Revenue Sharing in a Heterogeneous Federation

Jean Hindriks†  Susana Peralta‡  Shlomo Weber§

September 21, 2005

Abstract

This paper models revenue sharing between asymmetric regions. One of the regions, the core, is a more attractive location for capital than the periphery. When regions compete in capital taxes, we show that, due to the positive effect of limiting harmful tax competition, the revenue sharing is always desirable for the periphery, and is desirable for the core only if the degree of revenue sharing is significant (the so-called J-curve effect of revenue sharing). We then introduce public investments and demonstrate that there is a strategic incentive for under-investment to limit tax competition. While, due to a common pool effect, revenue sharing raises the degree of under-investment, it is, however, still desirable in most cases. We also examine the agglomeration effect that stems from public investment that increase the flow of capital to the region. We show that when the mobility of capital is sufficiently high an extreme agglomeration may arise at equilibrium and the revenue sharing can mitigate this effect.

Keywords: Fiscal Federalism, Revenue Sharing, Common Pool, Agglomeration Forces.

JEL Classification: C72, H23, H70.

---

†CORE, Université Catholique de Louvain.
‡Universidade Nova de Lisboa.
§CORE, Université catholique de Louvain, SMU and CEPR.

*This research was partially supported by the ARC project on Heterogeneity in economics. We gratefully acknowledge helpful comments from Kristian Behrens, Georges Casamatta, Giordano Mion, Pierre Picard, Jacques Thisse and participants of the CORE-IDEI conference on Public Economics (Toulouse, May 2005), the Summer School in the Analysis of Heterogeneity in Social Organizations (CORE, June 2005) and the Workshop on Fiscal Federalism (Barcelona, June 2005)
1 Introduction

Many federal countries have arranged equalization payments schemes by which a central government transfers resources between jurisdictions. These equalization payments are enshrined in the Canadian constitution (see Smart 1996). Similar schemes are used in Australia, Denmark and Switzerland (see Ahmad and Thomas, 1996) and in many developing countries (see Shah, 1994). They also underlie the European Union’s Structural Funds (the Regional Development Fund, the Social Fund, the Financial Instrument for Fisheries Guidance and the European Agricultural Guidance and Guarantees Fund) which amounts to one third of the EU budget between 2000 and 2006 (European Communities, 2004).

Another example is Germany, where in addition to the transfers from the Federal to State governments, there exists a scheme of transfers amongst states. Payments out of states with more than average revenue per capita into those with less than average amount for 50 million DM in 1996 (Spahn and Föttinger, 1997). In the US, the state tax sharing is one of two forms of state intergovernmental aid to local governments. The other form consists of appropriated categorical grant-in-aid. Data show that state intergovernmental aid by each state to its local governments (combined city and county) is the largest element of state expenditures. In 2000 the share of state intergovernmental expenditures in state general revenue was on average 33.2% in the US, and the average for the Southern states was 29.9%. This intergovernmental expenditures includes grant-in-aid, shared taxes and reimbursement for the cost of certain programmes carried out by localities. From 1985 to 2000, payments to local governments have remained at an almost constant percentage of total general expenditures (32% to 35 %).

The alleged purpose of these scheme is an attempt to equalize the citizens access to public services across jurisdictions, i.e., correct fiscal imbalances. Another reason, outlined in a seminal contribution by Boadway and Flatters (1982), is that fiscal equalization schemes can generate efficiency gains by internalizing the fiscal externality (through federal transfers equal to the difference between a jurisdiction’s actual source-based revenue and the average level of the federation). While Boadway and Flatters (1982) assumed the lack of jurisdictions’ incentives to alter tax rates in response to equalization policies, it has been later shown that the efficiency gains are sustained even if this assumption is relaxed. Then the federal planner may design intergovernmental transfers to implement the efficient tax rates at the local level. However, unless there exist lump sum transfers at the federal level, there is no guarantee that all jurisdictions benefit from such transfers and would implement it on a voluntary basis. To address the issue of voluntary inter-regional transfers, Hindriks and Myles (2003) have

---

1 Note that this figure does not include the Common Agricultural Policy.
3 See also Stiglitz (1983) and Dahlby and Wilson (1994).
5 There is also some empirical literature on the relationship between intergovernmental transfers and local tax effort: Baretti et al. (2000), Von Hagen and Hepp (2000), Jha et al. (1999), among others. A more theoretical paper is Bordignon et al. (1996) who show how intergovernmental transfers affect tax enforcement.
shown that symmetric jurisdictions, while competing for a mobile tax base, can voluntary agree to share revenue as a strategic device to limit harmful tax competition. When regions are heterogenous, notably in terms of fiscal revenue, it is no longer clear that they could all benefit from revenue sharing arrangements. Those with low fiscal revenue would benefit while those with high fiscal revenue would bear disproportionate shares of the fiscal burden. This is the first issue we will address in this paper.

The second issue is related to the form of competition between regions. How to attract capital and investments in one jurisdiction? Lighter tax burdens, coupled with public expenditures on infrastructure or other public services that raise fiscal revenue ability, provide a set of policy tools that can influence investment and capital flows. With several policy instruments it is less clear that competitive pressures will lead to a race to the bottom between jurisdictions. For example, while state and local governments in the US vary widely in their public services provision and taxation levels, the competition among these governments has not led to “withering away” of taxation and public expenditures. As for Europe, Baldwin and Krugman (2002) have found that tax revenue in percent of GDP differs substantially between the so-called “core” and “periphery” countries. Their explanation is that firms in the core countries are more likely to pay higher taxes in return for better infrastructure and proximity to a larger market. Using firm level effective tax rates for large Belgian firms, Vandenbussche et al. (2005) find large regional disparities, where the “peripheral” region of Wallonia charges a much lower effective tax rate than the “core” region of Flanders. Head and Mayer (2004) study the choice of location of Japanese firms across Europe and conclude that taxes are far from being the main determinant: the market size and agglomeration externalities play an important role.

In this paper we model the regional competition for capital through taxation and public investments under the heterogenous degree of regional attractiveness for investment. We consider an economy with a total stock of capital that is to be divided between two localities. Each locality provides local public investments. As the public investment increases, the productivity of capital is increased and the locality will attract more capital. This assumption implies that fiscal revenue can be written an increasing function of public investment. There is also a cost to increasing public investments. It is assumed that localities choose their public investments before setting taxes. This simple model shows that flows of capital between localities will induce inefficient choices of taxes and public investments. The reason for this is that the movement of capital between localities reduces the tax base of the locality they leave and increases the tax base they join. These non-market linkages lead to the inefficiency. An important feature of our model is the tax-public investment interaction. In fact, localities that offer a slightly more appealing set of local public investments may appear more attractive even if they tax more capital. If the two issues become entangled in this way then the equilibrium outcome is less clear. Further difficulties with the hypothesis arise when the localities are heterogeneous ex-ante in their capacity to attract capital, as they will in general choose different policies.

