Dynamical Life Tables

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Abstract

A life table represents data about mortality in a given population. Using the data for long term assurance plans assumes stability of that population. In particular, the life expectancy, \( e_x \), of a life aged \( x \) is fixed. In fact \( e_x \) changes and the population is not stable. We suggest a model to derive a future life table, based on a given life table and some assumptions on the change of \( e_x \). These future life tables for various periods combines to a single life table. This is the Dynamical-Life-Table (DLT). In the study of long term life assurance schemes, the DLT provides a tool. For policies that are now issued we get premiums and reserves that accounts to some predictions about the change in \( e_x \). For past policies that are still in force we may evaluate the profit or deficit that are due to the change that occurred in \( e_x \).

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1. A problem in the current life tables

Calculating assurance or pension premium rely on the existence of a suitable life table [Neill 89]. The mathematical background for those calculations can be found in numerous books. We chose to mention two. A book by Nesbitt et al. [Ness.86] that contains a broad mathematical view of the subject and a book by R.C. Elandt-Johnson and Norman Johnson [Elan.80] that deal with the subject along with many more aspects of population modeling. Nevertheless, well-founded funds rely on having accurate knowledge of its members mortality rates. This kind of information is attained by modeling the population of interest and constructing the desired life table. Further elaboration can be found in a book by A. S. Puzev [Puze].

In our work we used the English Life Tables [ELT 12],[ELT 13],[ELT 14]. These tables describe the mortality rates in England and Wales. As can be seen, tables are updated once every ten years. The need for periodic updates is due to changes in mortality over time. This however, means that life tables can not give an accurate description of mortality rates over time. Notice, for example, that a group of sole aged 30 last birthday at the calendaric year 1960 will be expected to obey to the mortality rates appearing in ELT12. This is the best approximation that we can give until 1970. In that year we will be able to use ELT13 and so on. In fact, if we want to assure this population we can expect that it will be subject to more then four

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different life tables. This is if the life expectancy in that age $e_x$ is 41.06 years- as it appears in ELT12. Of course, in 1960 we only have ELT12. Still, is it enough even for the next 10 years?

It is clear that the change in mortality varies in time. This means that for that observed population, we should change the table every year!

A step in that direction is the work done by Society of Actuaries UP-94 Task Force [Task96]. Their idea was to take two adjacent life tables, use them as "poles", and generate a series of contiguous life tables for each year between the two existing tables.

Observe that in order to achieve the desired approximation, one must possess two life tables. This enables a fund director to retrospectively observe existing premiums against the ones calculated using the contiguous life tables. As a result reserves should be reassessed. One shortfall in this method is that it is an interpolation-based method. Extrapolation of mortality rates is expected to loose accuracy rapidly. Furthermore, since the current table is used together with the previous one to approximate past mortality, only a retrospective point of view is possible. Another shortfall is that the method is based upon numerically bridging two adjacent life tables rather then constructing a theorem for a dynamically changing mortality.

In our work, we constructed a mathematical model that can describe such a dynamic population. Using this model, a future estimation of the mortality can be constructed. This enables a more suitable premium calculation as well as retrospective reserves reassessing of existing policies. Moreover, changing the model parameters enables the conduction of a sensitivity analysis in a click of an Excel button.

### 2. Dynamical-Life-Tables

Life tables, as the English Life Tables (ELT), represent a brief "snapshot" of the mortality in an observed population. Our first goal is to try to understand the implications of a "dynamically changing mortality". In order to do so, a second survey is being held in the next year. Notice that the "basic" data from the survey is roughly ratio between soles age $x$ last birthday and the number of death at that year. After a few more manipulations, such as graduation, we get $q_x$ - the rate of mortality. One can think of $q_x$ as the number of death out of a population of $l_x$ soles aged $x$ last birthday.

Since mortality rates are assumed "time dependent", the new table is slightly different from the initial one. If we held a third pole in the following year, we get a mortality table that is yet slightly different from the preceding ones. Continuing this process will produce a series of consecutive mortality tables. When plotted together in a $(x,t,q_x)$ three dimensional space, we get a surface that describes the mortality. We refer to it as the Mortality-Surface (MS). The table containing mortality rates for different ages at different years is the Dynamical-Life-Table (DLT). This table forms a description of the mortality variation over time. Conducting a yearly-based survey is neither feasible nor desired since it will not be able to produce predictions of future mortality.

Instead, a time dependency is introduced to $q_x$, the rate of mortality. In order to produce a well founded, widely applicable model, a few guidelines and restrictions were introduced.
First, the DLT is based upon a given life table. This means that at the time of the survey the given life table and the DLT coincide. Additionally, changing the initial life table will result in generating a new table for a different population. Second, the introduced time-dependency should not change the fact that $q_x$ represent, from a probability point of view, the probability of "success" in a Bernoulli trail. Finally, in order to characterize the change in mortality over time we choose to use the average change in life expectancy at age, namely $e_x$. This process was tested using ELT12 as a basis for the DLT while an average change of year-per-decade was chosen to describe the mortality change over time. The resulting DLT was compared with ELT13 after ten years and ELT14 after twenty. The relative difference between the DLT and ELT13 was 2% in the age interval 30 to 65 and 10% for ages 66 to 90. When tested against ELT14, the relative difference was even smaller: 0.9% in the age interval 30 to 65 and 7.6% for ages 66 to 90. The effect on premium and reserves, based on ELT13 or ELT14, compared with DLT is smaller due to the affect of interest.

3. On the implementation of the DLT

To generate a Dynamical-Life-Table you actually need two things: an original life table and an estimate of the average change expected in life expectancy at age $x$ ($e_x$). The original table connects the DLT to a certain population while the change in life expectancy, defines the table's behavior in time. Changing the original table will result in fitting the table to a different population. Changing the estimate of the average change in life expectancy at age $x$ will enable a fund director to conduct a sensitivity analysis. Observe that one may estimate increase in $e_x$ for some period and a decrease in other.

Once a life table is at hand, calculating premiums and reserves is merely a matter of choosing the appropriate mortality data that fits the insured's age group and following the regular process of premium or reserves calculation. In fact, a tool, implemented as an Excel chart, is available. This tool receives as input an original life table, an estimated average change in life expectancy at age $x$ and an estimate of yearly-based interest rates. As output, premium for whole life assurance and pension plans is calculated. Again, by changing the original life table one can fit the table to any desired population. Alternatively varying the estimate of the average change in life expectancy at age $x$ will result in conducting a sensitivity analysis for premium rates.

Constructing a mathematical model for dynamical mortality rates is an important step towards fully understanding the dynamical nature of the population variation. The Dynamical-Life-Table and the Mortality-Surface represents an implementation of the model for concrete population. Using the DLT and the MS enables us to observe the effects of the change in life expectancy at age $x$ on premium and reserves. Finally, the premium calculation tool is an important managing tool that lets a fund manager the opportunity to construct sensitivity analysis for desired populations.
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