RISK ANALYSIS IN ASSET-LIABILITY MANAGEMENT FOR PENSION FUND

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ABSTRACT

The management of a Pension Fund must take into account the temporal evolution of its assets and its liabilities. The variables we work here are the factors and returns representing assets and liabilities; as illustration purpose, numerical data have been taken from the Spanish Market. The past performance of these variables is analysed statistically and we deduce a Vector Error Correction Model (VECM) to model their behaviour. In the process of scenario generation we simulate different trajectories for each of the variables (return or factor) with a probability of occurrence associated to each trajectory.

Risk is related to adverse results and to a first approximation is measured by the Value at Risk (VaR) over a given planning horizon together with the calculation of the Expected Shortfall. Finally it is also considered necessary that the portfolio managers should be able to introduce into the calculation their own preferences in the face of risk, so that we complete the analysis by evaluating the fund in terms of the Expected Utility.

The practical application of risk analysis carried out according to the results of a scenario generation model constitutes the main contribution of the present work.

Keywords: Asset-liability management, Scenario generation, Risk, Value at Risk, Expected shortfall, Expected utility.

JEL Classification: G023, G024
INTRODUCTION

Asset-Liability Management according to the Society of Actuaries (1998) “… is the practice of managing a business so that decisions on assets and liabilities are coordinated. It can be defined as the ongoing process of formulating, implementing, monitoring and revising strategies related to assets and liabilities in an attempt to achieve financial objectives for a given set of tolerances and constraints..... ALM is relevant to, and critical for, the sound management of the finances of any institution that invest to meet liabilities”.

The factors and yields which condition the behaviour of the assets and liabilities are generated from a great number of scenarios, which reflect some behaviour of the environment in which decisions have to be taken. Within this context, our work centres on the a posteriori analysis that must be made of the scenario generation prior to using the mathematical programming models. This analysis is performed by evaluating, through such measures of risk as the Value at Risk (henceforth VaR), the Expected Shortfall, and the Expected Utility, the consequences of the future behaviour of factors and yields on a set of investment strategies.

The methods used to obtain the scenario generation model may be based on cascade structures following the work of Mulvey & Thorlacius (1998), and of Wilkie (1986, 1995), but in the present work we fit the time series with an Autoregressive Vector Error Correction Model (henceforth VECM), following the line set out by Boender et al. (1995, 1998), Dert (1995, 1998), Kim & Mina (2000) and Kim et al (1999).

The coordination between assets and liabilities for each scenario may be analysed by the surplus, understood as the difference between the values of assets and of residual liabilities (Mulvey & Vladimirou, 1992), or by the funding level understood as the ratio between those two values (Daykin et al., 1991). In the present work, we chose the first option that allows us to quantify the capital needs or surpluses in monetary units. We analyse a set of investment strategies in order to determine their risk levels, incorporating the future information on yields and factors obtained from the generated set of multi-period scenarios.

Risk is related to adverse results, and to a first approximation may be measured by the VaR. We shall here analyse this process empirically for the calculation of a fund’s VaR, over a given planning horizon, together with the calculation of the Expected Shortfall. It is also considered necessary that the portfolio managers should be able to introduce into the calculation their own preferences in the face of risk, so that we complete the analysis by evaluating the fund in terms of the Expected Utility. The practical application of risk analysis carried out according to the results of a scenario generation model constitutes the main contribution of the present work. The intention is to improve analyses which have until now been performed only partially and with no practical implementation of the methods advanced.

1. SCENARIO GENERATION

The scenario generation model that we worked with is of the "Dynamic Modeling" type as is described by the Dynamic Financial Analysis Committee of the Casualty Actuarial Society (1999). The works of Boender et al. (1995,1998), Dert (1995, 1998) and the applications of Kim et al. by the group Riskmetrics (1999) also follow the line of obtaining a scenario generation model using a VECM that will be described in the following paragraphs.