We argue that the tax externality problem could be resolved together with the issue of fiscal equalization by using revenue sharing. The method of controlling externalities by
internalizing the fiscal externality ensures that private and social costs of taxation become the same. It seems a simple solution but it is not without its difficulties when there is also public investment decisions to be considered. In fact, revenue sharing has two opposite effects in terms of efficiency. The positive effect is to mitigate the harmful tax competition that leads to inefficiently low tax rates. This positive effect is greater the greater the mobility of the tax base. The negative effect is related to the common pool problem: regions exert efforts that enhance tax revenue which are discouraged by revenue sharing insofar as the benefits (in terms of larger tax revenue) are shared across regions. This negative effect is greater the greater the elasticity of public investment to revenue sharing.\footnote{Careaga and Weingast (2000) argue that when governments have reelection concerns, revenue sharing due to the common pool effect reduces governments incentive to invest in market fostering public goods, and favors transfers to special interest to gain political support.}

Take the extreme case of full revenue sharing. It will clearly eliminate any distortion in tax choices because each locality will internalize the revenue loss it inflicts on other by cutting tax to attract more capital. However this solution will reduce significantly the incentives to invest in local public investments, since all the gains would be shared equally with other localities, while the cost will fall only the locality that raises its public investment. The problem is compounded when localities raise different amounts of tax revenues since then full revenue sharing involves a net transfer from the high revenue to the low revenue localities.

We argue that revenue sharing is desirable under fiscal competition circumstances even if it discourages public investment and if regions are asymmetric. The main challenge is to examine the balance of the two opposite forces. Surprisingly enough, we find that when regions are symmetric (there are no transfers going on across regions in equilibrium), the positive effect always dominates, whatever the mobility of tax base and elasticity of public investment. The reason is that revenue sharing induces less competitive policy choices from the other region, namely more taxation and less public investment.

A further effect of revenue sharing is fiscal equalization: tax sharing is used to absorb fiscal imbalances among regions. Therefore we introduce regional heterogeneity in the model to see if the rich “core” region (with larger tax base) can benefit from sharing its revenue with the poor “periphery” region (with smaller tax base). We study first this issue by assuming away the public investment disincentive from revenue sharing. We then find that if the degree of heterogeneity is low, the fiscal revenue of each region is monotonically increasing with the rate of revenue sharing. However, under high degree of heterogeneity the effect of revenue sharing for the core region is ambiguous. A moderate revenue sharing would result in fiscal revenue loss while a more substantial revenue sharing would result in a gain. The latter is true even though revenue sharing involves redistribution from the core to the periphery.

Adding public investment choice does not change the result that total fiscal revenue increases with revenue sharing. However, public investments bring a surprising observation of a possibility of capital agglomeration. If capital is sufficiently mobile, one region may attract the entire stock of capital by taking advantage of increasing returns in public infrastructure investment. This result offers a novel argument for revenue sharing as a mitigating device to deal with agglomeration effects across regions.

The paper is organized as follows. The next section presents the basic model without...
public investments and derives the equilibrium outcome among heterogenous regions. Section 3 proceeds to the analysis of the impact of revenue sharing on fiscal revenue as a function of the degree of heterogeneity across regions. Section 4 extends the model to account for public investments. Section 5 addresses the issue of agglomeration and, finally, Section 6 concludes. The proofs of all propositions are relegated to the Appendix.

2 The Framework

Suppose there are two regions, called “core” and “periphery”. Both regional governments levy a tax on the mobile tax base (capital). The government in each region has to share a fraction $\alpha \in [0, 1)$ of its revenue with the other region. We assume that the regions correctly anticipate how their tax choices, $t^c$ and $t^p$, respectively, will affect the allocation of capital. Naturally, the core region has an a priori advantage vis-a-vis the periphery in the sense that with identical (effective) taxes the core attracts more capital than the periphery. The attractiveness of the core region is explicitly introduced through the heterogeneity parameter $\epsilon$ that quantifies the relative advantage of the core over the periphery. It has been shown empirically that capital is not perfectly mobile (Feldstein and Horioka, 1980). We capture this idea by using a Hotelling (linear) location model. The repartition of capital is obtained by equating the (effective) net of tax return adjusted for the mobility cost, $\delta > 0$. One can rationalize this mobility cost by supposing, for instance, that physical capital has spatially differentiated returns. Alternatively, capital owners may have a preference for investing close to their residence, e.g., because of an informational advantage. The core is located at 0 and the periphery at 1, such that the fraction of mobile factor in the core region, $x^c = x^c(t^c, t^p) \in [0, 1]$, is derived by the following equation:

\[
(1 - t^c) - \delta(x^c - \epsilon) = (1 - t^p) - \delta(1 - x^c),
\]

or

\[
x^c(t^c, t^p) = \frac{1 + \epsilon}{2} - \frac{t^c - t^p}{2\delta}.
\]

We also define $x^p(t^c, t^p) = 1 - x^c$. Note that if the taxes in both regions are identical, the fractions of the capital attracted is solely determined by the degree of regions asymmetry, i.e., $x^c(t, t) = (1 + \epsilon)/2$ for all tax levels $t$.

Given the revenue share $\alpha \in [0, 1)$, both regions anticipate the allocation of capital and

---

8 A clarification note is in order for the readers who are familiar with the Core-Periphery terminology in New Economic Geography (NEG). In NEG models, the core emerges endogenously as the region where the economic activity agglomerates. We define the core region a priori as the one with a locational advantage, hence, more likely to get a higher share of the economic activity in equilibrium.

