Information Linkages and Correlated Trading

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Abstract

In a market with informationally connected traders, the dynamics of volume, price informativeness, price volatility, and liquidity are severely affected by the information linkages every trader experiences with his peers. We show that in the presence of information linkages among traders, volume and price informativeness increase and liquidity improves. Moreover, we find that information linkages boost or damage the traders' profits according to whether these linkages convey negatively or positively correlated signals. Finally, our model predicts patterns of trade correlations consistent with those identified in the empirical literature: trades generated by “neighbor” traders are positively correlated and trades generated by “distant” traders are negatively correlated.

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Introduction

One pervasive feature in financial markets is the existence of information linkages among market participants. Traders and investors are socially connected and have access to comparable sources of information. Many writers describe the financial community as one of overlapping groups of people who share similar opinions, either because they are endowed with comparable signals about the fundamentals and/or communicate regularly with one another [e.g., Shiller (1984, 2005), Hertz (1998)], or simply because they are exposed to similar cultural biases [e.g., Guiso, Sapienza and Zingales (2006)]. Many information-based explanations of asset price movements hinge upon the assumption that investors do not experience information linkages at all. In this paper, we relax this assumption and explore the resulting implications along several dimensions: asset price volatility, liquidity, market efficiency, trading volume, correlation among trades and volumes generated by heterogenous traders and gains from informed trading.

Our notion of information linkages is tightly related to the recent empirical literature on the value of local information and social interactions in financial markets. For example, Coval and Moskowitz (1999) provide strong evidence that geographical proximity influences investors’ portfolio choices. Hong, Kubik and Stein (2005) document that US fund managers located in the same city exhibit similar portfolio choices. They argue that these correlated portfolio choices arise either (i) through peer-to-peer communication; or (ii) simply because fund managers in a given area commit themselves to investment decisions based upon common sources of information - such as a local newspaper or TV station. Similarly, Feng and Seasholes (2004) find that in the Chinese stock market, trades are positively correlated for geographically close investors, but negatively correlated for distant investors.

A rational explanation of these findings must necessarily rely on a pronounced heterogeneity in the investors’ information endowments. Thus, at the heart of our analysis is the idea that in asset markets, there are groups of traders whose signals and beliefs are more correlated with some and less correlated with other groups of traders. A general measure of informational distance between any two traders is the amount of information they share. To generate heterogeneity in informational distance, we consider a model with strategic traders who are locally connected to common sources of information about the long term value of an asset. We call these local connections “information linkages”. Accordingly, we define close traders as those who are connected through these linkages, and distant traders as those who are not. These local connections give rise to overlapping networks of traders, which may include only one’s closest neighbors or the entire market. Indeed, there are no obvious arguments suggesting whether information linkages
should be best thought of as being local or global. Our framework is kept as general as possible to allow for a wide spectrum of possibilities.

Why do we need a model to explain that traders with comparable information trade similarly? First, we wish to examine the implications of our model on traditional market variables such as volume, liquidity, or return volatility. For example, do information linkages among traders boost stock market volatility? What is the economic link between this volatility, volume and the existence of information linkages in the market?

Second, we are interested in the patterns of trade correlations among traders. In equilibrium, every trader makes use of the information available at the information linkages that he has with his local peers. He also knows, however, that by trading aggressively, he would reveal part of this information to distant peers. What is the ultimate effect on the correlation between “close” and “distant” trades? Our model generates sharp predictions about these patterns, which are consistent with the available empirical evidence.

Third, we wish to understand how information linkages impact on the gains from informed trading: do asset markets with many information linkages lead to an increase in traders’ profits? This question has important practical scopes. For example, suppose that the gains from informed trading deteriorates as the number of information linkages increases. In this case, we would expect to see a small number of traders in markets with many information linkages, and vice-versa.

More in detail, the predictions of our model can be streamlined as follows:

(i) The equilibrium price process, trading activity, return volatility, and traders’ profits are severely affected by the number of information linkages among traders.

(ii) Compared to a market without information linkages, a market with information linkages is characterized by higher liquidity, lower return volatility, and higher volume.

(iii) In a market with information linkages, the correlation among trades is heterogeneous, both temporally and spatially. More precisely, the correlation among trades is very high at the beginning of the trading period. The same correlation decreases with the unfolding of the trading period, exhibiting different patterns, according to the informational distance among traders. Such an additional heterogeneity exhibits the following features:

(iii.1) For traders who are sufficiently close (close neighbors), the correlation among opinions and trades is persistently high over the entire trading period.
(iii.2) Traders’ opinions and actual trading activity diverge with the traders’ relative informational distances. Eventually, the correlation between trades is negative for relatively distant traders. A significant and persistent divergence in trades occurs even when the number of information linkages is high enough to make any two traders’ opinions close at the beginning of the trading period.

(iv) Information linkages can boost the gains from informed trading if they convey negatively correlated signals. However, if the signals available at the information linkages are positively correlated, the mere existence of these linkages considerably and consistently damages the traders’ profits.

The first property makes our model economically meaningful. As we shall show, even a modest presence of information linkages among traders can induce a quite large effect on the equilibrium price and trading activity. Precisely how information linkages affect the price and trading behavior is summarized by the three remaining properties. Consider, for example, the model’s prediction about high volume (i.e., the second prediction). Its economic interpretation is simple. Heterogeneity in private information is a source of monopolistic power for traders. But information linkages destroy part of this monopolistic power. Consequently, every trader trades to preempt his peers, which makes market-wide volume increase. (Our predictions about liquidity and volatility can be understood in a similar vein.)

Next, consider the third property about the correlation among trades. This property is consistent with the empirical evidence. The economic interpretation for the positive correlation between close trades is intuitive: the linkages among the traders boost the correlation of the information endowments induced by the linkages among the traders. Indeed, in markets without information linkages, the correlation among trades eventually becomes negative [see Foster and Viswanathan (1996)].

Interestingly, our model also matches the empirical evidence on the negative correlation between distant trades [see our previous discussion of Feng and Seasholes (2004) findings]. The economic mechanism at work in the model is the following. Over time, the equilibrium asset price conveys more and more information about the traders’ average opinion about the asset value, not the single private opinions every trader has about this value. Therefore, over time, traders stand on opposite sides of the market, on average, which makes the correlation of the single trades decrease. This correlation tends to become negative, especially for distant traders. However, the presence of information linkages in the market “kicks in” for close traders. In
particular, we find that for close traders, the information sharing effect induced by the traders’ linkages dominates the negative correlation effect related to each trader standing on the opposite side of the market as a whole.

The fourth prediction about the gains from informed trading can be explained as follows. In our model, traders face a crucial trade-off. On the one hand, information linkages damage the traders’ monopolistic power. On the other hand, the very same linkages improve the quality of the traders’ inference about the fundamental value of the asset. If the signals available at the information linkages are positively correlated, then the losses generated by the first effect are smaller than the gains generated by the second effect. This property arises under a wide range of conditions on initial beliefs’ heterogeneity and the market structure, as summarized by the number of traders and batch auctions, as well as the initial correlation of the signals made available at the traders’ initial locations. As it turns out, these results are reversed if the signals available at the information linkages are negatively correlated.\(^1\)

The framework in this paper builds on the seminal papers of Foster and Viswanathan (1996) and Back, Cao and Willard (2000), who develop a multi-traders generalization of the Kyle’s (1985) model. In these two papers, every trader is endowed with one signal about the long-term value of an asset, each signal being different from the others. The correlation between any two signals, however, is the same for all traders. Our model relies on the same economic construct underlying these two papers. The information linkages, however, destroy the homogeneous correlation between the traders’ information endowments, and induces patterns of trade correlations varying with the traders’ informational proximity. As a result, some traders may agree more with some and less with other peers. For example, our model predicts that in some cases, two traders may not be connected to the same information linkages, but may still have highly correlated information endowments. This occurs when two traders share information with a third trader who in turn shares information with each of them.

Finally, note that there do exist rational explanations for patterns of heterogenous trades. Notably, Brennan and Cao (1997) consider a rational expectations model (not a model with strategic agents) in which the agents’ geographical distance allows them to estimate the value of an asset with different precisions. For example, a good piece of public news makes those traders

\(^{1}\)In the credit markets literature, Pagano and Jappelli (1993) identify conditions under which banks find it profitable to exchange information about their customers’ quality. Under uncertainty about the borrower’s quality, credit bureaus allow lenders to improve their knowledge about new customers, at the cost of giving up to competitors one’s informational rent about existing customers. While similar trade-offs enter into our profits calculations, in our model information sharing is not necessarily the result of any information exchange activity.
who are more precisely informed (local) react less than those who are less (foreign), which might lead to a negative correlation between distant trades. Feng and Seasholes (2004) point out that the Brennan and Cao mechanism can explain their empirical findings of a negative correlation among distant trades. Our paper offers an alternative mechanism, in which heterogenous correlations among trades occur even if the precision of the private signals is the same for all the traders.

Our paper also relates to the most recent theoretical literature. Stein (2006) develops a model in which competitors can find it fruitful to engage in truthful conversations when these conversations boost the quality of their initial estimates about an asset payoff. Thus, his model provides foundations to a particular mechanism leading to the information linkages we consider in our paper.

In independent work, Ozsoylev (2006) develops a model in which every investor observes the expectations of his neighbors. In this sense, his model is similar in spirit to our local information linkages mechanism. However, our work differs from Ozsoylev’s for two reasons. First, Ozsoylev considers a model in which the investors do not enjoy market power. The assumption of no-market power allows the author to investigate asymmetric networks of information. In our model, traders do enjoy a monopolistic market power and, hence, need to forecast the forecasts of others. To simplify this dimensionality issue, we consider a symmetric network. In this network, every trader relates to the remaining traders in the same manner, yet still shows a pattern of trade correlation varying with the proximity of his peers. The second difference with Ozsoylev’s model is that our model is dynamic. Ozsoylev’s work and ours therefore complement each other.

The paper is organized as follows. In the next section, we develop the information structure of the model. In Section II, we derive a dynamic equilibrium while in Section III, we analyze the properties of this equilibrium. Section IV concludes. The appendix contains all technical details omitted in the main text.

I. Information structure

A. The asset market and traders’ location

We consider a market for one risky asset in which trading takes place in \( N \geq 1 \) batch auctions. The asset pays a random payoff \( f \sim N(0, \sigma^2_{f,0}) \) at the end of the trading period. The crucial feature of the model is that a number of imperfectly competitive traders experience information linkages related to the asset payoff. Precisely, we assume that the traders are physically located around a circle. By convention, they are ordered clockwise, such that trader \( i \) has trader \( i + 1 \) to
his left and trader $i - 1$ to his right (see Figure 1). For reasons developed below, we assume that there are $M$ such traders, where $M$ is an odd number.

Signals about the fundamental value $f$ are available at each trader’s location. Let $s_0 = [s_{1,0}, \ldots, s_{M,0}]^\top$ be the $M \times 1$ vector of the signals available in the market. We assume that each signal $s_{i,0}$ is available at the $i$-th trader’s location and is observed by a subset of the $i$-th trader’s neighbors, on either side. Hence, we allow for “double-sided” information linkages. That is, the signal available at any trader’s location is observed by $G$ traders to the right and $G$ traders to the left of any given trader. For example, for $G = 1$, the $i$-th trader may share the signal at his location with traders $i - 1$ and $i + 1$ (see Figure 1). In this case, traders $i - 1$, $i$ and $i + 1$ observe $s_{i,0}$: trader $i$ observes $s_{i-1,0}$, $s_{i,0}$ and $s_{i+1,0}$, and so forth.

This network of information linkages leads the $i$-th trader’s information set to be $s_{i,0} = [s_{i-G,0}, \ldots, s_{i,0}, \ldots, s_{i+G,0}]^\top$, $G \in [0, (M - 1) / 2]$. To ease notation, we let $\hat{G} = 2G + 1$ be the number of signals every trader has access to. Let $\bar{s}_{i,0}$ denote the average signal available to the $i$-th trader,

$$\bar{s}_{i,0} = \hat{G}^{-1} \sum_{k=-G}^{G} s_{i+k,0}. \quad (1)$$

In the absence of information linkages, we have that $\hat{G} = 1$ and, hence, $s_{i,0} = \bar{s}_{i,0} = s_{i,0}$ for all $i$. In principle, the maximum number of information linkages is $\hat{G} - 1 = M - 1$, in which case $s_{i,0} = s_0$ for all $i$. However, such a complete information sharing market may fail to have a linear equilibrium as the number of auctions $N$ gets large and the uncertainty related to liquidity trades (to be introduced later) gets small, as initially conjectured by Holden and Subrahmanyam (1992) and shown by Back, Cao and Willard (2000). Therefore, we shall limit ourselves to analyze cases in which $\hat{G} < M$.

Our information structure can be interpreted in a variety of ways. For example, every signal $s_{i,0}$ can be thought of as being broadcast to the $i$-th trader’s location through a local newspaper or TV station. Then, informationally linked traders are those who have access to the same information source. In the limiting case in which $\hat{G} = 1$, every trader gathers information from a unique local news source, and there are no information linkages among them. As $\hat{G}$ increases, these sources of news overlap across traders. In this example, the number of information linkages every trader experiences with his peers, $2G$, is interpreted as the media coverage of information providers.

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2In this paper, we shall make an abuse in notation and write, for a given set $A$, $G \in A$ for $G \in A \cap \mathbb{N}$, where $\mathbb{N}$ is the set of integers.
Another interpretation of our information structure is that of social distance among traders. Social distance can relate to geographical, cultural, demographic or linguistic distance, and translates to differences in beliefs among traders. For example, repeated interactions among traders can lead them to sharing close opinions and views. At the other extreme, socially distant traders are less likely to interact and, hence, more likely to have relatively more independent opinions and views. Then, our model can be understood as one that focuses on the asset pricing implications of a given architecture of opinion formation. Stein (2006) lays down the foundations for an honest exchange of ideas to arise among strategic players. This exchange arises due to complementarities in the production of ideas. For example, to produce a useful idea about an asset payoff, it is necessary to have access to the peer’s previous ideas on the same topic. In our model, information linkages can be the result of such fruitful conversations among close traders.  

B. The distribution of signals, and trading behavior

Next, we describe the distribution of the initial signals available at the traders’ location, \( s_0 \). To keep the model as general and tractable as possible, we follow Foster and Viswanathan (1996) and assume that all the signals are jointly normally distributed with mean zero and variance-covariance matrix equal to

\[
\Psi_0 = E([s_{1,0}, \cdots, s_{M,0}]^\top [s_{1,0}, \cdots, s_{M,0}]).
\]

The unconditional distribution of the signals is symmetric in that: (i) each signal has variance \( \Lambda_0 \), (ii) the covariance between any two signals is \( \Omega_0 \), and (iii) the covariance between each signal and the asset value is \( c_0 \). The joint distribution of the vector \([f, s_0]^\top\) is given by:

\[
\begin{bmatrix}
  f \\
  s_0
\end{bmatrix}
\sim N
\begin{bmatrix}
  0 \\
  0
\end{bmatrix},
\begin{bmatrix}
  \sigma_f^2 & c_01^\top \\
  c_01 & \Psi_0
\end{bmatrix},
\Psi_0 =
\begin{bmatrix}
  \Lambda_0 & \Omega_0 & \cdots & \Omega_0 \\
  \Lambda_0 & \Omega_0 & \cdots & \Omega_0 \\
  \vdots & \vdots & \ddots & \vdots \\
  \Lambda_0 & \Omega_0 & \cdots & \Omega_0
\end{bmatrix},
\tag{2}
\]

where we assume that \( \Psi_0 \) is invertible, which it is, provided \( \Lambda_0 > -(M-1)\Omega_0 \), a restriction we maintain throughout the paper.

