., t.
\]

The test is based on a statistical test that compares \( \hat{C}(z) \) with an estimated \( C_\theta(z) \), of the theoretical copula \( C_\theta \). This statistical test is given by,

\[
\hat{T} = t \int_{[0,1]^n} \left\{ \hat{C}(z) - C_\theta(z) \right\}^2 d\hat{C}(z) = \sum_{j=1}^{t} \left\{ \hat{C}(z_j) - C_\theta(z_j) \right\}^2.
\] (1)

A bootstrap procedure is required to compute the \( p \)-value of the test (1). The steps of the bootstrap technique are detailed in Berg (2009) and could be easily adapted to the parametric approach. For the univariate distributions we do the standard goodness-of-fit tests.

We work with a sample of the automobile portfolio of an insurer operating in Portugal. It has monthly observations of claim counts from 2000 to 2008. As expected, the estimated values of the Kendall’s \( \tau \) presented in Table 1 reveal dependence among the three groups. The descriptive statistics presented in Table 2 show some negative skewness for the material own damages risk group which maybe be due to the existence of a franchise and an upper capital limit (value of the vehicle). For details, please see Santos (2010).

<table>
<thead>
<tr>
<th>Risk group</th>
<th>TPL property damages</th>
<th>TPL bodily injury</th>
<th>Material own damages</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPL property damages</td>
<td>1</td>
<td>0.437</td>
<td>0.492</td>
</tr>
<tr>
<td>TPL bodily injury</td>
<td>1</td>
<td></td>
<td>0.299</td>
</tr>
<tr>
<td>Material own damages</td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1: Kendall’s \( \tau \) matrix of the claim counts

<table>
<thead>
<tr>
<th>Risk group</th>
<th>TPL property damages</th>
<th>TPL bodily injury</th>
<th>Material own damages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>4 028</td>
<td>337</td>
<td>1 198</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>568.93</td>
<td>54.07</td>
<td>121.46</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
<td>14%</td>
<td>16%</td>
<td>10%</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.13</td>
<td>0.38</td>
<td>-0.19</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-1.05</td>
<td>0.06</td>
<td>-0.61</td>
</tr>
</tbody>
</table>

Table 2: Descriptive statistics of the data
Figure 1: TPL property damages - Gamma and normal vs empirical distribution.

Figure 2: TPL bodily injury - Gamma and normal vs empirical distribution.

Figure 3: Material own damages - Gamma and normal vs empirical distribution.
2. CONTINUOUS MODELLING

We consider gamma and normal distributions approximations to model the marginal behaviour for the claim counts of each risk group. The former can be viewed as a continuous version of the negative binomial distribution, when the random variables do not take zero values and do not have a large number of repeated values. The latter is taken from the Central Limit Theorem.

Parameters estimation is carried out using the Inference For Margins method (IFM). It is a two-step method that first estimates the marginal parameters and then calibrates the copulae parameters. It does \( n \) separate optimizations of the univariate likelihoods, followed by an optimization of the multivariate likelihood as a function of the dependence parameter vector. Table 3 shows the maximum likelihood parameter estimates (ML) for the gamma and normal approximations as well as the \( p \)-values of the Kolmogorov-Smirnov tests for the three risk groups. Accordingly, Figures 1-3 show plots of the empirical and approximating distributions.

<table>
<thead>
<tr>
<th>Risk group</th>
<th>Gamma distribution</th>
<th>Normal distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>TPL property damages</td>
<td>50.39</td>
<td>0.013</td>
</tr>
<tr>
<td>TPL bodily injury</td>
<td>39.60</td>
<td>0.117</td>
</tr>
<tr>
<td>Motor own damages</td>
<td>95.91</td>
<td>0.080</td>
</tr>
</tbody>
</table>

Table 3: ML estimates of gamma and normal fits and \( p \)-values

We tried five different trivariate copula families and estimated their parameters considering both gamma and normal distributions. These results are shown in Table 4. We tried the \( t \)-copula but the estimated degrees of freedom were high and thus the \( t \)-copula comes close to the Gaussian copula. According to Embrechts et al. (2003) a robust estimator for the components of the correlation matrix \( R \) of the Gaussian copula is given by \( R_{ij} = \sin(\pi \tilde{\tau}_{ij}/2) \).