The variables used in the present work are a subset of those mentioned above: rates of return on cash (C), on deposits (henceforth D), on stocks (henceforth SM), on bonds (henceforth B), on mortgage loans (henceforth ML), the actuarial discount rate (henceforth L), and consumer price inflation (henceforth CPI). The proxies used for the analysis of the factors and yields relating to the Spanish market are: the daily interbank market rate (C), the three-month rate (D), the General Index of the Madrid Bourse (SM), the
index of the yield of a portfolio of bonds (B), the mean mortgage loan rate at more than three years (ML),
the rate at issue of five-year Treasury Bonds (L), and finally the consumer price inflation rate (CPI).

An analysis of the overall historical behaviour of these variables from January 1989 to December
2001 shows a strong short-term correlation between the different interest rates and also between these and
the consumer price inflation rate. The relationship between them and the returns on stocks and on bonds
is slight and negative between all the interest rates and the return on stocks.

We analysed the relationships between the variables in the short-term, and incorporated the long-
term relationships by means of a co-integration analysis. The trace and maximum value tests indicated the
existence of three co-integration relationships. These were introduced into the model by means of an Error
Correction Mechanism (henceforth ECM). The work of Dominguez (2001) shows how to solve it. The
resulting VECM, which provides information on both short-term and long-term relationships, for the period
t and with the variables that were analysed in the present work, the following:

\[
\begin{bmatrix}
C_{t}^{(12)} \\
D_{t}^{(12)} \\
L_{t}^{(12)} \\
B_{t}^{(12)} \\
SM_{t}^{(12)} \\
CPI_{t}^{(12)} \\
ML_{t}^{(12)}
\end{bmatrix} = \begin{bmatrix}
0.00238 & 0.6719 & 0 & 0 & -0.8659 & -0.46483 & 0 & -0.80502 \\
-0.0025 & 1.0837 & -1.4586 & -1.5317 & 0 & -0.10734 & -0.1369 & 0.89607 \\
-0.00041 & 0.1085 & 0 & 1.2571 & 0 & 0.89155 & 1.0487 & 1.0724 \\
0.00524 & 0.3099 & 0.1651 & -1.4637 & 0.8691 & -0.38233 & -0.22644 & 0.4067 \\
0.00064 & 0.0692 & 0.4094 & -0.9173 & 0.8204 & 0.886 & -1.7761 & 0.2898 \\
-0.0012 & -0.2856 & -0.0571 & 0.1494 & 0.3719 & -1.3464 & 2.5214 & 0.1974 \\
0.00009 & -0.1626 & 0.3240 & 0 & 0.9149 & -0.4227 & -0.5104 & 0.9842
\end{bmatrix}
\]

\[+\]

\[
\begin{bmatrix}
C_{t-1}^{(12)} \\
D_{t-1}^{(12)} \\
L_{t-1}^{(12)} \\
B_{t-1}^{(12)} \\
SM_{t-1}^{(12)} \\
CPI_{t-1}^{(12)} \\
ML_{t-1}^{(12)}
\end{bmatrix}
\]

The variance and covariance matrix of the residuals takes the following values:

\[\text{The database is obtained from the Statistical Bulletin of the Bank of Spain.}\]
After applying Cholesky factorization to the residuals of the estimated VECM, which are characterized by being $N(0, \Sigma)$, they are transformed into residuals $N(0,1)$ and one obtains different values of the variables in the future via a Monte Carlo simulation. Each of the simulations has the same probability of occurrence.

Scenario generation consists in obtaining different trajectories of behaviour of the entire set of variables out to a pre-set planning horizon. $S$ will represent the set of scenarios so that any scenario $s$ generated forms part of the said set ($s \in S$).

For the practical case that will be described in the following section, we generated 625 scenarios, $s=\{1,2,...,625\}$, each with the same probability of occurrence, for a planning horizon of two periods. The representation of a scenario is given by $s$, with $s \in S$. The superscript $s$ will be added to any variable, which depends on the values of the scenario $s$.