9 We thank Pierre Picard for suggesting us these interpretations.

10 It is worth pointing out that the asymmetry between regions makes capital more attracted to one region but does not affect the perceived elasticity of the tax base for the different regions. In most asymmetric fiscal competition models it is the asymmetry in the perceived elasticity of the tax base which is the driving force in generating the asymmetric equilibrium outcome (see Haufler, 2001).
independently choose their tax rates so as to maximize revenue:\footnote{The assumption of revenue maximizing government is a shortcut for describing a situation where residents care sufficiently about the provision of public goods that are financed by tax revenues (see Kanbur and Keen, 1993). What is the appropriate objective function for the principal is ultimately an empirical question. However, it can be argued that if the government maximizes a social welfare function with redistributive objective in mind, then, under revenue constraints, in some cases the optimal policy must be net revenue maximizing. This is true if the welfare gains from higher net revenue are sufficient to offset the losses in welfare due to a net revenue maximizing policy (see Chander and Wilde, 1998).}

\[
\begin{align*}
R_c(t_c, t_p) &= (1 - \alpha)t_c x_c + \alpha t_p (1 - x_c) \\
R_p(t_c, t_p) &= (1 - \alpha)t_p (1 - x_c) + \alpha t_c x_c.
\end{align*}
\]

It is easy to verify that the revenue functions are concave, thus, yielding the following tax response functions,

\[
\begin{align*}
t_c(t_p) &= \frac{(1 + \epsilon}\delta}{2} + \frac{t_p}{2(1 - \alpha)} \\
t_p(t_c) &= \frac{(1 - \epsilon)\delta}{2} + \frac{t_c}{2(1 - \alpha)}.
\end{align*}
\]

Note that the tax choice exhibits strategic complementarity which is reinforced by the revenue sharing. Figure 1 displays the tax best replies. It is immediate to observe that if \(\alpha = \frac{1}{2}\) then the equilibrium fails to exist in absence of fiscal externalities. Moreover, if \(\alpha > \frac{1}{2}\) then the equilibrium tax rates are negative, that indicates “race below the bottom”, where both regions end up with negative revenues. In order to eliminate these pathological situation, we assume that \(\alpha < \frac{1}{2}\) throughout the rest of the paper.

Solving the system of two equations yields the Nash equilibrium in taxes:

\[
t_c^* = \frac{(1 - \alpha)\delta}{1 - 2\alpha} + \frac{(1 - \alpha)\delta\epsilon}{3 - 2\alpha}
\]
Thus, as expected, the core region levies a higher capital tax in equilibrium. Note that our core-periphery set up can be viewed as the case of a wealthy Western country and its poorer Eastern counterpart. Indeed, higher taxation levels in the core region are consistent with the fact that Western Europe countries have higher taxation levels than in the new members of the European Union (see Baldwin and Krugman, 2004 and Krogstrup, 2003).

The equilibrium allocation of capital $x^*_c = x^c(t^*_c, t^*_p)$, is given by

$$x^*_c = \frac{1 + \epsilon}{2} - \frac{\epsilon(1 - \alpha)}{3 - 2\alpha}.$$  

It is easily verified that the fraction $x^*_c$ (and, therefore $x^*_p$) is between 0 and 1 for any value of $\alpha < \frac{1}{2}$ whenever $\epsilon < 2$, which we assume hereafter. It would be useful to summarize the relationship between the cost shares and the equilibrium tax rates:

**Proposition 2.1** Both equilibrium tax rates are increasing in $\alpha$. However, the difference between equilibrium taxes rates $t^*_c - t^*_p$, while positive, declines with $\alpha$.

Thus, the extension of revenue-share arrangements raises the taxes levels in both regions and in the same time mitigates the tax competition between them.

Proposition 2.1, together with (1), immediately implies that the fraction of the capital captured by the core region increases in $\alpha$. Indeed, the gap between the tax rates declines improving the attractiveness of the core region. Thus, the “pre-sharing” fiscal revenue of the core region, $\pi^*_c = t^*_c x^*_c$, is also an increasing function of $\alpha$. To complete our discussion on equilibrium allocation of capital we add:

**Proposition 2.2** The “pre-sharing” fiscal revenue of the peripheral region, $\pi^*_p = t^*_p x^*_p$, increases in $\alpha$. Moreover, the difference between the pre-sharing fiscal revenues $x^*_c t^*_c - x^*_p t^*_p$, while positive, increases with $\alpha$.

We now proceed with the examination of the fiscal revenue implications of the revenue sharing mechanism. Indeed, at an (interior) equilibrium the payoff function of the regions is given by

$$R^*_c(\alpha) = (1 - \alpha)\pi^*_c + \alpha\pi^*_p$$

and

$$R^*_p(\alpha) = (1 - \alpha)\pi^*_p + \alpha\pi^*_c.$$  

Since

$$R^*_c(\alpha) + R^*_p(\alpha) = \pi^*_c + \pi^*_p, \quad \text{and} \quad R^*_c(\alpha) - R^*_p(\alpha) = (1 - 2\alpha)(\pi^*_c - \pi^*_p),$$

Proposition 2.2 immediately yields the following important observation:

**Proposition 2.3** For any degree of mobility and any degree of heterogeneity between regions, the total fiscal revenue increases with revenue sharing whereas the fiscal revenue gap shrinks. In particular, the fiscal revenue of the periphery increases.
We now turn our attention to the core region. Obviously, the fact that total fiscal revenue increases with revenue sharing is not a sufficient condition for an improvement in the core region. We show, however, that if the degree of regions heterogeneity, $\epsilon$, is not too high, then the core (and the periphery) benefit from revenue sharing. However, if the degree of heterogeneity is sufficiently high, then the lower degree of revenue sharing is harmful to the core region. In order to benefit from the revenue sharing, the degree of sharing should be sufficiently high. The intuition here is as follows. Note that the core fiscal revenue is the sum of its own pre-sharing revenue and the net transfer, i.e., $R_c^\alpha = \pi_c^* - \alpha(\pi_c^* - \pi_p^*)$. One sees immediately, by Propositions 2.1. and 2.2, that the first term is increasing, and the second decreasing, in the degree of revenue sharing. When $\epsilon$ is small, then the gap between the core and periphery is small as well, and the impact of the net transfer is not very important. When $\epsilon$ is large enough, then pre-sharing fiscal revenue gap between the two regions is large and the impact of the net transfer is high, especially, by Proposition 2.3, for high $\alpha$. However, the increase in the core pre-sharing revenue more than compensates for the negative impact of the net transfer when $\alpha$ is high enough. Indeed, the higher the degree of internalization of the fiscal externality, the faster the core’s capital share increases with revenue sharing.