Let \((x_{i,n})_{n=1}^N\) be the sequence of trades submitted by the \( i \)-th trader over the trading period. Trades are chosen so as to maximize the expected profits, viz

\[
W_{i,n} = \max_{(x_{i,t})_{t=m}^N} E \left[ \sum_{t=m}^N (f - p_t) x_{i,t} \left| F_{i,n} \right. \right], \quad n = 1, \cdots, N,
\tag{3}
\]

For example, conversations and complementarities in the production of ideas might allow traders to turn totally uninformative pieces of information into signals with finite variance, leading to the vector of signals \( s_0 = [s_{1,0}, \cdots, s_{M,0}]^\top \) available at the traders’ location.
where $F_{i,n}$ is the trader $i$ information set at the $n$-th batch auction. On top of these informed orders, there is a sequence of liquidity trades $(u_n)_{n=1}^N$, where $u_n$ is independent and identically distributed as a standard normal variable, with mean zero and variance $\sigma_u^2$.

The aggregate order flow is given by:

$$y_n = \sum_{i=1}^{M} x_{i,n} + u_n, \quad n = 1, \ldots, N. \quad (4)$$

Finally, the $(M+1)$-th market participant is the market maker, who commits himself to offset the order flow according to the Semi-Strong efficiency rule:

$$p_n = E(f|y_1, \ldots, y_n), \quad n = 1, \ldots, N. \quad (5)$$

As we shall explain in the next section, our information structure simplifies each trader’s dynamic inference about other traders’ signals. It allows us to focus on linear equilibria, thereby avoiding dimensionality issues related to forecasting the forecasts of others. This simplification arises for two reasons:

(i) The distribution of $[f\ s_0]^{\top}$ in Eq. (2) is still symmetric, as in Foster and Viswanathan (1996). In particular, the average of the individual average signals, $\bar{s} = M^{-1} \sum_{i=1}^{M} \bar{s}_{i,0}$, is a sufficient statistic for the full information liquidation value, in the sense that,

$$E(f|s_0) = \theta \bar{s}, \quad (6)$$

where $\theta = c_0 M \left( \Lambda_0 + (M - 1) \Omega_0 \right)^{-1}$.

(ii) The circular location of the traders makes the patterns of signals’ correlation the same for each of these traders.

At the same time, our information structure allows us to study a market with differentially informed traders who experience heterogeneous patterns of signals’ correlation.

C. Average signals

The average signal every trader has access to, $\bar{s}_{i,0}$, plays a key role for each trader forecasting problem and, hence, for the trading strategies, as we shall show below. We now derive the distribution of these average signals $(\bar{s}_{i,0})_{i=1}^{M}$, under the assumptions made so far.
We denote the unconditional variance-covariance matrix of the average signals with \( \bar{\Psi}_0 = E([\bar{s}_1, \cdots, \bar{s}_M, \bar{s}_0] \top [\bar{s}_1, \cdots, \bar{s}_M, \bar{s}_0]) \). The elements of this matrix depend on the number of information linkages in the market. Accordingly, we set \( \bar{\Psi}_0 = \bar{\Psi}_0 (G) \), where

\[
\bar{\Psi}_0 (G) = \begin{bmatrix}
\bar{\Lambda}_0 (G) & \bar{\Omega}_0 (1, G) & \cdots & \bar{\Omega}_0 \left( \frac{M-1}{2}, G \right) & \cdots & \bar{\Omega}_0 (-1, G) \\
\bar{\Lambda}_0 (G) & \ddots & \ddots & \ddots & \ddots \\
\cdots & \ddots & \ddots & \ddots & \ddots \\
\bar{\Lambda}_0 (G) & & & & \\
\end{bmatrix},
\]

and the elements

\[
\bar{\Lambda}_0 (G) = \text{var}(\bar{s}_i, 0), \quad \bar{\Omega}_0 (k, G) = \text{cov}(\bar{s}_{i+k} , 0, \bar{s}_i, 0), \quad k = \mp 1, \mp 2, \cdots, \mp \frac{M-1}{2}
\]
denote the unconditional variance of the average signals available to any trader (\( \bar{\Lambda}_0 (G) \)), and the unconditional covariance between the average signals of any two traders who are located \( k \) positions apart (\( \bar{\Omega}_0 (k, G) \) \( k \neq 0 \)). Naturally, \( \bar{\Lambda}_0 (G) \) is the same for each trader, due to the symmetric unconditional distribution of the signals in Eq. (2). Furthermore, we have that \( \bar{\Omega}_0 (k, G) = \bar{\Omega}_0 (-k, G) \), which follows by both the circular information structure and the double-sided nature of the information linkages in this market.

D. Correlations

How are the information endowments correlated across traders? Let \( \rho_0 (k, G) = \bar{\Lambda}_0 (G) / \bar{\Omega}_0 (k, G) \) be the correlation between the average signals available to any two traders who are located \( k \) positions apart. First, the unconditional variance of the average signals available at each location is,

\[
\bar{\Lambda}_0 (G) = \frac{\Lambda_0 + 2G\bar{\Omega}_0}{G}.
\]

Consider, next, any two traders \( i \) and \( j = i + k \) with \( k \neq 0 \), and the unconditional covariance between the average signals these two traders have access to, i.e. the off-diagonal elements \( \bar{\Omega}_0 (k, G) \) in Eq. (7). The covariance between the average signals depends on: (i) the overall number of information linkages in the market and, hence, on \( G \); and (ii) \( k \), the distance between the \( i \)-th trader and \( (i + k) \)-th trader. This dependence on \( k \) arises because the number of signals every trader shares with the remaining traders depends on their relative position on the circle. For example, assume that \( 2G < (M - 1)/2 \). In this case, trader \( i \) shares \( 2G \) signals with trader \( i + 1 \), \( 2G - 1 \) signals with trader \( i + 2 \) and in general \( 2G + 1 - k \) signals with trader \( i + k \).
Eventually, trader \( i \) shares no signals with trader \( i + 2G + 1 \) and beyond. As Figure 2 illustrates, the covariance between the average signals of any two traders does in general depend on the relative distance of the traders, \( k \).

To derive explicitly the covariance between any two traders’ information endowments, we need to distinguish between two situations, arising according to whether the number of information linkages is large or small, compared to the number of traders. Intuitively, if the number of information linkages, \( 2G \), is greater than the number of traders each trader has on either side, \( (M - 1)/2 \), some pieces of information might “go through” over and above the dimension of the information network any trader belongs to. Technically, then, we need to analyze the following two cases.

(i) Small number of information linkages: \( 2G \leq (M - 1)/2 \). In this case, any two traders can not share any signal, provided they are located sufficiently apart, as in Figure 2. However, if two traders are sufficiently close, they share information. In Appendix A, we show that,

\[
\Omega_0(k, G) = \begin{cases} 
\hat{\Lambda}_0(G) - \hat{G}^{-2} (\Lambda_0 - \Omega_0) k, & \text{for } k \in [1, 2G + 1] \\
\Omega_0, & \text{for } k \in [2G + 1, \frac{M-1}{2}] 
\end{cases}
\]

(ii) Large number of information linkages: \( 2G \geq (M - 1)/2 \). In this case, the large number of information linkages might lead any two traders to share information even when they do not share the signals available at their location, a property of the model we label “double overlap”. Consider, for example, Figure 3. In this example, trader \( i \) shares the signal available at his own location with trader \( i - \ell \), but he does not share this signal with trader \( i + k_2 \). However, traders \( i + k_2 \) and \( i - \ell \) share at least the signals that are available at their location. Hence, traders \( i \) and \( i + k_2 \) do share common signals (at least the signal \( s_{i-\ell,0} \)), even if they are not connected to the same information linkage. In the appendix, we show that the “double overlap” modifies the correlation structure in (9) as follows:

\[
\Omega_0(k, G) = \begin{cases} 
\hat{\Lambda}_0(G) - \hat{G}^{-2} (\Lambda_0 - \Omega_0) k, & \text{for } k \in [1, 2 \left( \frac{M-1}{2} - G \right)] \\
2\hat{\Lambda}_0(G) - \hat{G}^{-2} M (\Lambda_0 - \Omega_0) - \Omega_0, & \text{for } k \in \left[ \frac{2}{\left( \frac{M-1}{2} - G \right)}, \frac{M-1}{2} \right]
\end{cases}
\]

To summarize, the elements of the variance-covariance matrix \( \Psi_0(G) \) in Eq. (7) depend on the number of information linkages. By Eq. (8), the elements on the main diagonal, \( \hat{\Lambda}_0(G) \), are the same. The off-diagonal elements, instead, are decreasing in the traders’ relative distance \( k \), according to the pattern in Eqs. (9)-(10).
E. Forecasts

We now describe how, conditionally upon the information set $s_i,0$, any trader $i$ forecasts (i) the final liquidation value, and (ii) the sum of the remaining traders’ average signals. The traders need these forecasts to implement their trading strategies, as we shall explain in the next section.

By Eq. (2),

$$ E(f | s_i,0) = \frac{\text{cov}(f, \bar{s}_{i,0})}{\text{var}(\bar{s}_{i,0})} \bar{s}_{i,0} = \hat{G} \eta_1 \bar{s}_{i,0}, \tag{11} $$

where $\eta_1 = c_0 (\Lambda_0 + 2G\Omega_0)^{-1}$.

Next, define the unconditional covariance between the average signal available to any trader $i$, $\bar{s}_{i,0}$, with the remaining traders’ average signals as:

$$ \bar{\Gamma}_0(G) = \text{cov} \left( \sum_{k \neq i} \bar{s}_{k,0}, \bar{s}_{i,0} \right) = \sum_{k \neq i} \Omega_0(k,G). \tag{12} $$

Due to the information structure in this market, $\bar{\Gamma}_0(G)$ is the same for each trader. In the appendix, we prove that, $\bar{\Gamma}_0(G)$ is the same for each trader. In the appendix, we prove that,

$$ \bar{\Gamma}_0(G) = (M-1) \Omega_0 + \frac{2G}{G_0} (\Lambda_0 - \Omega_0), \quad \text{for all } G \in [0, \frac{M-1}{2}]. \tag{12} $$

The correlation between the average signals of any two traders varies with their relative location [see Eqs. (9)-(10)]. Therefore, every trader’s expectation of any other trader’s average signal depends on the relative distance $k$ between the trader and his peers. However, the expectation of the sum of all remaining traders’ average signals is independent of $k$. In the appendix, we show that it equals

$$ E \left( \sum_{j \neq i} \bar{s}_{j,0} | s_i,0 \right) = E \left( \sum_{j \neq i} \bar{s}_{j,0} | \bar{s}_{i,0} \right) = (M-1) \phi_1 \bar{s}_{i,0}, \tag{13} $$

where the regression coefficient, $\phi_1$, is

$$ \phi_1 = \frac{\bar{\Gamma}_0(G)}{(M-1) \bar{\Lambda}_0(G)}, $$

and $\bar{\Lambda}_0(G)$ and $\bar{\Gamma}_0(G)$ are given by Eqs. (8) and (12). The fact that the unconditional covariance $\bar{\Gamma}_0(G)$ is the same for every trader simplifies the traders’ inference about the forecasts of others. It leads to equilibria in which both the market maker and the traders are not concerned with forecasting each individual average signal, but only the average of the average signals, $M^{-1} \sum_{i=1}^{M} \bar{s}_{i,0}$, similarly as in Foster and Viswanathan (1996) and Back, Cao and Willard (2000).
II. Equilibrium price and trades

This section develops a dynamic model which relies on the information structure described in the previous section. We derive the equilibrium price and traders’ strategies (Proposition 1). We also provide a characterization of the trading strategies (in Corollary 1), which helps understand the correlation of trades and volume we uncover in Section III.

A. Market maker’s inference

Let \( z_{i,t} = y_t - x_{i,t} \) be the residual order flow, from the perspective of the \( i \)-th trader. We let \( F_{i,n} = \{ s_{i,0}, (z_{i,t})_{t=1}^{n-1}, (x_{i,t})_{t=1}^{n-1} \} \) and \( F_{M+1,n} = \{ (y_t)_{t=1}^{n} \} \) denote the information sets available to the \( i \)-th trader and the market maker at the \( n \)-th batch auction. We denote the market maker’s estimate of the average signal available to any trader \( i \) as

\[
 t_n = E(\bar{s}_{i,0}|F_{M+1,n}) = \hat{G}^{-1} \sum_{k=-G}^{G} E(s_{i+k,0}|F_{M+1,n}) = E(s_{i,0}|F_{M+1,n}),
\]

which is independent of the \( i \)-th trader’s specific location, due to the symmetric location of the traders. Let \( s_{i,n} \) denote the \( i \)-th trader residual informational advantage (relative to the market maker) on the signal made available to his location after \( n \) rounds of trading,

\[
 s_{i,n} = s_{i,0} - E(s_{i,0}|F_{M+1,n}) = s_{i,0} - t_n.
\]

Trader \( i \) informational advantage on his average signal, \( \bar{s}_{i,n} \), has a similar interpretation. By Eq. (14), it is:

\[
 \bar{s}_{i,n} = \bar{s}_{i,0} - E(\bar{s}_{i,0}|F_{M+1,n}) = \bar{s}_{i,0} - t_n.
\]

Finally, let us define the market maker’s updates of the residual variances:

\[
 \sigma_{f,n}^2 = \text{var} \left[ E(f|s_0)|F_{M+1,n} \right]
\]

\[
 \Lambda_n = \text{var} \left( s_{i,0}|F_{M+1,n} \right) \]

\[
 \Omega_n = \text{cov} \left( s_{i,0}, s_{j,0}|F_{M+1,n} \right)
\]

(16)

where \( \sigma_{f,n}^2 \) is the residual variance of the full information asset value after \( n \) rounds of trading, and \( \Lambda_n \) and \( \Omega_n \) are the residual variance and covariance of the signals available at each trader’s location. Similarly, we define:

\[
 \bar{\Lambda}_n (G) = \text{var} \left( \bar{s}_{i,n}|F_{M+1,n} \right)
\]

\[
 \bar{\Omega}_n (k, G) = \text{cov} \left( \bar{s}_{i,n}, \bar{s}_{i+k,n}|F_{M+1,n} \right)
\]

\[
 \bar{\Gamma}_n (G) = \sum_{k \neq i} \bar{\Omega}_n (k, G)
\]
where \( \Lambda_n(G) \) is the market maker’s residual variance of the average signal available to each trader, and \( \Omega_n(k,G) \) is the market maker’s residual covariance between the average signals available to any two traders who are located \( k \) positions apart.