### 2. COORDINATION BETWEEN ASSET AND LIABILITIES

In this section, the goal is to analyse the degree of coordination between the liabilities that a life insurance portfolio has to satisfy, and the assets in which it is invested, taking into account the information provided by the scenario generation model described in the previous section. The variable used to evaluate the coordination between assets and liabilities will be called the surplus, and is defined as the difference between the values of the assets and liabilities for each period. In the present case, this period will be monthly, and will be expressed as $t$.

Every month decisions are made relative to investment policy, and investments are made for each period according to the market conditions existing at that time. The alternatives the manager has available in making investment decisions are: cash, deposits, bonds, stocks, and mortgage loans. The monthly amount assigned to each alternative in each period $t$, under scenario $s$, is represented by $X_{Ct}^{(12)s}$, $X_{Dt}^{(12)s}$, $X_{Bt}^{(12)s}$, $X_{SMt}^{(12)s}$ and $X_{MLt}^{(12)s}$, respectively, and their sums gives the total value of the assets in each period. These amounts represent a percentage ($\alpha_{Ct}$, $\alpha_{Dt}$, $\alpha_{Bt}$, $\alpha_{SMt}$, and $\alpha_{MLt}$) of the total Available Fund in each period $t$ under scenario $s$, $F_0^{(12)s}$. The investment decision may be modified in each period. While there may thus be various investment strategies in the two time periods, the actual selection of strategies is that summarised in the six strategies $e=\{1,2,3,4,5,6\}$ specified in Table 1.
Table 1. Investment strategies as a function of each asset mix.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>January 2002</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_B$ : 100%</td>
<td>$\alpha_C$ : 10.67%</td>
<td>$\alpha_D$ : 4.27%</td>
<td>$\alpha_B$ : 62.05%</td>
<td>$\alpha_C$ : 4%</td>
<td>$\alpha_D$ : 3%</td>
<td>$\alpha_{SM}$ : 100%</td>
</tr>
<tr>
<td>$\alpha_D$ : 4.27%</td>
<td>$\alpha_C$ : 10.67%</td>
<td>$\alpha_D$ : 4.27%</td>
<td>$\alpha_B$ : 25%</td>
<td>$\alpha_C$ : 4%</td>
<td>$\alpha_D$ : 3%</td>
<td>$\alpha_{ML}$ : 8%</td>
</tr>
<tr>
<td>$\alpha_{SM}$ : 23%</td>
<td>$\alpha_{SM}$ : 60%</td>
<td>$\alpha_{ML}$ : 60%</td>
<td>$\alpha_{ML}$ : 8%</td>
<td>$\alpha_{SM}$ : 60%</td>
<td>$\alpha_{ML}$ : 8%</td>
<td>$\alpha_{ML}$ : 0.01%</td>
</tr>
<tr>
<td>$\alpha_{ML}$ : 0.01%</td>
<td>$\alpha_{ML}$ : 0.01%</td>
<td>$\alpha_{ML}$ : 0.01%</td>
<td>$\alpha_{ML}$ : 0.01%</td>
<td>$\alpha_{ML}$ : 0.01%</td>
<td>$\alpha_{ML}$ : 0.01%</td>
<td>$\alpha_{ML}$ : 0.01%</td>
</tr>
</tbody>
</table>

The impact of the behaviour of the factors on the company's results is evaluated by the difference between the values of the assets and liabilities in each period of the planning horizon. The assets consist of the investment at each moment, while the liabilities represent the necessary endowment in reserves in each period. This reserve is calculated in our case prospectively, as the difference between the present actuarial value of the liabilities and the present assets value of future premiums.