**Proposition 2.4** There exists a threshold degree of heterogeneity, $\epsilon_0$, such that:
(i) if $\epsilon < \epsilon_0$, the fiscal revenue of the core region always increases with $\alpha$,  
(ii) if $\epsilon \geq \epsilon_0$, there is a critical revenue share $\overline{\alpha}(\epsilon) \in (0, 1/2)$ such that the fiscal revenue of the core region is decreasing for $\alpha < \overline{\alpha}(\epsilon)$ and increasing for $\alpha \geq \overline{\alpha}(\epsilon)$. Moreover, the threshold $\overline{\alpha}(\epsilon)$ is increasing in the degree of heterogeneity $\epsilon$.

Figure 2 exhibits the fiscal revenue of the core region as a function of revenue sharing for the (maximal) heterogeneity $\epsilon = 2$. The value of $\delta$ is irrelevant since all equilibrium variables are scaled-up by $\delta$ and is set equal to 1. The graph is a J-curve implying that the fiscal revenue of the core region rises when revenue sharing is pushed far enough. This result suggests that large revenue sharing could be accepted by rich regions whenever small ones would be resisted. This argument can support the rejection of the gradual revenue share by the Eastern European countries during the negotiations for their entry to the European Union last year. Indeed, as our argument shows, a low degree of revenue sharing could be harmful to the core countries.

To summarize, revenue sharing raises the total fiscal revenue and reduces the gap between two regions, implying that the periphery benefits from revenue sharing. This is not so surprising as we have assumed away the disincentive effect of revenue sharing on public investments. Perhaps more surprising is the result that the core region can also benefit from revenue sharing, even if its fiscal capacity is much larger, provided that revenue sharing is pushed far enough. In this case the efficiency gain (i.e., relaxing harmful tax competition) from revenue sharing outweighs the cost of transferring resources to the periphery.

We are now in a position to introduce public investments into the analysis. The motivation is threefold. First revenue sharing can discourage public investments which can make it less desirable. Second, local effort is considered a major factor in the assessment of the fairness of revenue sharing arrangements. Third, public investment can create agglomeration forces with all capital located in the region with a larger investment in public infrastructure.
3 Public Investments

We now extend the model to account for public investments. Each regional government levies a tax on a mobile tax base (capital) and invests in public inputs which can raise the productivity of capital. Public investment in infrastructures is a long-term decision variable: we capture this feature by modelling a two-stage game, where in the first stage regions choose non-cooperatively public investment levels \((g^c, g^p)\). The cost of public investment, \(I(g)\), and the productivity of public investment, \(f(g)\), are twice continuously differentiable and increasing. We naturally assume that the cost function is convex while the productivity benefit is concave. To guarantee that the framework of this section is consistent with the analysis of the previous, we assume that \(f(0) = 1\), and in absence of the public investment, the region can rely only on the value of their own capital allocation. We also assume that the level of investment is bounded from above, so the range of possible investments is given by the interval \([0, g_{max}]\).

Both regions anticipate correctly how their public investment levels will affect the tax rate choice \((t^c, t^p)\) of the second period. As we shall see this will give incentive to regions to under-invest (relative to the first-best) in order to reduce the fiscal competition in the next stage.

Both regional governments reckon that the repartition of capital between regions varies with public investment and taxes. Let \(x^c(t^c, t^p, g^c, g^p) \in [0, 1]\) be the fraction of mobile factor invested in the domestic region. Again, by utilizing the Hotelling (linear) location model, we obtain the value of the capital repartition by equating the net of tax productivity adjusted for a regional attachment. That is, \(x^c = x^c(t^c, t^p, g^c, g^p)\) solves

\[
(1 - t^c)f(g^c) - \delta(x^c - \epsilon) = (1 - t^p)f(g^p) - \delta(1 - x^c),
\]

or

\[
x^c = \frac{1 + \epsilon}{2} + \frac{(1 - t^c)f(g^c) - (1 - t^p)f(g^p)}{2\delta}, \tag{2}
\]

where, as before, the parameter \(\epsilon > 0\) captures the initial regional heterogeneity between core and periphery.
Given the revenue sharing arrangement, regions maximize tax revenue net of the cost of public investments. These expressions are given by:

\[ R_c(t^c, t^p, g^c, g^p) = (1 - \alpha)x^c t^c f(g^c) + \alpha(1 - x^c)t^p f(g^p) - I(g^c), \quad (3) \]

\[ R_p(t^c, t^p, g^c, g^p) = (1 - \alpha)(1 - x^c)t^p f(g^p) + \alpha x^c t^c f(g^c) - I(g^p). \quad (4) \]

In this setting, revenue sharing has three different effects. First, it helps to internalize the fiscal externality, thus reducing the negative consequences of fiscal competition. Second, it discourages public investment and, hence, decreases the overall fiscal revenue. The third effect is redistributive: when one region has a higher fiscal revenue, it pays a net transfer to the other.

We first examine the benchmark optimal policy \((t^c_o, t^p_o, g^c_o, g^p_o)\), where a benevolent planner chooses both public investment and taxes in the two regions in order to maximize their joint fiscal revenue. That is,

\[ (t^c_o, t^p_o, g^c_o, g^p_o) = \arg \max_{(t^c, t^p, g^c, g^p)} x^c t^c f(g^c) + (1 - x^c)t^p f(g^p) - I(g^c) - I(g^p). \quad (5) \]

Given that the tax base is inelastic within the federation, it is optimal to fully tax the base (i.e., to set taxes equal to 1). Thus, the share of capital in the core and periphery is \((1 + \epsilon)/2\) and \((1 - \epsilon)/2\), respectively. Therefore, both regions simply equalize their marginal investment benefits of and marginal investment cost. Let us denote by \(\phi(g)\) the ratio of the marginal cost to the marginal productivity of the public investment:

\[ \phi(g) = \frac{I'(g)}{f'(g)}. \]

We conclude that:

**Proposition 3.1** The optimal policy \((t^c_o, t^p_o, g^c_o, g^p_o)\) is attained when

(i) both regions charge the maximal tax \(t^c_o = t^p_o = 1\),
(ii) the optimal investment levels \(g^c_o\) and \(g^p_o\) are determined by

\[ \phi(g^c_o) = \frac{1 + \epsilon}{2} \quad \text{and} \quad \phi(g^p_o) = \frac{1 - \epsilon}{2}. \]

Note that since the function \(\phi\) is increasing, the optimal solution yields a larger investment in the core region, \(g^c_o > g^p_o\).

Using the optimal policy as a benchmark, we now proceed to determine the (subgame perfect) Nash equilibrium in taxes and public investment levels when decisions are devolved to regions.