### B. Dimensionality issues

We focus on equilibria in which each trader’s forecasts of the asset value \( f \) and the forecasts of others are linear in the trader’s average signal. In these equilibria, all higher order forecasts of other traders’ forecasts are also linear in the same average signal. Consequently, average signals constitute sufficient statistics for both the asset value and the forecasts of others. Furthermore, we focus on equilibria independent of the forecasts’ history. We look for a linear equilibrium in which (i) each trader strategy is linear in his residual average signal, viz

\[
x_{i,n} = \hat{G} \beta_n \bar{s}_{i,n-1},
\]

and (ii) the market maker’s learning about the asset value satisfies

\[
p_n = p_{n-1} + \lambda_n y_n,
\]

for some deterministic sequences \( \beta_n \) and \( \lambda_n \) to be determined in equilibrium. We shall come back to the economic interpretation of the linear strategy \( x_{i,n} \) in Eq. (17) below (see Corollary 1).

Foster and Viswanathan (1996) focus on the same equilibrium conditions summarized by Eqs. (17) and (18). They show that any trader \( i \) can manipulate the remaining traders’ beliefs about the asset value only through the aggregate order flow. In the appendix (see Lemma 3), we show that the presence of information linkages does not destroy this property in our model. Consequently, the residual order flow, \( (z_{i,t})_{t=1}^{n-1} \), constitutes a redundant piece of information in the information set of the \( i \)-th trader at the \( n \)-th batch auction, and we set \( F_{i,n} = \{ s_{i,0}, (y_t)_{t=1}^{n-1} \} \).

As a result, any trader \( i \) forecasts the asset value as follows,

\[
E(f - p_{n-1} | F_{i,n}) = \hat{G} \eta_n \bar{s}_{i,n-1},
\]

for some deterministic sequence \( \eta_n \) to be determined in equilibrium. That is, \( \bar{s}_{i,n-1} \) is sufficient for any trader \( i \) to forecast the asset value, before submitting his order at time \( n \). Eq. (19) is the dynamic counterpart to the projection in Eq. (11). Similarly, any trader \( i \) forecasts (the sum of the) other traders’ informational advantage and, hence, their trades, according to,

\[
E(\sum_{j \neq i} s_{j,n-1} | F_{i,n}) = (M - 1) \phi_n \bar{s}_{i,n-1},
\]
for some deterministic sequence $\phi_n$. Eq. (20) is the dynamic analogue to Eq. (13). As is clear, focusing on the linear strategies in Eq. (17) plays a key role in resolving the dimensionality issue, since it allows to conclude that the forecasts of the trades of others are linear in each trader’s average signal.

C. Deviations

When we consider deviations from the optimal play by the $i$-th trader, the average signal, $\bar{s}_{i,n-1}$, is no longer a sufficient statistic to predict the asset value and the other traders’ forecasts, as in Eqs. (19)-(20). The average signal, $\bar{s}_{i,n-1}$, would be a sufficient statistic only if the $i$-th trader played the linear strategy in Eq. (17) in the first $n-1$ trading rounds.

In a Bayesian Nash equilibrium, the optimal strategy for the $i$-th trader is best no matter which past strategy this same trader might have played. Hence, we need to figure out deviations from the equilibrium trades occurring in the first $n-1$ batch auctions. We follow closely Foster and Viswanathan (1996), and conjecture that (i) the $i$-th trader deviation, $x'_{i,n}$ say, coincides with the equilibrium strategy in Eq. (17), plus an additional term reflecting the price deviation induced by suboptimal play in the previous $n-1$ rounds,

$$x'_{i,n} = \hat{G}_n \bar{s}_{i,n-1} + \gamma_n \left(p_{n-1} - p'_{n-1}\right),$$

where $p'_{n-1}$ is the price process that would emerge should the $i$-th trader have decided to deviate in the previous rounds of trading; and (ii) the past suboptimal play of the $i$-th trader shows up in the value function in Eq. (3),

$$W_{i,n} = \alpha_n \bar{s}^2_{i,n} + \psi_n \bar{s}_{i,n} \left(p_n - p'_{n} \right) + \mu_n \left(p_n - p'_{n}\right)^2 + \delta_n.$$  

In Eqs. (21) and (22), $\alpha_n$, $\psi_n$, $\mu_n$ and $\delta_n$ are some deterministic sequences to be determined in equilibrium. As in Foster and Viswanathan (1996), the necessary and sufficient conditions for an equilibrium hinge upon the mutual consistency between the conjectured value function in Eq. (22) and the traders’ deviation in Eq. (21).
We have:

**Proposition 1.** There exists a Bayesian Nash equilibrium in which trading strategies and prices are as in Eqs. (17)-(18); \( \lambda_n \) is the unique real, positive solution to:

\[
0 = \frac{\theta(M - \hat{G})(\Lambda_n - \Omega_n)\sigma_u^4\lambda_n^4 + \sigma_u^2\psi_n\Lambda_n}{GM^2\sigma_{f,n}^4} - \frac{\theta\sigma_u^2[2\Lambda_n + (M - 1)\Omega_n - \frac{2G}{G}(\Lambda_n - \Omega_n)]}{M\sigma_{f,n}^2} + \frac{\psi}{\alpha_n} \Lambda_n \lambda_n + \frac{\sigma_{f,n}^2}{\theta}
\]

and the trading strategy coefficients \( \beta_n \) and \( \gamma_n \) are given by:

\[
\beta_n = \frac{\theta\lambda_n\sigma_u^2}{GM\sigma_{f,n}^2}
\]

\[
\gamma_n = \frac{(1 - 2\lambda_n\mu_n)[1 - \theta^{-1}\hat{G}(M - 1)\beta_n\lambda_n]}{2\lambda_n(1 - \lambda_n\mu_n)}
\]

The value function coefficients satisfy the recursions:

\[
\alpha_{n-1} = \alpha_n \left[ 1 - \theta^{-1}\hat{G}(1 + (M - 1)\phi_n)\beta_n\lambda_n \right]^2 + \hat{G}^2\beta_n[\eta_n - \beta_n\lambda_n(1 + (M - 1)\phi_n)]
\]

\[
\psi_{n-1} = \psi_n \left[ 1 - \lambda_n\gamma_n - \theta^{-1}\hat{G}(M - 1)\beta_n\lambda_n \right] \left[ 1 - \theta^{-1}\hat{G}(1 + (M - 1)\phi_n)\beta_n\lambda_n \right] + \hat{G} \left\{ \gamma_n[\eta_n - \beta_n\lambda_n(1 + (M - 1)\phi_n)] - \beta_n\gamma_n\lambda_n + \beta_n \left[ 1 - \theta^{-1}\hat{G}(M - 1)\beta_n\lambda_n \right] \right\}
\]

\[
\mu_{n-1} = \mu_n \left[ 1 - \lambda_n\gamma_n - \theta^{-1}\hat{G}(M - 1)\beta_n\lambda_n \right]^2 + \gamma_n \left[ 1 - \lambda_n\gamma_n - \theta^{-1}\hat{G}(M - 1)\beta_n\lambda_n \right]
\]

\[
\delta_{n-1} = \delta_n + \theta^{-2}\alpha_n\lambda_n^2\sigma_u^2 + \theta^{-2}\hat{G}^2\alpha_n\lambda_n^2\beta_n^2\var\left( \sum_{j\neq i} \bar{s}_{j,n-1} \right) F_{i,n}
\]

where \( \alpha_N = \psi_N = \mu_N = \delta_N = 0 \) and

\[
\phi_n = \frac{\Gamma_{n-1}}{(M - 1)\bar{\Lambda}_{n-1}}
\]

\[
\eta_n = \frac{\theta(\Gamma_{n-1} + \bar{\Lambda}_{n-1})}{GM\bar{\Lambda}_{n-1}}
\]

\[
\var\left( \sum_{j\neq i} \bar{s}_{j,n-1} \right) F_{i,n} = M[\Lambda_{n-1} + (M - 1)\Omega_{n-1}] - \left[ 1 + \phi_n^2(M - 1)^2 \right] \bar{\Lambda}_{n-1} - 2\Gamma_{n-1},
\]

where \( \bar{\Lambda}_n \equiv \Lambda_n(G) \) and \( \bar{\Gamma}_n \equiv \Gamma_n(G) \). Furthermore, the following inequality must hold:

\[
\lambda_n (1 - \lambda_n\mu_n) > 0,
\]
and the following recursion on the full information residual variance must hold:

\[ \sigma_{f,n}^2 = \left(1 - \theta^{-1} \hat{G} M \beta_n \lambda_n \right) \sigma_{f,n-1}^2. \]

In our model, not only are traders concerned with learning from the information that other traders possess. This learning process is also complicated by every trader’s geographical location and the number of information linkages every trader has with his neighbors. As Proposition 1 reveals, trading strategies and value functions are heavily affected by the heterogeneous correlation structure arising from all these information linkages - a fact we shall examine in detail in Section III.

D. Value estimates and subjective mispricings

Back, Cao and Willard (2000) show that in continuous time, the equilibrium trading strategies in Foster and Viswanathan (1996) are linear in the subjective mispricing perceived by any trader \(i\), defined as the trader’s estimate of the asset value minus the market price. This property survives in our model with information linkages. In this subsection, we produce the formal argument as we shall need this property to develop the economic interpretation of the model’s predictions (see the next section).

Define the subjective mispricing perceived by any trader \(i\) as the difference \(E(f|F_{i,n}) - p_{n-1}\), and let \(\Delta_n\) denote the ratio of the market maker’s residual variance of the average signal in the market, \(\text{var}(\bar{s}|F_{M+1,n})\), to the residual variance of the individual average signal, \(\text{var}(\bar{s}_i,0|F_{M+1,n})\), viz

\[ \Delta_n = \frac{\text{var}(\bar{s}|F_{M+1,n})}{\text{var}(\bar{s}_i,0|F_{M+1,n})}. \]

Finally, let \(\varsigma_{f,n}^2\) denote the trader \(i\) residual variance of the asset value after \(n\) trading rounds,

\[ \varsigma_{f,n}^2 = \text{var}[E(f|s_0)|F_{i,n+1}]. \]

In terms of the ratio \(\Delta_n\), the equilibrium traders’ strategies in Eq. (17) are as in the following corollary:

**Corollary 1.** Any trader \(i\) strategy in Eq. (17) is linear in the subjective mispricing \(E(f|F_{i,n}) - p_{n-1}\), i.e.

\[ x_{i,n} = \frac{\hat{G}_n \beta_n}{\Delta_{n-1} \theta} [E(f|F_{i,n}) - p_{n-1}], \]
and
\[ \Delta_n = \frac{\sigma^2_{f,n} - \varsigma^2_{f,n}}{\sigma^2_{f,n}}. \]  

(32)

Moreover,
\[ E \left( \sum_{j \neq i} x_{i+j,n} \left| F_{i,n} \right. \right) = \frac{G\beta_n}{\Delta_{n-1} \theta} (M \Delta_{n-1} - 1) \left[ E \left( f \left| F_{i,n} \right. \right) - p_{n-1} \right], \] 

and
\[ E (x_{i+k,n} \left| F_{i,n} \right.) = \frac{G\beta_n}{\Delta_{n-1} \theta} \bar{\rho}_{n-1} (k, G) \left[ E (f \left| F_{i,n} \right. \right) - p_{n-1} \right] \] 
\[ = E \left( \sum_{j \neq i} x_{j,n} \left| F_{i,n} \right. \right) - \frac{G\beta_n}{\Delta_{n-1} \theta} \left( \sum_{\ell \notin \{i,i+k\}} \bar{\rho}_{n-1} (\ell, G) \right) \left[ E (f \left| F_{i,n} \right. \right) - p_{n-1} \right], \] 

(34)

where \( \bar{\rho}_{n} (k, G) = \Omega_n (k, G) / \bar{\Lambda}_n (G) \) is the correlation between the average signals available to any two traders who are located \( k \) positions apart.

Corollary 1 generalizes, in a discrete time setting, Lemma 6, and other results, in Back, Cao and Willard (2000). As in Back, Cao and Willard, Eq. (31) indicates that any trader buys/sells the asset if he believes the asset is undervalued/overvalued. Moreover, Eq. (32) reveals that \( \Delta_n \) is a measure of the relative “tightness of beliefs” between the market maker and the traders, i.e. the percentage of the market maker’s residual uncertainty that is accounted for by each trader’s information.

Eq. (33) implies that any trader expects to be on the same side of the market if and only if the market maker’s residual uncertainty is sufficiently large, i.e. \( \Delta_{n-1} > M^{-1} \). This property generates a “rat race”. If \( \Delta_{n-1} < M^{-1} \), any trader expects to be on the opposite side of the market, which generates a “waiting game”. The next section demonstrates that in our model, the dynamic properties of the “tightness of beliefs”, \( \Delta_n \), are severely affected by the number of information linkages every trader has with his peers, \( 2G \).

Finally, Eq. (34) highlights the role information linkages and traders’ distance play in our model. Consider, first, a market without information linkages, i.e. \( G = 0 \). In this case, trader \( i \) information set collapses to the signal available at his location and, hence, \( \bar{\rho}_{n} (k, 0) = \rho_{n} \), for all \( k \). By Eq. (34), then, the \( i \)-th trader expects any of the remaining traders to submit the same order, i.e. \( E (x_{i+k,n} \left| F_{i,n} \right.) = E (x_{i+j,n} \left| F_{i,n} \right. \right), \) for all \( k, j \), which implies that,
\[ E (x_{i+k,n} \left| F_{i,n} \right.) = \frac{E \left( \sum_{j \neq i} x_{j,n} \left| F_{i,n} \right. \right)}{M-1}, \] 

for all \( k \neq 0 \).
Therefore the $i$-th trader believes to be on the same side of the market whenever $\Delta_n > M^{-1}$, and to be so with respect to any single trader.

In contrast, Eq. (34) reveals that in the presence of information linkages, any trader $i$ can simultaneously trade in the same direction of the market but against some of his peers. Indeed, Eq. (34) implies that when $G > 0$, the $i$-th trader’s expects different orders from different traders, i.e. $E(x_{i+k,n} | F_{i,n}) \neq E(x_{i+j,n} | F_{i,n})$, for all $k, j, |j| \neq k$. As we explained in Section I.D, the correlation between the traders’ information endowments decreases with their relative distance [see Eqs. (9)-(10)], which means that each trader agrees more with his neighbors and less with his distant peers. Therefore, if the $i$-th trader is on the same side of the market, he has to trade in the same direction of his close neighbors, although he might trade against peers who are sufficiently far apart. Whether two traders are on the same side of the market depends on their relative distance and the number of information linkages, as we shall show in the next section.

**III. Predictions**

This section analyzes the properties of the equilibrium predicted by the model. We take the asset value, $f$, to equal the sum of all the signals disseminated in the economy,

$$f = \sum_{i=1}^{M} s_{i,0}. \tag{35}$$

Moreover, we assume that the variance of the asset value equals the volatility of the liquidity trades across all batch auctions, i.e. we set $\sigma_f^2 = 1$ and $\sigma_u^2 = N^{-1}$.