The coverage of a retirement payment is by means of a deferred capital sum of 100,000 € to be paid out at retirement age (65 years). The signing date of the pension plan is January 2002 and the participant is 60 years old. The contributions are paid monthly at the beginning of each month, and consist of a fixed percentage of the individual's wages over the length of his/her active life. The wages are 1.875 € per month (in December 2001), increasing monthly in the same percentage as the estimated inflation rate.

The survival tables: GRM/F-95. These are one of the official Spanish survival tables and they are static table. Nevertheless the Treasury Department admit the tables PERF/M2000 to and they are dynamic table. The actuarial interest rate is the maximum applicable rate will be 60% of the weighted arithmetic mean of the last three years of the mean interest rates of the last quarter of each fiscal year of 5-year or longer Treasury Bonds. The weighting is 50%, 30%, and 20% of each of the interest rates of the last three years.

Since the evaluation of the investment strategies is performed at the beginning of January 2002, the value of the surplus obtained in each period $t$ has to be updated. In this practical application, we obtained a value of the surplus for each scenario $s$ for the two periods of January 2002 ($t=1$) and February 2002 ($t=2$), and like we are interested in obtained one measurement with information about two periods we calculate the Aggregated Surplus. The Aggregated Surplus under scenario $s$ and strategy $e$, at the moment that is doing the valuation, initial moment, is:
\[
Surplus_{0}^{s,e} = \frac{Surplus_{1}^{s,e}}{(1 + i_{1}^{(12)s})} + \frac{Surplus_{2}^{s,e}}{(1 + i_{2}^{(12)s})(1 + i_{2}^{(12)s})} \\
\forall s = \{1, 2, \ldots, 625\}; \ e = \{1, 2, 3, 4, 5, 6\}
\]

where:

\(Surplus_{0}^{s,e}\) : Aggregated Surplus attained with strategy \(e\) discounted to the initial moment of the temporal horizon under scenario \(s\).

\(Surplus_{1}^{s,e}\) : Excess or surplus obtained in January 2002 under scenario \(s\) and strategy \(e\).

\(Surplus_{2}^{s,e}\) : Excess or surplus obtained in February 2002 under scenario \(s\) and strategy \(e\).

\(i_{1}^{(12)s}\) : Monthly actuarial discount rate in January 2002, under scenario \(s\).

\(i_{2}^{(12)s}\) : Monthly actuarial discount rate in February 2002, under scenario \(s\).

3. RISK ANALYSIS

In this section we shall quantify the risks incurred with each of the investment strategies. This will provide information to use both in evaluating the capital needs of each strategy and in choosing the one which is best in tune with the portfolio manager's goals. The ultimate objective of every manager is clearly to obtain a profit as the result of his or her management. Hence a risk will have been incurred when a loss is the final result of the generated surplus. A lack of coordination, in the sense that the liabilities are greater than the assets, is to be regarded as a risk, while a lack of coordination in the sense of an excess of assets with which to cover the liabilities is not. In the present study, using an expected utility criterion, we shall rank the riskiest strategies by incorporating into the analysis the subjective component of the manager's aversion to risk.

We shall here focus on three types of measure: VaR, Expected Shortfall, and a criterion of Expected Utility. This last measure incorporates the risk aversion of the decision maker who selects the investments, whereas the other two measures do not. One aspect to bear in mind is that a multi-period analysis is performed, so that there exist two possibilities: either measure the risk in each period, i.e., period-by-period, or perform an analysis at a single moment in time so that the risk for all of the periods is included.

We chose at the beginning of January 2002 to perform the risk analysis at a single moment in time. Table 2 gives the absolute values of the \(VaR\) discounted to the present time, for the different strategies analysed, and for a 95% confidence level (\(\gamma = 0.95\)).