In the second stage of the tax subgame, regions take public investments \(g^c\) and \(g^p\) as given and independently choose taxes. The capital allocation that results from the Nash equilibrium in taxes is denoted by \(\bar{x}(g^c, g^p)\). The derivation of this allocation is presented in the Appendix, where it is shown that

\[ \bar{x}(g^c, g^p) = \frac{1}{2} + \frac{1}{(3 - 2\alpha)} \left( \frac{\epsilon}{2} + \frac{f(g^c) - f(g^p)}{2\delta} \right). \quad (6) \]
Comparing (2) and (6), it follows that with no revenue sharing ($\alpha = 0$), the equilibrium tax choices change the perceived elasticity of the tax base to public investments. Indeed $\partial x^c/\partial g^c = 3(1 - t^c)\partial \tilde{x}/\partial g^c$. Also the perceived elasticity of the tax base to public inputs will increase with revenue sharing.

In the first stage of the game, regions choose public investment levels correctly anticipating the outcome of the Nash equilibrium tax game. The equilibrium investments, capital allocation, and corresponding tax choices will be denoted by $\hat{g}^c$, $\hat{g}^p$, $\hat{x}^c$, $\hat{x}^p$, $\hat{t}^c$, $\hat{t}^p$, respectively. By using the first order conditions for investment choice (see Appendix for details), we obtain the following inequality for every level of revenue sharing $\alpha \in [0, 1/2)$:

$$\phi(\hat{g}^c) + \phi(\hat{g}^p) = \frac{2(1 - \alpha)^2}{3 - 2\alpha} < 1.$$ 

Since, by Proposition 3.1,

$$\phi(g^c_o) + \phi(g^p_o) = 1,$$

we can state the following result:

**Proposition 3.2** The (subgame perfect) Nash equilibrium involves under-investment even in absence of revenue sharing. The effect of revenue sharing is to further reduce public investments.

It is noteworthy that there is under-investment even with no revenue sharing. The reason is that investment exacerbates competition in the tax subgame. Indeed, by increasing capital productivity, public investments increase the stake of tax competition. This causes the competing region to set a more aggressive (lower) tax rate. Indeed, using (8) in the Appendix, we obtain

$$\frac{dt_i^i}{dg_j^i} = -\frac{f'(g^j)}{\delta} < 0, \quad \text{for} \quad i, j = p, c, \quad i \neq j.$$ 

That is, due to a common pool effect, tax sharing further reduces the investment incentives.

What about equilibrium investment in each region? The following proposition states how the equilibrium public investment levels are related to the distribution of capital and fiscal revenues.

**Proposition 3.3** In equilibrium the region spending more on public investment attracts more capital and generates a higher level of fiscal revenue.

The fact that when the core invests more it has more capital is a consequence of optimal tax choices, as clear from (6). For the periphery region, however, to attract more capital it has to invest sufficiently more to offset its initial deficit. What the proposition says is that, it will only be worth to invest more for the periphery if this extra investment gives it a tax base advantage. It is somewhat surprising that in equilibrium the periphery may end up with more capital and revenue than the core.
4 Impact of revenue sharing

Proposition 3.2 indicates the investment-reducing effect of revenue sharing. As mentioned in the introduction, revenue sharing has also fiscal internalization and fiscal equalization effects. Hence, it is not obvious a priori the impact of revenue sharing on the total fiscal revenue of the federation, as well as on each region individually.\footnote{One may argue that there are costs (e.g. information and administration) to implement the revenue sharing agreement that are not taken into account in our analysis. Nevertheless, if revenue sharing improves fiscal revenue, the regions would have an incentive to spend resources on the implementation of the scheme.}

We now point out that in the symmetric version of our model (when $\epsilon = 0$) each region benefits from revenue sharing. The reason is that, in a symmetric equilibrium, revenue sharing does not involve redistribution between regions and it leads one region to change its policy (which by an envelope argument has only a second order effect on its payoff), while it induces the competing region to act less aggressively by taxing more and making less public investment (with first order effect on payoff):

Proposition 4.1 In a symmetric game, each region benefits from revenue sharing regardless of the degree of mobility and the elasticity of public investment to revenue sharing.

Since public investment creates a negative externality in the other region that is not properly taken into account by the region, one could think that the equilibrium public investment level is too high and thus that revenue sharing is desirable because it brings down the public investment level closer to the optimal level. As we have already shown, this is not true. This finding is illustrative of those in the second-best analysis which say that reducing the number of distortions is not necessarily a good thing. Indeed revenue sharing raises fiscal revenue even though it distorts public investment choice. The reason is that revenue sharing also induces improved tax choices. Two small distortions are preferable to a large one.

This symmetric analysis illustrates clearly the efficiency gains of revenue sharing on tax and public investment levels. We now proceed to the analysis of the asymmetric model where the equity issues will be present. The objective is to see if the efficiency gains of revenue sharing can outweigh its cost for the rich region which then has to transfer more to the poor region than it receives.

In order to illustrate the effect of revenue sharing in an asymmetric game, consider that the public investment productivity is linear, $f(g) = g$ with a quadratic cost function, $I(g) = g^2/2$. The expressions for equilibrium taxes and public investments are found in the Appendix. We can prove analytically that revenue sharing is increases total fiscal revenue(see Appendix for details). When it comes to regional revenue (and regional gap), we use numerical simulations to illustrate the effect of revenue sharing. Figures 3 and 4 show a few illustrative cases for the cases in which the core, respectively the periphery, invests more and attracts more capital. Fiscal revenue levels and gap are shown here; taxes, investment levels and equilibrium capital allocation are found in the appendix.

We may draw two main conclusions from the numerical simulations. Firstly, it is still possible that both regions benefit from revenue sharing in this enriched setting. Secondly, the interaction of investment, tax choices and revenue sharing yields somehow unexpected
results, like revenue sharing exacerbating regional inequalities or not benefitting the poorest region. Figure 3 shows two examples of interior equilibria. In the first case, revenue sharing amplifies regional inequality and is not always beneficial for the poorest region. Looking at equilibrium taxes and investment levels in Figure 6 (in the appendix), we may grasp the intuition for this result. As revenue sharing gets higher, the periphery’s tax rate becomes
Figure 4: Fiscal revenue and revenue sharing when periphery invests more

\[
\epsilon = 0.7, \delta = 0.12 \\
Core Fiscal Revenue
\]

very high and its investment very low, as compared to the policy choices of the core. This causes capital to fly very rapidly to the core and the periphery ends up being worse off. Figure 4 shows two examples of catch-up equilibria, i.e., situations in which the periphery catches up the core by investing more in infrastructures. In both cases, revenue sharing decreases regional inequality and in the first one the periphery (the richest region) benefits
from revenue sharing while in the second the relation is u-shaped. As will be made clear in the next section, equilibria where the periphery attracts more capital are unstable and not unique (there exist two further agglomerated equilibria, one where all capital is invested in the core and another where it is invested in the periphery).