Eq. (35) implies two immediate properties: (i) the covariance between any signal and the asset value equals $c_0 = \Lambda_0 + (M - 1) \Omega_0$, which implies that the parameter $\theta$ in the full information liquidation value [see Eq. (6)] equals the number of traders, $M$; and (ii) the correlation between any two signals available at two distinct locations equals,

$$\rho = \frac{1}{M (M - 1)} \left( \frac{1}{\Lambda_0} - M \right). \tag{36}$$

Moreover, by Eq. (35) and by $\sigma_f^2 = 1$, then, $M \Lambda_0 + M (M - 1) \Omega_0 = 1$, where, obviously, $\Omega_0 = \rho \Lambda_0$. Thus, as Eq. (36) reveals, the unconditional variance of each signal, $\Lambda_0$, is known, once we specify $\rho$ and $M$. Therefore, the free parameters of the model are: the correlation of the signals available at the information linkages, $\rho$, the number of traders, $M$, the number of information linkages each trader has with his peers, $2G$, and the length of the trading period, $N$. We set
$M = 7$ and consider $N = 10$ batch auctions.\footnote{We experimented all combinations of $M \in \{7, 51, 101\}$ and $N \in \{10, 20, 40\}$, and obtained results qualitatively very similar to those we report below.} We solve the model when the signals correlation is negative, with $\rho = -15\%$,\footnote{As explained in Section I.B, the variance-covariance matrix $\Psi_0$ is invertible if $\Lambda_0 > - (M - 1) \Omega_0$. With $M = 7$, this restriction is equivalent to $\rho > -0.167$.} and when it is positive, with $\rho = 10\%$ (low correlation) and $\rho = 90\%$ (high correlation). Finally, we analyze the cases in which $G = 0$ (no information linkages), $G = 1$ (information linkages) and $G = 2$ (many information linkages and double overlap).

\section*{A. Volume, liquidity, and volatility}

We assess how information linkages among traders affect the trading volume, liquidity and asset return volatility. As in Admati and Pfleiderer (1988), we decompose the expected volume at the $n$-th auction into the three components generated by the $M$ traders, the market maker and the liquidity traders. We estimate each of these components through the conditional standard deviation of the trades initiated by the $M$ traders ($Vol_{I,n}$ say), the market maker ($Vol_{M,n}$) and the liquidity traders ($Vol_{U,n}$):

$Vol_{I,n} = \hat{G} \beta_n \sqrt{M \left[ \Lambda_{n-1} + (M - 1) \Omega_{n-1} \right]}$, $Vol_{M,n} = \sqrt{G^2 \beta_n^2 M \left[ \Lambda_{n-1} + (M - 1) \Omega_{n-1} \right] + \sigma_u^2}$, and $Vol_{U,n} = \sigma_u$. Finally, we compute the return volatility by conditioning on the market maker’s information set. By Eqs. (17)-(18),

$$\text{var} \left( p_n - p_{n-1} | F_{M+1,n-1} \right) = \lambda_n^2 Vol_{M,n}^2.$$  \hfill (37)

Figure 4 depicts the trading activity generated by the informed traders, $Vol_{I,n}$.\footnote{For space reasons, we do not plot the volume generated by the market maker, which exhibits a similar pattern.} The central prediction of the model is its ability to generate high volume in the presence of information linkages. The explanation for this finding is that information linkages act as an incentive for any trader to anticipate his peers and trade more aggressively, which leads to an overall high level of volume. Precisely, traders engage in a rat race as long as their beliefs are far from those of the market maker. As we discussed in Section II.D, the rate race occurs precisely when the measure of tightness of beliefs in Eq. (29), $\Delta_n$, is sufficiently large, i.e. when $\Delta_n > M^{-1}$. As Figure 5 illustrates, information linkages result in a higher duration of this rat race. This finding always holds, even when the correlation between the signals at any two locations, $\rho$, is negative. Moreover, information linkages exert their largest effects on the duration of the rat race when the correlation $\rho$ is low.
Figure 4 also reveals that for a given number of information linkages, the incentives for any trader to preempt his peers become larger as the correlation $\rho$ increases. This is because high values of $\rho$ translate to a small monopolistic power for any trader, which increases the competition among traders and, then, volume. Similarly, the presence of information linkages reduces each trader’s monopolistic power, thereby inducing a higher overall level of volume. Note that this effect is more pronounced when the correlation of the individual signals at each location, $\rho$, is low.

Figure 6 depicts the process of price discovery, i.e. the dynamics of the residual variance of the asset value, $\sigma^2_{f,n}$. The decay rate of this variance increases as the number of information linkages gets larger. In other words, information linkages are sources of enhanced market efficiency. Moreover, such an improvement in market efficiency is more pronounced when the correlation $\rho$ is low. These findings are directly related to the dynamics of the tightness of beliefs depicted in Figure 6: information linkages make competition among informed traders more fierce. As a result, traders impound more information into their orders, thus reducing the market maker’s residual variance and boosting market efficiency.

The price responsiveness to the order flow, $\lambda_n$, is displayed in Figure 7. When the correlation $\rho$ is positive, then, overall, information linkages result in a reduced price responsiveness or, equivalently, an enhanced market liquidity. This is because information linkages lead to a higher trade aggressiveness and a higher information efficiency, which reduces the adverse selection faced by the market maker. As a result, the market maker reduces the price impact of the order flow, which makes the market more liquid.

When the correlation $\rho$ is negative, the previous conclusions are reversed: liquidity deteriorates with information linkages. This prediction arises as the result of two forces. On the one hand, information linkages enhance the traders’ estimates of the asset value, as usual. On the other hand, when $\rho < 0$, the tightness of beliefs between the traders and the market maker, $\Delta_n$, is considerably low (see Figure 5), which leads the traders to play a waiting game for the entire trading period (for $G = 1$) or for nearly all the batch auctions (for $G = 2$). Thus, the market maker faces relatively more informed traders who play a waiting game for most of the time and, hence, he raises the price responsiveness.

The dynamics of asset return volatility are shown in Figure 8. As Eq. (37) reveals, return volatility is affected by both the price responsiveness, $\lambda_n$, and the volume generated by the market maker, $Vol_{M,n}$, which exhibits a similar qualitative behavior as the informed order flow, $Vol_{I,n}$. When $\rho$ is positive, both $Vol_{I,n}$ and $\lambda_n$ are large during the first trading rounds and steadily decrease over time (see Figure 4 and 7), thus explaining the decreasing pattern of volatility. When
\( \rho \) is negative, and the number of information linkages is not large (i.e. \( G < 2 \)), volatility increases over the trading period, mirroring the behavior of both volume and price responsiveness. When \( G = 2 \), volatility exhibits a decreasing pattern because the increase in volume over time is more than offset by the decrease in \( \lambda_n \). Figure 8 reveals that overall, the presence of information linkages results in an increased return volatility during the first batch auctions, and a decreased return volatility afterwards.

**B. Correlated trading and volume**

Figure 9 depicts the correlation between the trades of each trader with his peers, close and distant,

\[
\text{corr} \left( x_{i,n}, x_{i+k,n} \mid F_{M+1,n-1} \right) = \bar{\rho}_{n-1} (k, G),
\]

where \( \bar{\rho}_{n}(k, G) \) is the correlation between the average signals of any two traders \( i \) and \( i+k \) (see Corollary 1).

Figure 9 shows that for a given number of information linkages, \( \bar{\rho}_{n} \) increases with the correlation \( \rho \), for all the batch auctions \( n \) and the peers’ distance \( k \). This feature of \( \bar{\rho}_{n} \) merely reflects the higher correlation between the signals available at each trader’s location. Moreover, for given \( k \) and \( G \), the trade correlation decreases over time. This pattern is related to the market maker’s learning process. As noted by Foster and Viswanathan (1996), the price conveys more and more information about the average signal, rather than about the individual signals available to traders. Similarly, in our model, the market maker learns more about the average of the average signals, \( \bar{s} \) (see Eq. (6)), rather than the private signals traders have about the asset value, \( \bar{s}_{i,0} \) (see Eq. (1)). This feature of the learning process implies that the market maker’s resolution of the residual uncertainty, \( \sigma_{f,n}^2 = \text{var} \left[ E (f \mid s_0) \mid F_{M+1,n} \right] \), takes place somewhat faster than the resolution of the traders’ residual uncertainty, \( \varsigma_{f,n}^2 = \text{var} \left[ E (f \mid s_0) \mid F_{i,n+1} \right] \). By Eq. (32), then, the tightness of beliefs, \( \Delta_n \), decreases over time. At some point, \( \Delta_n \) becomes so small that the market maker sets the price to a value quite close to the traders’ average opinion of the asset value. Inevitably, then, the traders expect to be on the opposite side of the market, as Corollary 1 suggests.

Although each trader expects to trade against the market, our model predicts that the correlation of trades varies across the market. In particular, information linkages induce close neighbors to trade in the same direction, and distant peers to trade in opposite directions. This property is illustrated in Figure 9, which shows that the trade correlation increases with \( G \), especially during the first trading rounds. For close traders, this correlation remains positive for the entire trading
period, reflecting the boost in the correlation between the information endowments. For distant traders, instead, information linkages are too weak to destroy the effects related to the market clearing mechanism, by which each trader expects to trade against the market. Note, also, that compared with the benchmark case of absence of information linkages ($G = 0$), it takes fewer batch auctions for the trade correlation between distant traders to become negative. Moreover, for distant traders, the trade correlation falls dramatically in the presence of information linkages. For example, when $\rho = 10\%$, the trade correlation among distant traders at the end of the trading period is only $−11\%$, in the absence of information linkages ($G = 0$). However, the trade correlation among distant traders reaches a value of $−70\%$ in the presence of information linkages ($G = 1$), and a value of $−30\%$ in the presence of many information linkages ($G = 2$). Note, the trade correlation is higher for $G = 2$ than for $G = 1$. This property arises quite naturally, as many information linkages induce an additional boost in the correlation between the information endowments of all traders, which then translates to an increased trade correlation, even for distant traders. Still, the trade correlation for distant traders is significantly lower with $G = 2$ than in the complete absence of information linkages.

How does volume correlate among traders? Figure 10 depicts the correlation among the volumes generated by the single traders (for brevity, the volume correlation), over the trading period.\textsuperscript{7} The dynamics of volume correlation are related to, albeit distinct from, those of the trade correlation. Absent any information linkages, individual trades are nearly uncorrelated when the correlation $\rho$ is low (see Figure 9), which translates to a similar property for the volumes generated by the traders. However, individual trades are positively correlated when $\rho$ is high, very strongly so during the first auctions, and quite weakly so during the last auctions.

In the presence of information linkages, we noted that the correlation among individual trades is positive for close neighbors, while it decays to negative values for distant traders (see Figure 9). Moreover, this decay is more pronounced for $G = 1$ than for $G = 2$. For close neighbors, the volume correlation features a pattern quite similar to that of the trade correlation. However, as the distance between traders increases, volume correlation becomes U-shaped and few information linkages ($G = 1$) lead to stronger volume correlation than more information linkages ($G = 2$). This is because for $G = 1$, the increased correlation of individual trades induced by the presence of information linkages does not offset the correlation decay arising from the market maker’s learning process. Individual trades are strongly negatively correlated for most of the trading rounds and volume correlation peaks up. When $G = 2$, the increase in the trade correlation

\textsuperscript{7}We compute the correlation between the individual traders’ volume through simulations, as we do not have a closed-form solution for it (see Appendix C for details).
resulting from the information linkages dominates the decay induced from the market maker’s learning process. As a result, individual trades and volume are weakly correlated for most of the batch auctions.

C. Traders’ profits

How do information linkages affect the profits from informed trade? Figure 11 plots the total expected profits to all informed trades, obtained varying the correlation of signals at two locations $\rho$, and the number of information linkages every trader has access to, $2G$. Figure 11 illustrates two main results. First, information linkages damage the expected profits when the correlation $\rho$ is positive, but enhance them when the correlation is negative. Second, for any fixed $G$, the expected profits are nonmonotonic in the correlation $\rho$. More generally, for any fixed tuple $(M, N, G)$, there exists a value of the initial correlation $\rho$ that maximizes the expected profits.

What are the origins of these findings? Changes in the correlation $\rho$ generate two effects. First, as $\rho$ increases, the signals available at each location become closer to each other, thus reducing each trader’s monopolistic power. Second, as Eq. (36) reveals, an increase in $\rho$ obviously implies a drop in $\Lambda_0$, which makes the traders’ estimates of the asset value more precise. In the absence of information linkages, the losses in the monopolistic power dominate over the precision gains, when $\rho$ is high or negative. When, instead, $\rho$ is low, the losses in the monopolistic power are more than offset by the precision gains.

In our model, an increase in the number of information linkages $2G$ captures similar effects. On the one hand, a trader with many information linkages has a better estimate of the asset value. On the other hand, these information linkages make our trader lose part of his monopolistic information power. Figure 11 shows that when the correlation $\rho$ is negative, e.g. $\rho = -0.15$, information linkages boost the expected profits. As noted earlier, this is precisely what we expect if $\rho$ had to increase from $-0.15$ to, say, $-0.10$, and were there no information linkages. However, while there exists a level of initial correlation maximizing the traders’ expected profits when $G = 0$, we do not have, here, such an “optimal” number of information linkages maximizing the traders’ expected profits.

IV. Conclusion

Why do individual investors engage in correlated trading behavior? One information-based explanation in the empirical literature is that proximity affects portfolio choice, which makes infor-
mationally close investors more likely to trade in the same direction, and informationally distant investors more likely to exhibit quite distinct portfolio choices. Although this explanation is simple and appealing, it is not clear whether it can be made consistent with rational behavior. Nor are the implications of such a trading behavior clear, both empirically and theoretically. For example, what should we expect such a correlated trading behavior to imply, in terms of liquidity, volume, market efficiency, market volatility, or profitability from informed trading? Also, under which conditions would we expect correlated trading and volume to arise in a market and, eventually, disappear from it? Finally, how differently should we expect informationally distant investors to trade?

This paper contributes to providing answers to these questions. We consider a theoretical framework in which a number of strategic traders belong to overlapping information networks, connected by what we term information linkages. We show that these information linkages have a significant and permanent impact on trading strategies and also on market variables. They raise volume and price informativeness, allow for better liquidity conditions and lower market volatility, and significantly affect the cross section of trades and the gains from informed trading. The model predicts that the traders’ expected profits increase when the information linkages bring quite distinct signals about the fundamentals, and decrease when these signals are positively correlated. Finally, the model predicts patterns of trade correlations that depend on the traders’ location: “neighbor” trades are positively correlated and “distant” trades are negatively correlated. This prediction is consistent with the robust evidence documented in the empirical literature, and could not be accommodated by previous models, in which traders do not have access to information networks. In fact, one important feature of our model is that the traders belong to an information network, which we took to be exogenous. Future research might attempt to examine the questions we studied in this paper, within a framework of endogenous network formation.
**Technical appendix**

**A. Derivation of selected formulae and preliminary results**

**Derivation of Eqs. (8)-(12).** To derive Eq. (8), we use the definition of the average signal in Eq. (1). A simple computation leaves:

\[ \text{var}(\bar{s}_{i,0}) = \frac{\hat{G} \Lambda_0 + 2G \hat{G} \Omega_0}{G^2}, \]

or equivalently (8). Next, we derive Eqs. (9) and (10). These equations correspond to two cases: (i) \(2G \leq (M - 1)/2\), and (ii) \(2G \geq (M - 1)/2\). We study these two cases separately. We introduce the following piece of notation

\[ \tilde{s}_{i,0} = \{s_{i-G,0}, \ldots, s_{i,0}, \ldots, s_{i+G,0}\}, \]

to denote the set of signals each trader \(i\) has access to, as a result of the presence of information linkages.