<table>
<thead>
<tr>
<th>Strategy</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(VaR_{0.95})</td>
<td>-1680</td>
<td>-1423</td>
<td>-2778</td>
<td>-1732</td>
<td>-3356</td>
<td>-4801</td>
</tr>
</tbody>
</table>

Table 2 shows that the strategies with the lowest \(VaR_{0.95}\) in absolute terms are numbers 1 and 2. These both invest the most of them in bonds.
This information allows us to state, with 95% probability that under the market conditions estimated with our scenario generation model the investment strategy 2 does not involve a loss generated during the two periods analysed of more than 1423 €. In the extreme cases of no diversification, i.e., of investing everything in either bonds or stocks, the maximum loss (with 95% probability) rises to 1680 € or 4801 €, respectively.

Table 3 gives the results of the $VaR$ in absolute value for the different allocations of assets that were analysed for each of the two periods (for a 95% confidence level).

Table 3. $VaR_{0.95}$ for the strategies analysed for each period and for a 95% confidence level.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$VaR_{0.95}$ (January, 2002)</td>
<td>-1485</td>
<td>-927</td>
<td>-1878</td>
<td>-1621</td>
<td>-2443</td>
<td>-3994</td>
</tr>
<tr>
<td>$VaR_{0.95}$ (February, 2002)</td>
<td>-272</td>
<td>-347</td>
<td>-1050</td>
<td>-587</td>
<td>-1087</td>
<td>-1037</td>
</tr>
</tbody>
</table>

In this last case the measure of risk $VaR$ is applied period-by-period within the temporal horizon analysed, since it may be of interest to the portfolio manager how the risk varies over time.

One of the deficiencies that is attributed to this measure of risk is that one has no information available in the case that there are losses which are greater than the amount established in the $VaR$. Hence a new measure that is used is the Expected Shortfall (henceforth $CVaR$ for Conditional $VaR$) defined as the conditional expectation of losses above the $VaR$, i.e., those which are in the tail of $(1−γ)$% of probability. In the practical case, for a 95% confidence level, that we are working with, the ranking established by the $VaR_{0.95}$ and $CVaR_{0.95}$ coincides.

Table 4. Ranking according to the $VaR_{0.95}$ and to $CVaR_{0.95}$ criteria.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>$VaR_{0.95}$</th>
<th>$CVaR_{0.95}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-1423</td>
<td>-1824</td>
</tr>
<tr>
<td>1</td>
<td>-1680</td>
<td>-2285</td>
</tr>
<tr>
<td>4</td>
<td>-1732</td>
<td>-2419</td>
</tr>
<tr>
<td>3</td>
<td>-2778</td>
<td>-3259</td>
</tr>
<tr>
<td>5</td>
<td>-3356</td>
<td>-4031</td>
</tr>
<tr>
<td>6</td>
<td>-4801</td>
<td>-5760</td>
</tr>
</tbody>
</table>

The $CVaR$ is regarded as a more coherent measure of risk since it takes into account the values within the tail of the distribution of losses (5% in this case).

Finally the practical case is completed by introducing a new measure which is different in kind from those analysed up to now. This is to introduce the portfolio manager's attitude to risk, so that
the resulting selection is in consonance with that criterion. Risky portfolios will be selected when the utility function reflects the risk-loving nature of the investor, and more conservative portfolios when he or she is averse to risk. The utility function that we worked with has also been applied by various workers\(^2\), and is, generically, the following:

\[
u(S_{t}^{s,c}) = \frac{1}{c} \left[ 1 - e^{-cS_{t}^{s,c}} \right]
\]

\(\forall s = \{1, 2, \ldots, 625\}; \ c = \{1, 2, 3, 4, 5, 6\}\)

Table 5 presents the values of the expected utility of the Surplus for each of the investment strategies analysed, as a function of different risk aversion coefficients of the portfolio manager and the ranking that results according to the investor's aversion to risk. We show results for \(c\) positives because normally one always assumes to have some aversion to risk.