5 Agglomeration vs Dispersion

One of the consequences of investing in public infra-structures is that the stability of equilibria is not guaranteed. This is because public investment on infrastructure attracts capital which in turn raises the fiscal revenue ability of such investments. Consider a small perturbation from a symmetric allocation of capital; it will raise the incentive of the region with more capital to invest in public infrastructures and reduces the incentive of the other region for public investments. This in turn will trigger further reallocation of capital towards the region investing more, increasing further its capital share. A chain reaction is then in place which leads to the progressive agglomeration of capital in a single region. It is important to note that such agglomeration forces are independent of the initial asymmetry between regions. We now turn to the examination of the agglomeration forces. First, we assume away revenue sharing and, for simplicity, utilize the example of the last section with a linear investment productivity, \( f(g) = g \), and a quadratic investment cost function, \( I(g) = g^2/2 \).

Increased spatial concentration of capital

The first effect of the agglomeration forces is the polarization of the spatial distribution of capital. That is, the region with more capital gets to have more than its respective “natural” share ((1 + \( \epsilon \))/2 and (1 - \( \epsilon \))/2 for the core and the periphery, respectively). To see this, use (15) in the Appendix to obtain

\[
\hat{x}_c = \frac{1 + \epsilon}{2} + \frac{1 - 3\delta}{9\delta - 2}, \\
\hat{x}_p = \frac{1 - \epsilon}{2} + \frac{1 - 3\delta}{2 - 9\delta}, \\
\hat{t}_c = \hat{t}_p = 3\delta.
\]

Denote 
\( \tilde{\delta} = \frac{1}{9}, \hat{\delta} = \frac{2}{9}, \bar{\delta} = \frac{1}{3} \).

Equilibrium existence is ensured for \( \delta > \tilde{\delta} \) and the tax solution is interior when \( \delta < \bar{\delta} \). Additionally, under relatively low degree of capital mobility, \( \delta > \hat{\delta} \), there is more capital

---

13 In the New Economic Geography (NEG) literature, agglomeration emerges for low values of the transportation cost. This is sometimes taken as a proxy of infrastructure development (e.g. railways or highways that decrease the cost of distance). This is not to be confounded with the public investments in infrastructure we refer to in this paper, which increases capital productivity, not its mobility. Indeed, the correct analogy is between transport costs in NEG and the mobility cost of capital here. As will be made clear, agglomeration arises for sufficiently low \( \delta \).
invested in the core ($\hat{x}^c > 1/2$) and the reverse occurs when capital mobility is low. It is now readily verified that whenever $\hat{x}^c > 1/2$ we also have $\hat{x}^c > (1 + \epsilon)/2$. Obviously, when the periphery has more capital it is immediate to observe that $\hat{x}^p > 1/2 > (1 - \epsilon)/2$.

Tax and investment public policies increase spatial inequality. This may constitute a case for introducing revenue sharing on equity grounds.

**Full agglomeration of capital**

The second effect of the agglomeration forces is the possibility of full agglomeration of capital in one of the regions, be it the core or the periphery. This may also happen in the context with symmetric regions ($\epsilon = 0$).\(^\text{14}\)

Given the equilibrium taxes, from expression (12) in the Appendix, we determine public investment levels

$$g^c = \frac{2}{3} \hat{x}$$
$$g^p = \frac{2}{3} (1 - \hat{x}).$$

Using (6) gives the best reply functions

$$g^c(g^p) = \frac{3\delta}{9\delta - 1} - \frac{g^p}{9\delta - 1},$$
$$g^p(g^c) = \frac{3\delta}{9\delta - 1} - \frac{g^c}{9\delta - 1},$$

where, to guarantee concavity of the revenue functions, $\delta \geq \hat{\delta}$.

We can state now that (i) public investments are strategic substitutes, and (ii) the slope of the reaction functions is smaller than 1, if $\delta > \hat{\delta}$. When this latter condition is violated, the interior equilibrium is unstable. Figure 5 shows the best replies in both cases.

We summarize our results below.

\(^{14}\) Despite the regional symmetry, we will keep the denominations *core* and *periphery*, for simplicity.
When $\delta \geq \overline{\delta}$, the interior equilibrium is stable and given by $\hat{g}^c = \hat{g}^p = \frac{1}{3}$.

Plugging $\hat{g}^c$ and $\hat{g}^p$ in (6) with $f(\hat{g}^c) = \hat{g}^c$ and $f(\hat{g}^p) = \hat{g}^p$, we obtain: $\hat{x} = \frac{1}{2}$. As for the tax equilibrium, from (7) we have $t^c \hat{g}^c = 2\delta \hat{x}$ and $t^p \hat{g}^p = 2\delta (1 - \hat{x})$. Substituting for $\hat{g}^c$, $\hat{g}^p$ and $\hat{x}$ we have $\hat{t} = \hat{t}^p = 3\delta$.

Therefore, fiscal harmonization emerges as a Nash equilibrium. It follows that an interior solution in taxes requires $\delta \leq \overline{\delta}$.

When $\overline{\delta} < \delta < \delta$, the interior equilibrium is unstable and there are two other equilibria with the complete agglomeration in one of the regions. The equilibrium public investments are given by either $\hat{g}^c = \frac{3\delta}{9\delta - 1} > \hat{g}^p = 0$ or $\hat{g}^p = \frac{3\delta}{9\delta - 1} > \hat{g}^c = 0$.

Plugging $\hat{g}^c$ and $\hat{g}^p$ in (6) with $f(\hat{g}^c) = \hat{g}^c$ and $f(\hat{g}^p) = \hat{g}^p$, we obtain either $\hat{x} = 1$ or $\hat{x} = 0$. Thus, there is complete agglomeration of capital in one region.