**Case (i) \((2G \leq (M - 1)/2)\).** Consider traders \(i\) and \(j = i + k, k \neq 0\). We have: \(s_{i+k,0} \notin \tilde{s}_{i,0}\) for all \(|k| > G\). Therefore

\[ \Omega_0(k,G) = \text{cov}(\tilde{s}_{i,0}, \tilde{s}_{i+k,0}) = \hat{G}^{-2} \sum_{l=-G}^{G} \sum_{m=-G}^{G} \text{cov}(s_{i+l,0}, s_{i+k+m,0}) = \Omega_0, \]

for all \(|k| > 2G\), which is the second line in (9). If instead \(|k| \leq 2G\), \(s_{i+k,0} \in \tilde{s}_{i,0}\) for all \(|k| \leq G\) and \(\tilde{s}_{i+k,0} \cap \tilde{s}_{i,0} \neq \emptyset\). In particular, trader \(i\) shares \((2G + 1 - k)\) signals with trader \(i + k\). Each of these signals contributes for \((\Lambda_0 + 2G \Omega_0) / (2G + 1)^2\) to \(\Omega_0(k,G)\). Shared signals thus contribute for

\[ \frac{\Lambda_0 + 2G \Omega_0}{(2G + 1)^2} \cdot (2G + 1 - k) \]

to \(\Omega_0(k,G)\). The remaining (not shared) \(k\) signals contribute for

\[ \frac{(2G + 1) \Omega_0}{(2G + 1)^2} \cdot k \]

to \(\Omega_0(k,G)\). Therefore,

\[ \Omega_0(k,G) = \frac{(\Lambda_0 + 2G \Omega_0)(\hat{G} - k) + \hat{G} \Omega_0 k}{\hat{G}^2}. \]

Grouping terms in the previous expression yields the first line in (9).
Case (ii) \((2G \geq (M - 1)/2)\). This case differs from the previous one due to the double overlap discussed in the main text (see Figure 3). In this case, the number of signals shared by traders \(i\) and \(i + k\) is:

\[
L(k, G) = 2G + 1 - k + n(k, G), \quad k = 1, \ldots, \frac{M-1}{2}.
\]  

(A1)

The term \(n(k, G)\) arises because traders located on the trader \(i\)'s right hand side semicircle might be sharing signals with traders located between \(i + 1\) and \(i + (M - 1)/2\) on the left hand side semicircle (see Figure 3); and obviously the \(i\)-th trader shares signals with traders located between \(i - 1\) and \(i - G\) as well. The double overlap occurs if and only if trader \(i + k\) on the left hand side semicircle and trader \(i - \ell\) with \(\ell \in [1, \frac{M-1}{2}]\) on the right hand side semicircle are such that \(\ell\) and \(k\) satisfy:

\[
\begin{align*}
M - 1 \leq (\ell - 1) + \frac{M - 1}{2} - k &\leq G \\
G &\geq \ell \geq 1 \\
\frac{M - 1}{2} &\geq k \geq 1
\end{align*}
\]

The first inequality in the previous restrictions requires trader \(i - \ell\) to share his signal with trader \(i + k\). The second and third constraints restrict trader \(i - \ell\) to be on the right hand side semicircle and trader \(i + k\) to be on the left hand side semicircle relative to trader \(i\). Thus, for fixed \(k\) \((1 \leq k \leq (M - 1)/2)\), the double overlap occurs if and only if

\[
G \geq \ell \geq M - G - k, \quad k = 1, \ldots, \frac{M-1}{2},
\]

and \(\ell \geq 1\). Clearly, \(\min_k (M - G - k) = \frac{M-1}{2} - G + 1 \geq 1\). Hence, the constraint that \(\ell \geq 1\) is redundant. By the previous inequalities, it follows that:

\[
n(k, G) = \max [G - (M - G - k) + 1, 0].
\]

By replacing this result into Eq. (A1) leaves:

\[
L(k, G) = \begin{cases} 
4G + 1 - (M - 1), & \frac{M-1}{2} \geq k \geq 2 \left(\frac{M-1}{2} - G\right) \\
2G + 1 - k, & 1 \leq k \leq 2 \left(\frac{M-1}{2} - G\right)
\end{cases}
\]

For all \(k \in [1, 2 \left(\frac{M-1}{2} - G\right)]\), \(\Omega_0(k, G)\) is thus exactly as in case \(a\) for \(k \in [1, 2G]\), and the first line of Eq. (10) follows. For all \(k \in [2 \left(\frac{M-1}{2} - G\right), \frac{M-1}{2}]\), tedious but straightforward computations lead to the second line of Eq. (10).

Finally, we demonstrate that Eq. (12) holds true. As usual, we consider the two cases in which \(2G \geq (M - 1)/2\). If \(0 \leq 2G \leq (M - 1)/2\), there are \([M - (4G + 1)]\) traders \(i + k\) such
that $\bar{s}_{i+k,0} \cap \bar{s}_{i,0} = \{0\}$. In correspondence of these indexes, $\text{cov}(\bar{s}_{i+k,0}, \bar{s}_{i,0}) = \Omega_0$. Therefore,

$$\bar{\Gamma}_0 (G) = 2 \sum_{k=1}^{2G} \Omega_0 (k, G) + [M - (4G + 1)] \Omega_0. $$

The $2G$ covariances in the summation can be computed through the first line in (9). Eq. (12) follows by the expression of $\bar{\Lambda}_0 (G)$ in Eq. (8). Next, consider the case $(M - 1)/2 \leq 2G \leq M - 1$. We have:

$$\bar{\Gamma}_0 (G) = 2 \left\{ \sum_{k=1}^{M-1-2G} \Omega_0 (k, G) + \left( 2G - \frac{M - 1}{2} \right) \left[ 2\bar{\Lambda}_0 (G) - \frac{M (\Lambda_0 - \Omega_0)}{G^2} - \Omega_0 \right] \right\}. $$

By plugging Eqs. (10) and (8) into the previous equation, we find that the expression of $\bar{\Gamma}_0 (G)$ is the same as the one obtained in the case $0 \leq 2G \leq (M - 1)/2$, and Eq. (12) follows. ■

**Derivation of Eq. (13).** Consider the projection of $s_{i+k,0}$ onto $s_{i,0}$ for $|k| \leq G$. We have $s_{i+k,0} \in s_{i,0}$ for $k = 0, \mp 1, \cdots, \mp G$. Hence,

$$E(s_{i+k,0} | s_{i,0}) = s_{i+k,0}, \quad k = 0, \mp 1, \cdots, \mp G. \quad (A1)$$

Next, consider a signal $s_{i+k,0}$ a given trader $i$ does not have access to, i.e. $s_{i+k,0}$, for $k = \mp (G + 1), \cdots, \mp \frac{M-1}{2}$. Let $\Psi_{0,G} = E(s_{i,0} s_{i,0}^\top)$ be the $\hat{G} \times \hat{G}$ variance-covariance matrix of the vector $s_{i,0}$. $\Psi_{0,G}$ is a $\hat{G} \times \hat{G}$ submatrix extracted from $\Psi_0$, and its inverse can be obtained with the same strategy of proof as in Foster and Viswanathan (1996) (p. 1479). Let $K = \left[ (\Lambda_0 - \Omega_0) (\Lambda_0 + 2G\Omega_0) \right]^{-1}$. Then,

$$\Psi_{0,G}^{-1} = K \cdot \begin{bmatrix} \Lambda_0 + (2G - 1) \Omega_0 & -\Omega_0 & \cdots & -\Omega_0 \\ -\Omega_0 & \Lambda_0 + (2G - 1) \Omega_0 & \cdots & -\Omega_0 \\ \vdots & \vdots & \ddots & \vdots \\ -\Omega_0 & -\Omega_0 & \cdots & \Lambda_0 + (2G - 1) \Omega_0 \end{bmatrix}. $$

For $|k| > G$, we have that $\text{cov}(s_{i+k,0}, s_{i,0}) = \Omega_0 \mathbf{1}_{\hat{G}}$, and by the Projection Theorem,

$$E(s_{i+k,0} | s_{i,0}) = \Omega_0 \mathbf{1}_{\hat{G}}^\top (\Psi_{0,G})^{-1} s_{i,0}$$

$$= \frac{G\Omega_0}{\Lambda_0 + 2G\Omega_0} s_{i,0}, \quad k = 0, \mp 1, \cdots, \mp G. \quad (A2)$$
Therefore, gathering Eqs. (A1) and (A2),

\[
E(s_{i+k,0}|s_{i,0}) = \begin{cases} 
  s_{i+k,0} & \text{for } k = 0, \mp 1, \cdots, \mp G \\
  \frac{G \Omega_0}{\Lambda_0 + 2G \Omega_0} \bar{s}_{i,0} & \text{for } k = \mp (G + 1), \cdots, \mp \frac{M-1}{2}
\end{cases}
\]

We now compute, for a given trader \( i \), his forecast of the sum of the remaining traders’ average signals onto \( s_{i,0} \). We obviously have \( \sum_{j=1}^{M} \bar{s}_{j,0} = \sum_{j=1}^{M} s_{j,0} \). Therefore,

\[
E\left(\sum_{j \neq i} \bar{s}_{j,0} \Big| s_{i,0}\right) = \sum_{j=1}^{M} E\left( s_{j,0} \Big| s_{i,0}\right) - \bar{s}_{i,0} = \theta t_{i,n} \]

Eq. (13) follows by rearranging terms in the last equality, and by the definition of \( \phi_1 \) in the main text.

We shall repeatedly use the results recorded in the following three lemmas, which are easy generalizations of results given in Foster and Viswanathan (1996).

**Lemma 1.** The relation between the market maker’s updated estimate of the asset value, \( p_{n} \), and the market maker’s updated estimate of the average signal, \( t_{n} \), is given by

\[
p_{n} = \theta t_{n}. \tag{A3}
\]

**Lemma 2.** Let \( \bar{\Lambda}_{n}(G) = \text{var}(\bar{s}_{i,n}|F_{M+1,n}) \), \( \bar{\Gamma}_{n}(G) = \sum_{k \neq i} \Omega_{n}(k, G) \), and \( \Omega_{n}(k, G) = \text{cov}(\bar{s}_{i,n}, \bar{s}_{i+k,n}|F_{M+1,n}) \). We have,

\[
\sigma_{f,n}^2 = \frac{\theta^2}{M} [\Lambda_{n} + (M - 1) \Omega_{n}] . \tag{A4}
\]

Furthermore, the following recursions hold,

\[
\begin{align*}
\Omega_{n-1} - \Omega_{n} & = \Lambda_{n-1} - \Lambda_{n}; \\
\sigma_{f,n-1}^2 - \sigma_{f,n}^2 & = \theta^2 (\Lambda_{n-1} - \Lambda_{n}); \\
\bar{\Lambda}_{n-1}(G) - \bar{\Lambda}_{n}(G) & = \Lambda_{n-1} - \Lambda_{n}, \quad \text{all } G; \\
\bar{\Omega}_{n-1}(k, G) - \bar{\Omega}_{n}(k, G) & = \Lambda_{n-1} - \Lambda_{n}, \quad \text{all } k, G; \\
\bar{\Gamma}_{n-1}(G) - \bar{\Gamma}_{n}(G) & = (M - 1) (\Lambda_{n-1} - \Lambda_{n}), \quad \text{all } G. 
\end{align*}
\]
Lemma 3. The market maker learning about individual and average signals evolves according to

\[ t_n = t_{n-1} + \zeta_n y_n, \]  

(A6)

for some deterministic sequence \( \zeta_n \). The relation between the updating parameters \( \lambda_n \) and \( \zeta_n \) is given by

\[ \lambda_n = \theta \zeta_n. \]  

(A7)

Finally, the trading strategy of any trader \( i \) does not depend on the residual order flow \( (z_{i,t})_{t=1}^{n-1} \). Precisely, we have:

\[ x_{i,n} = \hat{G}_n \left( \bar{s}_{i,0} - \sum_{r=1}^{n-1} \zeta_r y_r \right). \]  

(A8)

Proof of Lemma 1. By Semi-Strong market efficiency, \( p_n = E(\theta \bar{s}|F_{M+1,n}) \). By Eq. (6), \( \bar{s} \) is a sufficient statistic for \( E(\theta \bar{s}|s_0) \). Therefore,

\[
p_n = E(\theta \bar{s}|F_{M+1,n}) \\
= E \left[ E(\theta \bar{s}|F_{M+1,n}) | F_{M+1,n} \right] \\
= \frac{\theta}{M} E \left( \sum_{i=1}^{M} \bar{s}_{i,0} \right) | F_{M+1,n} \\
= \theta t_n,
\]

where the last line follows by Eq. (14).