**Table 5. The Fund's expected utility according to each investment strategy for investor averse to risk**

<table>
<thead>
<tr>
<th>(c = 0)</th>
<th>(c = 0.5)</th>
<th>(c = 1)</th>
<th>(c = 1.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility</td>
<td>e</td>
<td>Utility</td>
<td>e</td>
</tr>
<tr>
<td>1361</td>
<td>6</td>
<td>2.89</td>
<td>4</td>
</tr>
<tr>
<td>923</td>
<td>5</td>
<td>1.33</td>
<td>5</td>
</tr>
<tr>
<td>610</td>
<td>1</td>
<td>1.13</td>
<td>6</td>
</tr>
<tr>
<td>488</td>
<td>4</td>
<td>1.10</td>
<td>3</td>
</tr>
<tr>
<td>415</td>
<td>3</td>
<td>0.75</td>
<td>2</td>
</tr>
<tr>
<td>353</td>
<td>2</td>
<td>0.36</td>
<td>1</td>
</tr>
</tbody>
</table>

We can observe the fact that the order of preference is different from \(VaR\) for certain values of \(c\). It is because \(VaR\) is a measurement of maximum loss with certain probability and the utility take account the aversion to risk. \(VaR\) and \(CVaR\) are objective measurements of risk whereas the utility is a subjective measurement of risk. With both of them we can make a complete analysis of risk.

With the information that we have obtained with this paper we can assert that the investment in stock imply negative and risky results. These results are consequence of the bad year in Stock Exchange, for two reasons: the crisis owing of terrorist attack of 11th of September and the Argentinian crisis at the beginning of 2002. In Spain, in accordance with Inverco (Spanish Institution of Collective Investment and Pension Fund), the Pension Fund that investment near to 75% in Stocks, have had a loss of 16.5% of their funds and only the Pension Fund that investment in bonds had profits.

The results of our analysis are very similar the real situation and these measurements are good instrument to evaluate the risk incurred with each of the investment strategies. If one investor, in December of 2001, had made an analysis similar to this work, being averse to risk, he would have decided investment in strategies 1 or 2, and had avoid bad results. With this work we try to give to the investor several measurements, objective and subjective, of risk to help him to “walk” in the random work of the management.

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CONCLUSIONS

The scenario generation model that was obtained here was based on a Vector Error Correction Model (VECM) structure with two lags. This generates values for the yields on the different investments (cash, deposits, bonds, stocks, mortgage loans) and of the factors that influence the liabilities (actuarial discount rate, and the consumer price index, which also conditions the future value of the assets).

The different trajectories of the behaviour of the generated yields and factors, together with their probabilities of occurrence, affect the value of the Fund that is obtained at the end of the temporal horizon.

The information obtained from the scenario generation model allows one to analyse, in terms of the VaR, different investment alternatives in order to estimate the portfolio losses with each of these possible strategies. In this way, the risk can be quantified and evaluated before making the investment decision. This measure gives the maximum expected loss at a determined confidence level \( \gamma \). This information is improved considerably by using the concept of Expected Shortfall. One will then also know the expected value of the loss within the \((1 - \gamma)\) tail of the probability. Finally the analysis of risk is completed by adding to these objective measures a criterion of the expected utility of the surplus, which includes the portfolio manager's preferences with regard to the acceptance of risk.

The practical case that we developed in the present work shows that allocating a greater percentage of assets to bonds leads to a smaller risk than an allocation to stocks, and that the capital requirements are greater in the case of not diversifying the portfolio and investing 100% of assets in stocks. This constitutes risk analysis in objective terms. If the portfolio manager's preferences are introduced via a utility function the risk analysis is in subjective terms.

These results are obtained on the basis of the market conditions given by the scenario generation model. Due to the randomness of the behaviour of factors and yields, these results may change if the trajectories of that behaviour are different.

Lastly, we can state that it is possible to quantify both objectively and subjectively the risk to which any investment strategy is subject to in the long term, by modelling the uncertainty through a series of behaviour trajectories to each of which is associated a probability of occurrence.
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