When $0 < \delta \geq \underline{\delta}$, the payoff function is convex and, given the public investment level of the other region, it has two local maxima: $g = 0$ and $g = g_{\text{max}}$. We now use (4) to compute the best replies. It turns out that

$$R(g_{\text{max}}; g^p) > R(0, g^p) \quad \text{if and only if} \quad g^p < \hat{g} \equiv \frac{g_{\text{max}}}{2} (1 - 9\delta) + 3\delta.$$ 

The core region prefers to provide a high infrastructure level if infrastructure in the periphery is low; otherwise, it sets $g^c = 0$. A similar reasoning holds for the periphery. Hence, the best replies are

$$t^c(g^p) = 0 \quad \text{if} \quad g^p > \hat{g} \quad \text{and} \quad t^c(g^p) = g_{\text{max}} \quad \text{if} \quad g^p < \hat{g},$$

$$t^p(g^c) = 0 \quad \text{if} \quad g^c > \hat{g} \quad \text{and} \quad t^p(g^c) = g_{\text{max}} \quad \text{if} \quad g^c < \hat{g}.$$ 

Provided that $g_{\text{max}} > \hat{g}$, there are two equilibria, $(0, g_{\text{max}})$ and $(g_{\text{max}}, 0)$.

Then we have the following result:

**Low mobility (Dispersion)** For $\delta \geq \overline{\delta}$, both regions impose the same tax $\hat{t}^c = \hat{t}^p$, make the same investment and raise the same revenue;

**High mobility (Agglomeration)** For $0 < \delta < 2\underline{\delta}$, there is complete agglomeration of capital in either of the regions.

If we now introduce revenue sharing and regional asymmetry in this game, the possibility of agglomeration still arises. With asymmetric regions, the parameter range for which the interior equilibrium is unstable coincides with the one where the periphery invests more and attracts more capital than the core. Again, this will happen for sufficiently high mobility. The expressions for equilibrium taxes and public investments are cumbersome and relegated to the appendix.
Denote
\[ \hat{\delta}(\alpha) = \frac{1 - \alpha}{(3 - 2\alpha)^2}, \]
where \( \hat{\delta}(\alpha) \) is increasing with \( \hat{\delta}(\alpha) = \hat{\delta} \) for \( \alpha = 0 \). Then a sufficient condition for existence of a public investment equilibrium is \( \hat{\delta} \geq \hat{\delta}(\alpha) \). The equilibrium outcome has the following features:

**Low mobility (Dispersion)** For \( \delta > 2\hat{\delta}(\alpha) \), \( \hat{x} > \frac{1}{2} \) and \( \hat{g}^c > \hat{g}^p > 0 \): the core invests more, attracts more capital and so raises more revenue than the periphery;

**High mobility (Agglomeration)** For \( 0 < \delta < 2\hat{\delta}(\alpha) \), either \( \hat{x} = 1 \) and \( \hat{g}^c > \hat{g}^p = 0 \) or \( \hat{x} = 0 \) and \( \hat{g}^p > \hat{g}^c = 0 \): there is complete agglomeration of capital in either the core or the periphery.

Again, agglomeration arises for a sufficient degree of mobility.

6 Conclusion

This paper tackles the issue of revenue sharing between heterogeneous regions. In our framework, revenue sharing has three effects. Two are efficiency-related (the internalization of the fiscal externality and the disincentive to invest in infra-structures) and the third one is redistributive. We show that revenue sharing is desirable in a variety of settings, both for the federation as a whole as for each region individually, even for the region which is a net contributor to the system.

Our framework features the interaction between two policy variables: public investment in infrastructure and capital taxation. These are very different in nature. On the one hand, infrastructure attracts capital while taxes push it away; on the other hand, revenue sharing has a negative impact on infrastructure and a positive one on taxes. Our main result is that, even in the absence of revenue sharing, there is strategic under-investment in infrastructures. This is because infrastructure raises the stake of tax competition and leads the competing region to set a more competitive (lower) tax. As regards revenue sharing, we show that its positive effect on taxes outweighs the negative one on infrastructures, hence it increases total federation fiscal revenue. Although we obtain the result in the symmetric setting (and provide simulations for the asymmetric one), the intuition here is quite general and should hold in general. The change induced by increased revenue sharing in one region’s own policies has no effect on its payoff, since they are chosen optimally. At the same time, increased revenue sharing induces the other region to choose less competitive policies: higher taxes and lower infrastructure investment. Finally, the inclusion of the infrastructure policy has important consequences regarding equilibrium capital allocation. On the one hand, it gives the periphery region the possibility to catch-up, i.e., attract more capital and have a higher fiscal revenue than the core one. On the other hand, we may have complete agglomeration of capital in either the periphery or the core. Agglomeration and catch-up co-exist as equilibria of the game when capital is sufficiently mobile.

The statement by the European Economic and Social Committee\textsuperscript{15} is instrumental in

\textsuperscript{15}Available online at http://www.esc.eu.int/documents/program_ifri_en.pdf
reinforcing the policy relevance of the analysis undertaken in this paper:

_Western members have expressed fear that the new members may represent too much of a burden for their own economies or the European budget._ (...) _The public debate now focuses on wage and tax competition, which would be used by new members to attract production facilities and jobs._ (...) _Companies re-organize (...) in order to benefit both from cost-efficient locations and easy access to expanding markets._

Our results suggest an argument against concerns of “older” EU members’ (core countries) related to the budget consequences of the recent EU enlargement. Indeed, the core is shown to benefit from revenue sharing in a variety of settings, in particular if the degree of sharing is sufficiently large (the so-called J-Curve effect of revenue sharing). On the other hand, the core countries’ should be concerned by the competition from the periphery regions to attract investment. In a context of increasing globalization, capital mobility is likely to increase and the outcome where the periphery gets to attract more investment becomes more likely. Nevertheless, this outcome is an unstable equilibrium and agglomeration may occur in either the core or the periphery.

**Appendix**

It would be convenient to introduce the parameter \( a \) defined by \( \frac{1}{\alpha - a} \). Note that \( a \) is increasing from 1 to 2 when \( \alpha \) increases from 0 to \( \frac{1}{2} \).