Proof of Lemma 2. First, we derive Eq. (A4). By the Law of Iterated Expectations,

\[ E(\theta \bar{s}|F_{M+1,n}) = E[E(\theta \bar{s}|s_0)|F_{M+1,n}] = E(\theta \bar{s}|F_{M+1,n}) = p_n. \]

Hence, by the first equation in (16) and Eq. (A3),

\[
\sigma_{f,n}^2 = \operatorname{var}(\theta \bar{s} - p_n|F_{M+1,n}) \\
= E \left[ \left( \frac{\theta}{M} \sum_{i=1}^{M} \bar{s}_{i,0} - \frac{\theta}{M} \sum_{i=1}^{M} t_{i,n} \right)^2 \right] | F_{M+1,n} \\
= \frac{\theta^2}{M^2} E \left[ \left( \sum_{i=1}^{M} \bar{s}_{i,n} \right)^2 \right] | F_{M+1,n} \\
= \frac{\theta^2}{M^2} E \left[ \left( \sum_{i=1}^{M} s_{i,n} \right)^2 \right] | F_{M+1,n} \\
= \frac{\theta^2}{M} [\Lambda_n + (M-1) \Omega_n].
\]

30
Next, we derive Eqs. (A5-a)-(A5-e). Let

\[ \tau_n = \text{cov}(s_{i,n-1}, y_n | F_{M+1,n-1}) \]  

(A9)

Let \( \Psi_n = E \left( [s_{1,0} - t_n, \ldots, s_{M,0} - t_n]^\top | F_{M+1,n} \right) \). By the Projection Theorem,

\[ \Psi_n = \Psi_{n-1} - \frac{\tau_n^2}{\text{var}(y_n | F_{M+1,n-1})} \mathbf{1} \mathbf{1}^\top, \]

which gives the recursions:

\[ \Lambda_n = \Lambda_{n-1} - \frac{\tau_n^2}{\text{var}(y_n | F_{M+1,n-1})} ; \]

\[ \Omega_n = \Omega_{n-1} - \frac{\tau_n^2}{\text{var}(y_n | F_{M+1,n-1})} ; \]

(A10)

or equivalently (A5-a). Taking one lag in Eq. (A4) yields:

\[ \sigma_{f,n}^2 = \theta^2 \left( \Lambda_{n-1} + (M - 1) \Omega_{n-1} \right) ; \]

giving the recursion:

\[ \sigma_{f,n}^2 - \sigma_{f,n-1}^2 = \frac{\theta^2}{M} [\Lambda_{n-1} - \Lambda_n + (M - 1) (\Omega_{n-1} - \Omega_n)] = \theta^2 (\Lambda_{n-1} - \Lambda_n) , \]

(A11)

where the last equality follows by Eq. (A5-a). Now consider the variance of average signals \( \bar{\Lambda}_n (G) \). By Eq. (8), \( \bar{\Lambda}_n (G) \) can be expressed in terms of the elements in the signals variance-covariance matrix \( \Psi_n \) as:

\[ \bar{\Lambda}_n (G) = \frac{\Lambda_n + 2G\Omega_n}{G} ; \]

and Eq. (A5-c) follows by Eq. (A5-a), and by simple computations. We now provide the update for \( \Omega_n (k,G) \), thus completing the specification of the variance-covariance matrix \( \Psi_n (G) = E \left( [\bar{s}_{1,0} - t_n, \ldots, \bar{s}_{M,0} - t_n]^\top | F_{M+1,n} \right) \). By Eq. (A5-a) and the expression of the off-diagonal elements in \( \Psi_n (G) \) [see Eqs. (9) and (10) evaluated at \( n \)],

\[ \Omega_{n-1} (k,G) - \Omega_n (k,G) = \Lambda_{n-1} - \Lambda_n, \quad \text{all } k,G. \]

Finally, the following recursion is readily obtained by Eq. (12),

\[ \Gamma_{n-1} (G) - \Gamma_n (G) = (M - 1) (\Lambda_{n-1} - \Lambda_n) . \]
Proof of Lemma 3. By the definition of $t_n$ and $s_{i,n}$,

$$
t_n - t_{n-1} = E(s_{i,0} - t_{n-1} | F_{M+1,n}) = E(s_{i,n-1} | F_{M+1,n}) = \zeta_n y_n,
$$

where $\zeta_n$ is the regression coefficient of $s_{i,n-1}$ on $y_n$, viz

$$
\zeta_n = \frac{\text{cov}(s_{i,n-1}, y_n | F_{M+1,n-1})}{\text{var}(y_n | F_{M+1,n-1})} = \frac{\tau_n}{\text{var}(y_n | F_{M+1,n-1})},
$$

(A12)

which proves Eq. (A6). As regards Eq. (A7), we have that, by Eqs. (A6) and (A3),

$$
0 = \theta t_n - \theta t_{n-1} - \theta \zeta_n y_n = p_n - p_{n-1} - \theta \zeta_n y_n, \quad \text{and Eq. (A7) follows by Eq. (18)}.
$$

Finally, we show Eq. (A8). By the assumption that trading strategies are linear, as in Eq. (17), and the market maker’s recursive update in Eq. (15) and (A6),

$$
x_{i,n} = \hat{G}\beta_n \bar{s}_{i,n-1} = \hat{G}\beta_n (\bar{s}_{i,0} - t_{n-1}) = \hat{G}\beta_n (\bar{s}_{i,0} - \sum_{r=1}^{n-1} \zeta_r y_r).
$$

Remark on notation. To easy notation, we now suppress the dependence of $\hat{\Lambda}_n (G)$ and $\hat{\Gamma}_n (G)$ on $G$.

Derivation of Eqs. (19)-(20). We have:

$$
E(f - p_{n-1} | F_{i,n}) = E[E(f - p_{n-1} | s_0) | F_{i,n}]
$$

$$
= E(\theta \bar{s} - p_{n-1} | F_{i,n})
$$

$$
= \frac{\theta}{M} E \left( \bar{s}_{i,n-1} + \sum_{j \neq i} \bar{s}_{j,n-1} | F_{i,n} \right)
$$

$$
= \frac{\theta}{M} \left( 1 + \frac{\hat{\Gamma}_{n-1}}{\hat{\Lambda}_{n-1}} \right) \bar{s}_{i,n-1},
$$

(A13)

where the first line follows by the Law of Iterated Expectations, the second by Eq. (6), the third by Eq. (A3) and the fact that $\theta \bar{s} - p_{n-1} = \frac{\theta}{M} \sum_{i=1}^{M} \bar{s}_{i,n-1}$ and the fourth from

$$
E \left( \sum_{j \neq i} \bar{s}_{j,n-1} \bigg| F_{i,n} \right) = E \left( \sum_{j \neq i} \bar{s}_{j,n-1} \bigg| \bar{s}_{i,n-1}, F_{M+1,n-1} \right) = \frac{\hat{\Gamma}_{n-1}}{\hat{\Lambda}_{n-1}} \bar{s}_{i,n-1}.
$$

(A14)

Eq. (A13) is Eq. (19) with $\eta_n = \theta (\hat{G}M)^{-1}(1 + \hat{\Lambda}_{n-1}^{-1}\hat{\Gamma}_{n-1})$. Finally, Eq. (A14) is simply Eq. (20) with $\phi_n = (M - 1)^{-1} \hat{\Lambda}_{n-1}^{-1}\hat{\Gamma}_{n-1}$. ■
B. Proofs of Proposition 1 and Corollary 1

B.1 Proof of Proposition 1

We proceed in three steps. In the first step, we derive a recursive expression for the price deviation induced by traders’ suboptimal play. In the second step, we derive the traders’ optimality conditions. In the third step, we compute market maker updates.

**Step 1: Price deviation**

Let us denote deviations from the equilibrium path with a prime ('). For example, the deviation of the $i$-th trader from the equilibrium play in Eq. (17) to $(x_{i,k}^0, y_{k}^0 = y_k - (x_{i,k} - x_{i,k}^0))_{k=1}^{n-1}$ during the first $n-1$ auctions would generate the aggregate order flow $(\bar{s}_{i,n} - t_{n-1}) = \sum_{k=1}^{n-1} \zeta_k y_k$. Since the market maker’s update of the estimate of the asset value and the average signals are linear in the order flow due to Eqs. (18)-(A6), a deviation by the $i$-th trader would tilt the market maker’s learning process as well, resulting in $(p_n^0)_{k=1}^{n-1}$ and $(t_n^0)_{k=1}^{n-1}$.

First, we show that the price deviation induced by a suboptimal play of the $i$-th trader has the following recursive structure:

$$p_n - p_n^0 = (p_{n-1} - p_{n-1}^0) \left[ 1 - \theta^{-1} \hat{G} (M - 1) \beta_n \lambda_n \right] + \hat{G} \lambda_n \beta_n \bar{s}_{i,n-1} - \lambda_n x_{i,n}'.$$

(B1)

Indeed, let $y_n' = \sum_{j \neq i} x_{j,n} + x_{i,n}^0 + u_n$ and $t_n'$ be the aggregate order flow and the market maker’s update when trader $i$ deviates to $x_{i,n}^0$. By Eq. (A6), $t_n = \sum_{k=1}^{n} \zeta_k y_k$. Similarly, $t_n' = \sum_{k=1}^{n} \zeta_k y_k'$. Therefore, by Eq. (15),

$$\bar{s}_{j,n-1} - \bar{s}_{j,n-1}^0 = (\bar{s}_{j,0} - t_{n-1}) - (\bar{s}_{j,0} - t_{n-1}^0)$$

$$= \sum_{k=1}^{n-1} \zeta_k y_k' - \sum_{k=1}^{n-1} \zeta_k y_k$$

$$= \frac{1}{\theta} \left( \sum_{k=1}^{n-1} \lambda k y_k' - \sum_{k=1}^{n-1} \lambda_k y_k \right)$$

$$= \frac{1}{\theta} \left( p_{n-1}^0 - p_{n-1} \right),$$

(B2)

where the third line follows by Eq. (A7), and the fourth line holds as Eq. (18) implies that $p_n = \sum_{k=1}^{n} \lambda_k y_k$. Thus, by Eqs. (15) and (A6),

$$\bar{s}_{i,n} = \bar{s}_{i,0} - t_n = \bar{s}_{i,n-1} - (t_n - t_{n-1}) = \bar{s}_{i,n-1} - \zeta_n y_n.$$
Substituting for the equilibrium order flow [see Eq. (17)], using Eq. (A7), and taking expectations yields:

\[
E(\bar{s}_{i,n}|F_{i,n}) = \bar{s}_{i,n-1} - \frac{\hat{G}\beta_n\lambda_n}{\theta} \left[ \bar{s}_{i,n-1} + E \left( \sum_{i\neq j} \bar{s}_{i,n-1} | F_{i,n} \right) \right]
\]

\[
= 1 - \frac{\hat{G}\beta_n\lambda_n}{\theta} (1 + (M - 1) \phi_n) \bar{s}_{i,n-1},
\]

(\text{B3})

where the last line follows by Eq. (20). Using the equilibrium strategy in Eq. (17) and the price recursion in Eq. (18), we find that the price deviation has the following expression:

\[
p_n - p_n' = p_{n-1} - p_{n-1}' + \lambda_n (y_n - y_n')
\]

\[
= p_{n-1} - p_{n-1}' + \lambda_n \left[ \sum_{j \neq i} \hat{G}\beta_n (\bar{s}_{j,n-1} - \bar{s}_{j,n-1}') + \hat{G}\beta_n \bar{s}_{i,n-1} - x_{i,n}' \right].
\]

Substituting for \((\bar{s}_{j,n-1} - \bar{s}_{j,n-1}')\) from Eq. (B2) in the previous equation leaves Eq. (B1).

\textbf{Step 2: Traders’ strategies}

First, we show that the value function in Eq. (22) and the strategy in Eq. (21) are mutually consistent. Any trader \(i\) faces the following recursive problem:

\[
W_{i,n-1} = \max_{x_{i,n}'} E \left[ (f - p_n') x_{i,n}' + W_{i,n} | F_{i,n} \right]
\]

\[
= \max_{x_{i,n}'} E \left[ (f - p_{n-1}' - \lambda_n x_{i,n}' - \lambda_n \sum_{j \neq i} x_{j,n}) x_{i,n}' + W_{i,n} | F_{i,n} \right].
\]

(\text{B4})

Given the value function conjectured in Eq. (22), and Eq. (B1), the optimality conditions of the previous problem lead to:

\[
0 = E \left( (f - p_{n-1}' | F_{i,n}) - \hat{G}\beta_n \lambda_n E \left( \sum_{j \neq i} \bar{s}_{j,n-1} | F_{i,n} \right) - 2\lambda_n x_{i,n}' - \lambda_n \psi_n E(\bar{s}_{i,n} | F_{i,n}) - 2\lambda_n \mu_n E( p_n - p_n' | F_{i,n}) \right) \text{ (first order conditions)};
\]

and

\[-\lambda_n + \lambda_n^2 \mu_n < 0 \text{ (second order conditions)}.\]

Since \((\bar{s}_{j,n-1}, (p_{n-1} - p_{n-1}') \in F_{i,n}\), the first order conditions can be reorganized as follows:

\[
0 = E \left( f - p_{n-1} | F_{i,n} \right) + (p_{n-1} - p_{n-1}') - \hat{G}\beta_n \lambda_n \sum_{j \neq i} (\bar{s}_{j,n-1}' - \bar{s}_{j,n-1})
\]

\[
- \hat{G}\beta_n \lambda_n E \left( \sum_{j \neq i} \bar{s}_{j,n-1} | F_{i,n} \right) - 2\lambda_n x_{i,n}' - \lambda_n \psi_n E(\bar{s}_{i,n} | F_{i,n}) - 2\lambda_n \mu_n E( p_n - p_n' | F_{i,n}) .
\]
By replacing Eqs. (B1), (B2) and (B3) in the previous equation, and by rearranging terms, we obtain Eq. (21), where $\gamma_n$ is as in Eq. (25) and

$$\beta_n = \frac{\eta_n - 1}{\lambda_n [1 + (1 - \theta^{-1} \lambda_n \psi_n) (1 + (M - 1) \phi_n)]}. \quad \text{(B5)}$$

Next, we use Eq. (21), and find that the expected profit in any single auction is:

$$E \left[ (f - p_n) x_{i,n}^{'} | F_{i,n} \right] = \tilde{G}^2 \beta_n [\eta_n - \beta_n \lambda_n (1 + (M - 1) \phi_n)] \tilde{s}_{i,n}^2 - \gamma_n \left[ 1 - \lambda_n \left( \gamma_n + \theta^{-1} \tilde{G} (M - 1) \beta_n \right) \right] (p_n - p'_{n-1})^2$$

$$+ \left\{ \gamma_n (\eta_n - 2 \beta_n \lambda_n) + \beta_n \left[ 1 - (M - 1) \lambda_n \left( \gamma_n \phi_n + \theta^{-1} \tilde{G} \beta_n \right) \right] \right\} \tilde{G} \tilde{s}_{i,n-1} (p_{n-1} - p'_{n-1}). \quad \text{(B6)}$$

By taking the conditional expectation of the value function in Eq. (22) leaves:

$$E (W_{i,n} | F_{i,n}) = \alpha_n E (\tilde{s}_{i,n}^2 | F_{i,n}) + \psi_n (p_n - p'_n) E (\tilde{s}_{i,n} | F_{i,n}) + \mu_n (p_n - p'_n)^2 + \delta_n. \quad \text{(B7)}$$

By Eqs. (B1) and (B3), both $E (\tilde{s}_{i,n} | F_{i,n})$ and $(p_n - p'_n)$ are linear in $(p_{n-1} - p'_{n-1})$ and $\tilde{s}_{i,n-1}$. To identify all coefficients of the value function, we are therefore left with finding the conditional expectation $E (\tilde{s}_{i,n}^2 | F_{i,n})$. By Eqs. (15) and (A7),

$$E (\tilde{s}_{i,n}^2 | F_{i,n}) = \tilde{s}_{i,n-1}^2 + \zeta_n \tilde{s}_{i,n-1} E (y_n^2 | F_{i,n}) - 2 \zeta_n \tilde{s}_{i,n-1} E (y_n | F_{i,n})$$

$$= \left[ 1 - \theta^{-1} \tilde{G} \beta_n \lambda_n (1 + (M - 1) \phi_n) \right]^2 \tilde{s}_{i,n-1}^2 + \theta^{-1} \lambda_n^2 \sigma_u^2 + \theta^{-1} \tilde{G} \beta_n^2 \lambda_n^2 \text{var} \left( \sum_{j \neq i} \tilde{s}_{j,n-1} | F_{i,n} \right), \quad \text{(B8)}$$

where

$$\text{var} \left( \sum_{j \neq i} \tilde{s}_{j,n-1} | F_{i,n} \right) = \text{var} \left( \sum_{j \neq i} \tilde{s}_{j,n-1} | \tilde{s}_{i,n-1} \right)$$

$$= E \left[ \sum_{j \neq i} \tilde{s}_{j,n-1} - (M - 1) \phi_n \tilde{s}_{i,n-1} \right]^2$$

$$= \text{var} \left( \sum_{j \neq i} \tilde{s}_{j,0} | F_{M+1,n-1} \right) - (M - 1)^2 \phi_n^2 \text{var} (\tilde{s}_{i,0} | F_{M+1,n-1})$$

$$= M [\Lambda_{n-1} + (M - 1) \Omega_{n-1}] - \left[ 1 + (M - 1)^2 \phi_n^2 \right] \Lambda_{n-1} - 2 \Gamma_{n-1}. \quad \text{(B9)}$$