<table>
<thead>
<tr>
<th>Copulas</th>
<th>Gamma distribution</th>
<th>Normal distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \theta_1 )</td>
<td>( \theta_2 )</td>
</tr>
<tr>
<td>Gumbel</td>
<td>1.5437</td>
<td>1.6242</td>
</tr>
<tr>
<td>Nelsen</td>
<td>1.0474</td>
<td>1.0923</td>
</tr>
<tr>
<td>Cook-Johnson</td>
<td>0.9482</td>
<td>-</td>
</tr>
<tr>
<td>Gaussian</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4: Copulae parameters estimates with gamma and normal distributions and \( p \)-values

According to the goodness-of-fit test, with a significance level greater than 10%, we came out with the following models to fit the claim counts: Gaussian copula with normal marginal distributions; Nelsen’s copula with gamma marginal distributions; Gaussian copula with gamma marginal distributions; Cook-Johnson’s copula with gamma marginal distributions.

Cook-Johnson’s copula assumes an equal level of association for all pairs of random variables which is a very restrictive property. For instance, according to the Kendall’s \( \tau \) in Table 1 we see that may not be true. Nelsen’s copula with gamma marginal distributions allows skewness in
the data whereas the Gaussian copula leads to a radial symmetric distribution. Since our data has some asymmetry, we conclude that Nelsen’s copula with gamma distribution marginals should be preferred to model the joint claim counts of the three risk groups.

3. DISCRETE MODELLING

In the discrete modelling we consider a mixed Poisson distribution with a structure gamma distribution for the marginals, leading to a negative binomial distribution. Table 5 shows the maximum likelihood parameter estimates for the negative binomial distribution as well as the $p$-values of the Chi-squared test. Figure 4 shows the negative binomial fit versus empirical distribution.

<table>
<thead>
<tr>
<th>Risk group</th>
<th>$\alpha$</th>
<th>$p$</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPL property damages</td>
<td>50.77</td>
<td>0.012</td>
<td>14.91%</td>
</tr>
<tr>
<td>TPL bodily injury</td>
<td>43.92</td>
<td>0.115</td>
<td>24.81%</td>
</tr>
<tr>
<td>Motor own damages</td>
<td>105.96</td>
<td>0.081</td>
<td>78.11%</td>
</tr>
</tbody>
</table>

Table 5: ML parameter estimates of negative binomial distribution and $p$-values

![Negative binomial vs empirical distribution](image)

Figure 4: Negative binomial vs empirical distribution

The discrete approach should be the natural method to fit the claim counts, however it has limitations since the copula theory has serious restrictions when the marginals are discrete. Thus, we can neither properly estimate the copula and its parameters nor fully test the fit. To overcome these limitations we estimate copulae for the structure distributions. Since the parameters of the copulae represent the dependence parameters we estimate them using the claim counts sample. However, since the structure variables are not observable we cannot perform a goodness-of-fit test.
for the copulae. Tables 6 shows the estimates for the structure gamma distributions and Table 7 shows the parameters estimates for the copulae.

<table>
<thead>
<tr>
<th>Risk group</th>
<th>α</th>
<th>$\beta = p/(1 - p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPL property damages</td>
<td>50.77</td>
<td>0.0126</td>
</tr>
<tr>
<td>TPL bodily injury</td>
<td>43.92</td>
<td>0.1303</td>
</tr>
<tr>
<td>Motor own damages</td>
<td>105.96</td>
<td>0.0884</td>
</tr>
</tbody>
</table>

Table 6: Parameter estimates for the structure gamma distribution

<table>
<thead>
<tr>
<th>Archimedean Copulas</th>
<th>Structure gamma</th>
<th>θ₁</th>
<th>θ₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gumbel</td>
<td>1.5011</td>
<td>1.5829</td>
<td></td>
</tr>
<tr>
<td>Nelsen</td>
<td>1.0195</td>
<td>1.0704</td>
<td></td>
</tr>
<tr>
<td>Cook-Johnson</td>
<td>0.8485</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Archimedean copulas parameter estimates for the structure gamma for marginals

Comparing the estimates obtained in the discrete modelling with the ones obtained in the continuous case we see that they are similar in the cases of Gumbel, Nelsen and Cook-Johnson’s copulas.

4. CONCLUSION

According to the automobile data illustration presented, as multivariate model to fit the claim counts between TPL property damages, TPL bodily injury and material own damages risk groups, Nelsen’s copula should be chosen with gamma marginals. The discrete approach presented seems to confirm this conclusion, and it is an interesting line of research for the future. Moreover, the results obtained for the degrees of freedom of the $t$-copula support the absence of a tail dependence in the risk groups. This seems reasonable because extreme events are not covered by these groups. See Santos (2010) for details.

We remark that this application has at least one limitation due to a possible existence of seasonality in the data. This is not captured by the IFM method that assumes independent observations.

References

Modelling claim counts with copulas