Next, plug Eq. (B9) into Eq. (B8), then plug the resulting expression into Eq. (B7). Also, replace the expression for $p_n - p'_{n-1}$ from Eq. (B1) into Eq. (B7). Finally, plug the resulting expression for Eqs. (B7) and (B6) into Eq. (B4) and identify terms to obtain the recursions for the coefficients $\alpha_n, \mu_n, \psi_n$ and $\delta_n$ in Eq. (26).
Finally, we consider the market maker’s problem. By plugging the equilibrium trades in Eq. (31) into the order flow in Eq. (4) gives

\[ y_n = \sum_{i=1}^{M} \hat{G}_n \bar{s}_{i,n-1} + u_n = \hat{G} M \beta_n \left[ \frac{1}{M} \sum_{i=1}^{M} (\bar{s}_{i,0} - t_{n-1}) \right] + u_n. \] (B10)

where the second line follows by the market maker’s update in Eq. (15). By the definition of \( \bar{s} \), and the equality \( t_{n-1} = \theta^{-1} p_{n-1} \) in Eq. (A3),

\[ y_n = \hat{G} M \beta_n \left( \bar{s} - \frac{p_{n-1}}{\theta} \right) + u_n = \frac{\hat{G} M \beta_n}{\theta} (\theta \bar{s} - p_{n-1}) + u_n. \] (B11)

By Eqs. (B11) and (6), and the Law of Iterated Expectations,

\[ \text{cov} (f, y_n | F_{M+1,n-1}) = \text{cov} (\theta \bar{s}, y_n | F_{M+1,n-1}) \] .

Therefore,

\[ \text{cov} (f, y_n | F_{M+1,n-1}) = \text{cov} (\theta \bar{s} - p_{n-1}, y_n | F_{M+1,n-1}) = \frac{\hat{G} M \beta_n}{\theta} \text{var} (\theta \bar{s} - p_{n-1} | F_{M+1,n-1}) = \frac{\hat{G} M \beta_n}{\theta} \sigma_{f,n-1}^2 \] (B12)

where the first line follows because \( p_{n-1} \in F_{M+1,n-1} \), the second line is obtained through the order flow in Eq. (B11), and the third line is due to the expression of the residual variance in Eq. (16). We now re-write the recursion of \( \Lambda_n \) in terms of equilibrium parameters. Using the order flow in Eq. (B10) and the definition of \( \tau_n \) in Eq. (A9)

\[ \tau_n = \hat{G} M \beta_n \text{cov} (s_{i,n-1}, \sum_{i=1}^{M} s_{i,n-1} | F_{M+1,n-1}) = \hat{G} M \beta_n (\Lambda_{n-1} + (M - 1) \Omega_{n-1}) = \frac{M \hat{G} M \beta_n}{\theta^2} \sigma_{f,n-1}^2, \] where the third line follows by the expression of the residual variances in Eq. (16), and the last line holds by the expression of \( \sigma_{f,n}^2 \) in Eq. (A4). Therefore, by Eqs. (A10), (A12), the expression for \( \tau_n \) found above, and Eq. (A7),

\[ \Lambda_n = \Lambda_{n-1} - \zeta_n \tau_n = \Lambda_{n-1} - \frac{M \hat{G} M \beta_n \lambda_n}{\theta^3} \sigma_{f,n-1}^2. \] (B13)
Again by the above expression for \( \tau_n \), Eqs. (A7) and (A12),
\[
\lambda_n = \theta \xi_n = \frac{\theta \tau_n}{\text{var} (y_n | F_{M+1,n-1})} = \frac{M \hat{G} \beta_n \sigma_{f,n-1}^2}{\theta \cdot \text{var} (y_n | F_{M+1,n-1})}.
\]

But
\[
\text{var} (y_n | F_{M+1,n-1}) = \left( \theta^{-1} \hat{G} M \beta_n \right)^2 \text{var} (\theta s - p_{n-1} | F_{M+1,n-1}) + \sigma_u^2
\]
\[
= \left( \theta^{-1} \hat{G} M \beta_n \right)^2 \sigma_{f,n-1}^2 + \sigma_u^2.
\]

Therefore, the price sensitivity in Eq. (18) can be represented as:
\[
\lambda_n = \frac{\theta M \hat{G} \beta_n \sigma_{f,n-1}^2}{\left( \hat{G} M \beta_n \right)^2 \sigma_{f,n-1}^2 + \theta^2 \sigma_u^2}.
\]

After \( n \) trading rounds, the full information asset value has residual variance given by:
\[
\sigma_{f,n}^2 = \sigma_{f,n-1}^2 - \lambda_n \text{cov} (f, y_n | F_{M+1,n-1}) = \left( 1 - \theta^{-1} \beta_n \hat{G} M \right) \sigma_{f,n-1}^2,
\]
where the last equality follows by Eq. (B12) and the first equality holds as \( E (y_n | F_{M+1,n-1}) = 0 \), and by the Law of Iterated Expectations,
\[
\text{cov} (f, y_n | F_{M+1,n-1}) = E (f \cdot y_n | F_{M+1,n-1}) = E \left[ E (f \cdot y_n | F_{M+1,n}) | F_{M+1,n-1} \right]
\]
\[
= E (p_n \cdot y_n | F_{M+1,n-1}) = E (p_{n-1} \cdot y_n | F_{M+1,n-1}) + E \left( \lambda_n \cdot y_n^2 | F_{M+1,n-1} \right)
\]
\[
= \lambda_n \cdot \text{var} (y_n | F_{M+1,n-1}).
\]

Next, we plug Eq. (B14) into Eq. (B15) and obtain:
\[
\sigma_{f,n}^2 = \frac{\theta^2 \sigma_u^2 \sigma_{f,n-1}^2}{\left( \hat{G} M \beta_n \right)^2 \sigma_{f,n-1}^2 + \theta^2 \sigma_u^2}.
\]

By combining Eqs. (B14) and (B16) we find an alternative expression for \( \lambda_n \),
\[
\lambda_n = \frac{\hat{G} M \beta_n \sigma_{f,n}^2}{\theta \sigma_u^2},
\]
or equivalently Eq. (24). By solving Eq. (B16) for $\sigma^2_{f,n-1}$ gives

$$\sigma^2_{f,n-1} = -\frac{\theta^2 \sigma_u^2 \sigma_{Gn}^2}{(GM\beta_n)^2 \sigma_{f,n}^2 - \theta^2 \sigma_u^2}.$$  

(B18)

By combining Eqs. (B13), (B17) and (B18), we find that $\Lambda_n$ solves,

$$\Lambda_{n-1} - \Lambda_n = -\frac{\sigma_u^4}{(GM\beta_n)^2 \sigma_{f,n}^2 - \theta^2 \sigma_u^2} \lambda_n^2 = -\frac{\lambda_n^2 \sigma_u^2 \sigma_{f,n}^2}{\theta^2 (\lambda_n^2 \sigma_u^2 - \sigma_{f,n}^2)},$$  

(B19)

where the last equality follows because $(GM\beta_n)^2 \sigma_{f,n}^2 = \theta^2 \lambda_n^2 \sigma_u^4 \sigma_{f,n}^{-2}$ [due to Eq. (24)]. Also, Eqs. (A5-c) and (A5-e) imply that

$$\begin{align*}
\Gamma_{n-1} & = \Gamma_n + (M-1)(\Lambda_{n-1} - \Lambda_n) \\
\Lambda_{n-1} & = \frac{\Gamma_{n-1}}{\Lambda_{n-1}} = (M-1) \phi_n = \frac{GM}{\theta} \eta_n - 1.
\end{align*}$$  

(B20)

Furthermore, by Eqs. (27)-(28):

$$\begin{align*}
\Gamma_{n-1} & = (M-1) \phi_n = \frac{GM}{\theta} \eta_n - 1.
\end{align*}$$  

(B21)

By combining Eqs. (24) and (B5), we find that

$$\theta \sigma_u^2 \lambda_n^2 \left[1 + (1 - \theta^{-1} \lambda_n \psi_n) (1 + (M-1) \phi_n)\right] = (GM \eta_n - \lambda_n \psi_n) M \sigma_{f,n}^2.$$  

Substituting Eqs. (27)-(28) in the previous equation leaves

$$\begin{align*}
\theta \sigma_u^2 \lambda_n^2 \left[1 + (1 - \theta^{-1} \lambda_n \psi_n) (1 + \frac{\Gamma_{n-1}}{\Lambda_{n-1}})\right] & = \left[\frac{\theta}{M} \left(1 + \frac{\Gamma_{n-1}}{\Lambda_{n-1}}\right) - \lambda_n \psi_n\right] M \sigma_{f,n}^2.
\end{align*}$$

The quartic equation $F(\lambda_n) = 0$ in Eq. (23) is obtained by substituting Eqs. (B19), (B20), (A4), (8) and (12) evaluated at $n$ into the previous equation, and by tedious computations. To show that Eq. (23) admits a unique positive solution, note that the constant and the coefficient of $\lambda_n^4$ are both positive, and that the coefficient of $\lambda_n^2$ is negative. (Let $\rho_n = \Omega_n / \Lambda_n$ be the correlation coefficient between individual signals. Since $|\rho_n| \leq 1$, then $\Lambda_n - \Omega_n \geq 0$ and the coefficient for $\lambda_n^4$ is non-negative. Moreover, $\Lambda_n + (M-1) \Omega_n > 0$ and, hence, $2 \Lambda_n + (M-1) \Omega_n > 0$.) On the other hand, the sign of $\psi_n$ determines the sign of the terms in $\lambda_n^2$ and $\lambda_n$. However, regardless of whether $\psi_n$ is positive or negative, there are only two sign changes. By Descartes’ rule, Eq. (23) has at most two real positive roots. By Eq. (B15), $\sigma_{f,n}^2 < \sigma_{f,n-1}^2 \leftrightarrow \theta^{-1} \beta_n \lambda_n GM < 1$. By Eq. (24) this restriction becomes $\lambda_n^2 < \sigma_{f,n}^2 \sigma_u^{-2}$ or equivalently $\lambda_n < \sigma_{f,n}^2 \sigma_u^{-1}$. By Eq. (23), $F(\lambda = 0) = \theta^{-1} \sigma_{f,n}^2 > 0$, $F(\lambda = \sigma_u^{-1} \sigma_{f,n}^2) = - (\theta M)^{-1} \sigma_{f,n}^2 < 0$ and $F(\lambda = +\infty) = +\infty$; hence, there is one and only one positive root between 0 and $\sigma_u^{-1} \sigma_{f,n}^2$. □
B.2 Proof of Corollary 1

To prove Corollary 1, we need the following preliminary result.

**Lemma 4.** Let $\Delta_n$ be as in Eq. (29). Then,

$$
\Delta_n = \frac{\bar{\Gamma}_n + \bar{\Lambda}_n}{\Lambda_n}.
$$

**(B22)**

**Proof.** Define,

$$
\bar{\Gamma}_n = (M - 1) \Omega_n + \frac{2G}{G} (\Lambda_n - \Omega_n).
$$

**(B23)**

Then,

$$
\text{var} \left( \bar{s} \left| F_{M+1,n} \right. \right) = \text{var} \left( M^{-1} \sum_{i=1}^M \bar{s}_{i,0} \middle| F_{M+1,n} \right)
$$

$$
= M^{-2} \text{var} \left( \sum_{i=1}^M \bar{s}_{i,0} \middle| F_{M+1,n} \right)
$$

$$
= M^{-2} \left[ M \bar{\Lambda}_n + M \cdot \text{cov} \left( \sum_{j \neq i} \bar{s}_{j,n}, \bar{s}_{i,n} \middle| F_{M+1,n} \right) \right]
$$

$$
= M^{-2} \left( M \bar{\Lambda}_n + M \bar{\Gamma}_n \right)
$$

$$
= \frac{(\bar{\Gamma}_n + \bar{\Lambda}_n)}{M},
$$

where the first line follows by the definition of $\bar{s}$. Then, Eq (B22) follows by the previous equality and the definition of $\Delta_n$ in Eq. (29). □

We now proceed with the proof of Corollary 1. First, by Lemma 4 and Eq. (28),

$$
\Delta_{n-1} = \frac{\hat{G} \eta_n}{\theta}.
$$

**(B23)**

By plugging Eq. (19) into Eq. (17) leaves

$$
x_{i,n} = \frac{\beta_n}{\eta_n} [E (f \middle| F_{i,n}) - p_{n-1}] = \frac{\hat{G} \beta_n}{\theta \Delta_{n-1}} [E (f \middle| F_{i,n}) - p_{n-1}],
$$

**(B24)**

where the last equality holds by Eq. (B23). This is Eq. (31).
Next, we prove Eq. (33). We have,

\[ E\left( \sum_{j \neq i} x_{j,n} \mid F_{i,n} \right) = E\left( \sum_{j \neq i} \hat{G}_n \beta_n \bar{s}_{j,n-1} \mid F_{i,n} \right) \]
\[ = \hat{G}_n (M - 1) \phi_n \bar{s}_{i,n-1} \]
\[ = \hat{G}_n \frac{\bar{\Gamma}_{n-1}}{\Lambda_{n-1}} \bar{s}_{i,n-1} \]
\[ = \frac{\bar{\Gamma}_{n-1}}{\Lambda_{n-1}} x_{i,n} \]
\[ = (M \Delta_{n-1} - 1) \bar{x}_{i,n} \]
\[ = \frac{\hat{G}_n}{\theta} (M \Delta_{n-1} - 1) E(f \mid F_{i,n}) - p_{n-1} \]

where the first line holds by the expression for the trading strategy in Eq. (17), the second line follows by the expression for the forecast of the traders’ informational advantage in Eq. (20), the third line follows by Eq. (27), the fourth line follows, again, by Eq. (17), the fifth line follows by Eq. (B22) in Lemma 4, and, finally, the sixth line holds by Eq. (B24).

Finally, we prove Eq. (32). By Eqs. (A11) and (B23),

\[ \bar{\Lambda}_n + \bar{\Gamma}_n = \Lambda_n + 2G\Omega_n \]
\[ + (M - 1) \Omega_n + \frac{2G}{G} (\Lambda_n - \Omega_n) = \Lambda_n + (M - 1) \Omega_n. \]

Hence, we can rewrite Eq. (A4) as

\[ \sigma^2_{f,n} = \frac{\theta^2}{M} (\bar{\Lambda}_n + \bar{\Gamma}_n). \] (B25)

In terms of Eq. (B25), Eq. (B9) is,

\[ \text{var} \left( \sum_{j \neq i} \bar{s}_{j,n} \mid F_{i,n+1} \right) = M \left( \bar{\Lambda}_n + \bar{\Gamma}_n \right) - \left[ 1 + (M - 1)^2 \phi_{n+1}^2 \right] \bar{\Lambda}_n - 2\bar{\Gamma}_n. \]

Next, we substitute \( \phi_{n+1} \) from Eq. (27) in the previous equation, and obtain,

\[ \text{var} \left( \sum_{j \neq i} \bar{s}_{j,n} \mid F_{i,n+1} \right) = M \left( \bar{\Lambda}_n + \bar{\Gamma}_n \right) - \frac{\bar{\Lambda}_n^2 + \bar{\Gamma}_n^2}{\Lambda_n} - 2\bar{\Gamma}_n. \] (B26)

We now compute \( \varsigma^2_{f,n} \) defined in Eq. (30),

\[ \varsigma^2_{f,n} = \text{var} \left( \theta \bar{s} \mid F_{i,n+1} \right) = \text{var} \left( \frac{1}{M} \sum_{i=1}^{M} \bar{s}_{i,0} \mid F_{i,n+1} \right) = \left( \frac{\theta}{M} \right)^2 \text{var} \left( \sum_{j \neq i} \bar{s}_{j,n} \mid F_{i,n+1} \right), \]
where we use \( s_{j,n} = s_{j,0} - t_n \) and the fact that \( t_n \in F_{i,n+1} \) to get the last equality. Plugging Eq. (B26) into the previous equation yields,

\[
\varsigma^2_{f,n} = \left( \frac{\theta}{M} \right)^2 \left[ M (\bar{\Lambda}_n + \bar{\Gamma}_n) - \frac{\bar{\Lambda}_n^2 + \bar{\Gamma}_n^2}{\Lambda_n} - 2\bar{\Gamma}_n \right].
\]  

(B27)

By Eqs. (B25) and (B27), we have

\[
\sigma^2_{f,n} - \varsigma^2_{f,n} = \left( \frac{\theta}{M} \right)^2 \left( \frac{\bar{\Lambda}_n + \bar{\Gamma}_n}{\Lambda_n} \right)^2.
\]

Eq. (32) follows by the previous equality, the expression for \( \sigma^2_{f,n} \) in Eq. (B25), and the expression for \( \Delta_n \) in Eq. (B22). □

C. Computation of the equilibrium

We solve for the equilibrium using backward induction. By Eq. (A5-a), \( \Lambda_n - \Omega_n = \Lambda_0 - \Omega_0 \). We fix a terminal value for \( \Lambda_N \) and compute \( \Omega_N = \Lambda_N + \Omega_0 - \Lambda_0 \). \( \sigma^2_{f,N} \) then follows by Eq. (A4). Since \( \alpha_N = \psi_N = \mu_N = \delta_N = 0 \), we solve for \( \Lambda_N \) in Eq. (23), which yields \( \beta_N \) and \( \gamma_N \) through Eqs. (24)-(25). To compute the value function coefficients as of at time \( N - 1 \), one needs to express \( \Lambda_{N-1} \) and \( \Omega_{N-1} \) in terms of variables known at time \( N \). Below, we show that:

\[
\Lambda_{n-1} = \frac{\theta \Lambda_n - \hat{G} (M-1) \lambda_n \beta_n (\Lambda_n - \Omega_n)}{\theta - GM \lambda_n \beta_n}.
\]

(C1)

Then, \( \Lambda_{N-1} \) is obtained by evaluating Eq. (C1) at \( n = N \), and \( \Omega_{N-1} \) is obtained by the equality \( \Omega_{N-1} = \Lambda_{N-1} + \Omega_0 - \Lambda_0 \). Finally, we retrieve the regression coefficients \( \phi_N \) and \( \eta_N \) through Eqs. (27)-(28) and the equality \( \hat{\Gamma}_{N-1} = \hat{\Gamma}_N + (M-1) (\Lambda_{N-1} - \Lambda_N) \) [see Eq. (A5-e)]. The value function coefficients at \( N - 1 \), then, are uniquely determined by Eq. (26).

The above procedure is applied at each trading round \( n \in [1, N] \), yielding the initial value of \( \Lambda_0 \) implied by the choice of the terminal value of \( \Lambda_N \). Then, the resulting initial value of \( \Lambda_0 \) is compared to that we posited as the initial parameter, and repeat the procedure for different choices of \( \Lambda_N \), until we achieve convergence.

Derivation of Eq. (C1). Taking one lag in Eq. (A4) and substituting the result into Eq. (B13) yields:

\[
\Lambda_{n-1} = \Lambda_n + \frac{\hat{G} \beta_n \lambda_n}{\theta} [\Lambda_n + (M-1) \Omega_{n-1}].
\]

Since \( \Omega_{n-1} - \Omega_n = \Lambda_{n-1} - \Lambda_n \) [see Eq. (A5-a)],

\[
\Omega_{n-1} = \Omega_n + \frac{\hat{G} \beta_n \lambda_n}{\theta} (\Lambda_{n-1} - \Omega_{n-1} + M \Omega_{n-1}) = \frac{\theta \Omega_n + \hat{G} \beta_n \lambda_n (\Lambda_{n-1} - \Omega_{n-1})}{\theta - MG \beta_n \lambda_n}.
\]
By solving for $\Omega_{n-1}$ we find that
\[
\Omega_{n-1} = \frac{\theta \Omega_n + \hat{G}\beta_n \lambda_n (\Lambda_n - \Omega_{n-1})}{\theta - \hat{G}\beta_n \lambda_n M}.
\] (C2)

Finally,
\[
\Lambda_{n-1} = \Lambda_n + \Omega_{n-1} - \Omega_n = \frac{\theta \Lambda_n - \hat{G}\beta_n \lambda_n [M (\Lambda_n - \Omega_n) - (\Lambda_{n-1} - \Omega_{n-1})]}{\theta - \hat{G}\beta_n \lambda_n M},
\]
where we have used Eq. (C2). Eq. (C1) follows by replacing $\Lambda_n - \Omega_n = \Lambda_{n-1} - \Omega_{n-1}$ in the previous equation. \[\Box\]

**Computation of the correlation among volumes.** By definition, the correlation among the volume generated by any two traders $i$ and $j$ is,
\[
\rho_{V,n}(i, j) = \frac{\text{cov}(|x_{i,n}|, |x_{j,n}|)}{\text{var}(|x_{i,n}|)}, \quad \text{for } i \neq j.
\] (C3)

To produce the results in Figure 11, we set $\rho = -0.15$, $\rho = 0.10$ or $\rho = 0.90$. We simulate $S = 5000$ values of the initial signals $(s_i, \theta)_{i=1}^M$, and compute the dynamics of the individual trades over the ten batch auctions. Thus, we obtain a sequence of trades $(x_{i,n}^s)_{s=1}^S$ for each trader $i$ and each batch auction $n$. We estimate $\rho_{V,n}(i, j)$ in Eq. (C3) through the estimator $\hat{\rho}_{S,V,n}(i, j)$ given below,
\[
\hat{\rho}_{S,V,n}(i, j) = \frac{\sum_{s=1}^S \left( |x_{i,n}^s| - \frac{1}{S} \sum_{s=1}^S |x_{i,n}^s| \right) \left( |x_{j,n}^s| - \frac{1}{S} \sum_{s=1}^S |x_{j,n}^s| \right)}{\sqrt{\sum_{s=1}^S \left( |x_{i,n}^s| - \frac{1}{S} \sum_{s=1}^S |x_{i,n}^s| \right)^2} \cdot \sum_{s=1}^S \left( |x_{j,n}^s| - \frac{1}{S} \sum_{s=1}^S |x_{j,n}^s| \right)^2}.}
\]
References


Figures

**Figure 1: Geographical location of traders**
This figure depicts an example of a network of information linkages among $M$ traders who are physically located around a circle. Each circle represents the signal available at the location of each trader. Each trader has $(M-1)/2$ traders to his left and $(M-1)/2$ traders to his right. In this example, every trader has $2G=2$ information linkages. The signal available at the location of the $i$-th trader (the empty circle) is also observed by one trader on his left and one trader on his right.
Figure 2: Overlapping information sets
This figure illustrates how signals overlap across the traders, when there are \( M \geq 11 \) such traders and each of them has \( 2G=4 \) information linkages with his peers. The empty circles denote the signals available at the location of the traders. The filled circles denote the signals every trader observes on top of the signal available at his location. The signals on the left (resp. right) of the empty circles are those that are available at the location of left (resp. right) neighbors.
Figure 3: “Double overlap”
This figure illustrates a “double overlap” in the signals available at the location of the traders. The “double overlap” arises when the number of information linkages among traders is large, i.e. $2G \geq (M-1)/2$. In this figure, the $i$–th trader does not share the signal available at his location with the $(i+k_2)$–th trader, but only with all the traders up to $(i+G)$ and $(i-G)$, e.g., with traders $(i+k_1)$ and $(i-k)$. Yet the $(i+k_2)$–th trader and the $(i - l)$–th trader share at least the signals available at their location. As a result, the $i$–th and the $(i + k_2)$–th traders do indeed share information.
Figure 4: Volume
The dynamics of the volume generated by informed trading in a market with information linkages, seven informed traders, one unit of initial variance of information, one unit of liquidity traders variance across all periods and ten trading rounds. The left-hand side panel depicts the dynamics of volume when the initial correlation among the signals available at the traders’ information linkages is negative (ρ=−0.15). The remaining panels depict the dynamics of volume when this correlation is positive but low (ρ=0.10) (middle panel) and high (ρ=0.90) (right-hand side panel). Every panel displays the dynamics of volume arising when each trader has a number of information linkages equal to 2G, with G=0,1 and 2.
Figure 5: Tightness of beliefs
The dynamics of $\Delta_n$, the tightness of beliefs between the market maker and any trader, in a market with information linkages, seven informed traders, one unit of initial variance of information, one unit of liquidity traders variance across all periods and ten trading rounds. The horizontal line is the inverse of the number of traders, $M^{-1}$. When the tightness of beliefs is sufficiently large, $\Delta_n > M^{-1}$, all traders engage in a rat race. When $\Delta_n < M^{-1}$, traders play a waiting game. Information linkages boost the duration of the rat race. The left-hand side panel depicts the dynamics of $\Delta_n$ when the initial correlation among the signals available at the traders’ information linkages is negative ($\rho=-0.15$). The remaining panels depict the dynamics of $\Delta_n$ when this correlation is positive but low ($\rho=0.10$) (middle panel) and high ($\rho=0.90$) (right-hand side panel). Every panel displays the dynamics of $\Delta_n$ arising when each trader has a number of information linkages equal to $2G$, with $G=0,1$ and 2.
Figure 6: Efficiency

The price-discovery process in a market with information linkages, seven informed traders, one unit of initial variance of information, one unit of liquidity trader variance across all periods and ten trading rounds. The left-hand side panel depicts the price-discovery process arising when the initial correlation among the signals available at the traders’ information linkages is negative (\(\rho=-0.15\)). The remaining panels depict price-discovery process arising when this correlation is positive but low (\(\rho=0.10\)) (middle panel) and high (\(\rho=0.90\)) (right-hand side panel). Every panel displays the price-discovery process arising when each trader has a number of information linkages equal to \(2G\), with \(G=0, 1\) and 2.
Figure 7: Price impacts

Price-impacts in a market with information linkages, seven informed traders, one unit of initial variance of information, one unit of liquidity trader variance across all periods and ten trading rounds. The left-hand side panel depicts the dynamics of price-impacts arising when the initial correlation among the signals available at the traders’ information linkages is negative ($\rho = -0.15$). The remaining panels depict the dynamics of price-impacts when this correlation is positive but low ($\rho = 0.10$) (middle panel) and high ($\rho = 0.90$) (right-hand side panel). Every panel displays the dynamics of price-impacts arising when each trader has a number of information linkages equal to $2G$, with $G = 0, 1$ and $2$. 
Figure 8: Volatility
Asset return volatility in a market with information linkages, seven informed traders, one unit of initial variance of information, one unit of liquidity trader variance across all periods and ten trading rounds. The left-hand side panel depicts the dynamics of return volatility arising when the initial correlation among the signals available at the traders’ information linkages is negative ($\rho=-0.15$). The remaining panels depict the dynamics of return volatility when this correlation is positive but low ($\rho=0.10$) (middle panel) and high ($\rho=0.90$) (right-hand side panel). Every panel displays the dynamics of return volatility arising when each trader has a number of information linkages equal to $2G$, with $G=0, 1$ and 2.
Figure 9: Heterogeneity in the correlation among trades
Correlation among trades in a market with information linkages, seven informed traders, one unit of initial variance of information, one unit of liquidity trader variance across all periods and ten trading rounds. The left-hand side panels depict the dynamics of correlation among trades arising when the initial correlation among the signals available at the traders’ information linkages is negative (ρ=-0.15). The remaining panels depict the dynamics of correlation among trades when this correlation is positive but low (ρ=0.10) (middle panels) and high (ρ=0.90) (right-hand side panels). The top panels depict the correlation dynamics between two close neighbors, i.e. the dynamics of trade correlation between traders i and i-1. The bottom panels depict the correlation dynamics between two distant traders, i.e. the dynamics of the correlation among the trade emanating from traders i and i-3. Every panel displays the dynamics of correlation arising when each trader has a number of information linkages equal to 2G, with G=0, 1 and 2.
Figure 10: Heterogeneity in the correlation among volumes

Correlation among volume in a market with information linkages, seven informed traders, one unit of initial variance of information, one unit of liquidity trader variance across all periods and ten trading rounds. The left-hand side panels depict the dynamics of correlation among volume arising when the initial correlation among the signals available at the traders’ information linkages is negative ($\rho = -0.15$). The remaining panels depict the dynamics of correlation among volume when this correlation is positive but low ($\rho = 0.10$) (middle panels) and high ($\rho = 0.90$) (right-hand side panels). The top panels depict the correlation dynamics between two close neighbors, i.e. the dynamics of volume correlation between traders $i$ and $i-1$. The bottom panels depict the dynamics of volume correlation between two distant traders, i.e. the dynamics of correlation among the volume generated by traders $i$ and $i-3$. Every panel displays the dynamics of the volume correlation arising when each trader has a number of information linkages equal to $2G$, with $G=0, 1$ and 2.
Figure 11: Traders’ expected profits
The traders’ expected profits in a market with seven informed traders, one unit of initial variance of information, one unit of liquidity trader variance across all periods and ten trading rounds. Displayed are the expected profits arising when each trader has a number of information linkages equal to \(2G\), with \(G=0,1\) and 2; and the initial correlations among individual signals equals \(\rho=-0.15, -0.10, 0.01, 0.1, 0.2, \ldots, 0.9\), and 0.99.