**Proof of Proposition 2.1:** The equilibrium tax rates and allocation of capital can be rewritten as

\[
\begin{align*}
t^c_c &= \frac{\delta}{2-a} + \frac{\delta \epsilon}{2+a}, \\
t^p_i &= \frac{\delta}{2-a} - \frac{\delta \epsilon}{2+a}.
\end{align*}
\]

It is straightforward to verify that both expressions increase in \( a \) when \( \epsilon < 2 \), whereas their difference, \( \frac{\delta \epsilon}{2+a} \), declines in \( a \). □

**Proof of Proposition 2.2:** Note that the pre-sharing revenue of the periphery, \( \pi^p_i \), is given by

\[
\pi^p_i = \left( \frac{\delta}{2-a} - \frac{\delta \epsilon}{2+a} \right) \left( \frac{1-\epsilon}{2} + \frac{\epsilon}{2+a} \right),
\]

which can be rewritten as:

\[
\frac{\delta}{2} \left( \frac{1-\epsilon}{2-a} - \frac{(1-\epsilon)\epsilon}{2+a} + \frac{2\epsilon}{4-a^2} - \frac{2\epsilon^2}{(2+a)^2} \right).
\]

By differentiating this expression with respect to \( a \), we obtain

\[
\frac{\delta}{2} \left( \frac{1-\epsilon}{(2-a)^2} + \frac{(1-\epsilon)\epsilon}{(2+a)^2} + \frac{4a\epsilon}{(4-a^2)^2} + \frac{4\epsilon^2}{(2+a)^3} \right).
\]
\[
\frac{\delta}{2} \left( \frac{(2+a)^3 - 4a(2+a)\epsilon + (2-a)^3\epsilon^2}{(2-a)^2(2+a)^3} \right).
\]

But for every \(0 \leq \epsilon \leq 2\) and \(1 \leq a < 2\), we have
\[
(2+a)^3 - 4a(2+a)\epsilon + (2-a)^3\epsilon^2 > (2+a)(2-a)^2 > 0,
\]
implying that the pre-sharing revenue of the periphery, \(\pi^p\), is increasing in \(a\), and in revenue shares as well.

Finally, the gap between pre-sharing revenues of the core and periphery, given by \(\pi^c - \pi^p = \frac{2\delta}{4-a^2}\), is increasing in \(a\).

\[\Box\]

**Proof of Proposition 2.3:** Denote by \(R^c_*(\alpha)\) and \(R^p_*(\alpha)\) the revenue at equilibrium for the core and the periphery, respectively. The substitution for the equilibrium taxes and allocation of capital yields
\[
R^c_*(\alpha) = (1 - \alpha) \left[ \frac{\delta}{2 - a} + \frac{\delta \epsilon}{2 + a} \right] \left[ 1 + \frac{\epsilon}{2} - \frac{\epsilon}{2 + a} \right] + \alpha \left[ \frac{\delta}{2 - a} - \frac{\delta \epsilon}{2 + a} \right] \left[ 1 - \frac{\epsilon}{2} + \frac{\epsilon}{2 + a} \right]
\]
\[
= \delta \left[ \frac{1}{2(2-a)} + \frac{\epsilon}{a(2-a)} + \frac{\epsilon a \epsilon^2}{2(2+a)^2} \right].
\]

\(R^p_*(\alpha)\) is obtained by replacing \(\epsilon\) by \(-\epsilon\), in the above expression. This yields a positive equilibrium gap
\[
R^c_*(\alpha) - R^p_*(\alpha) = \frac{2\delta \epsilon}{a(2+a)},
\]
which is decreasing in \(a\) (and \(\alpha\)). Thus, revenue sharing has equalizing effect on equilibrium regional fiscal revenue levels. The total revenue is given by
\[
R^c_*(\alpha) + R^p_*(\alpha) = \delta \left[ \frac{1}{(2-a)} + \frac{a \epsilon^2}{(2+a)^2} \right].
\]
Therefore, for all \(a \in [1, 2]\) (so \(\alpha \in [0, 1/2]\))
\[
\frac{\partial R^c_*(\alpha) + R^p_*(\alpha)}{\partial a} = \frac{\delta}{(2-a)^2} + \frac{(2-a)}{(2+a)^3} \delta \epsilon^2 > 0,
\]
implying that the total revenue is increasing in the revenue share. \(\Box\)

**Proof of Proposition 2.4** We will utilize the following lemma.

**Lemma 1** For any degree of mobility there exist a threshold degree of heterogeneity \(\tau(\alpha)\) which is increasing in \(\alpha\) with \(\tau(0) = \epsilon_0\) and \(\lim_{\alpha \to 1/2} \tau(\alpha) = \infty\) such that \(\frac{\partial R^c_*(\alpha)}{\partial \alpha} > 0\) for \(\epsilon < \tau(\alpha)\).
**Proof:** Denote
\[ \frac{dR^c}{d\alpha} = H(\epsilon, a) = \frac{1}{2} a^2 (2-a)^{-2} - 2(1+a)(2+a)^{-2} \epsilon + \frac{1}{2} a^2 (2-a)(2+a)^{-3} \epsilon^2 \]

We consider solution of the equation \( H(\epsilon, a) = 0, \bar{\epsilon}(a) \) on the rectangle \( \Omega = [0, K] \times [1, 2-\Delta] \) where \( K \) is a sufficiently large number and \( \Delta > 0 \) is sufficiently small.

By the Implicit Function Theorem we obtain
\[ \frac{d\bar{\epsilon}}{da} = -\frac{H_a(\bar{\epsilon}, a)}{H_t(\bar{\epsilon}, a)}. \]

We have
\[
H_a(\bar{\epsilon}, a) = \frac{1}{2} [2a(2-a)^{-2} + 2a^2(2-a)^{-3}] - 2 \left[ (2-a)^{-2} - 2(1+a)(2+a)^{-3} \right] \epsilon \\
+ \frac{1}{2} [2a(2-a)(2+a)^{-3} - a^2(2-a)^{-3} - 3a^2(2-a)(2+a)^{-4}] \epsilon^2 \\
= 2a(2-a)^{-3} + 2a(2+a)^{-3} \epsilon - 4a(a-1)(2+a)^{-4} \epsilon^2.
\]

The last expression is positive at \( \epsilon = 0 \), and has a positive derivative at that point. Moreover, at \( \epsilon = 2 \), it is equal to
\[
H_a(2, a) = 2a [(2-a)^{-3} + 2(2+a)^{-3} - 8(a-1)(2+a)^{-4}] \\
= 2a(2-a)^{-3} + 12a(2-a)(2+a)^{-4} > 0
\]
with \( H_a(2, a) \to \infty \) as \( a \to 2 \). Furthermore,
\[
H_t(\bar{\epsilon}, a) = -2(1+a)(2+a)^{-2} + a^2(2-a)(2+a)^{-3} \epsilon.
\]

This expression is increasing in \( \epsilon \) with \( H_t(0, a) < 0 \). At \( \epsilon = 2 \) it is equal to
\[
H_t(2, a) = -2(1+a)(2+a)^{-2} + 2a^2(2-a)(2+a)^{-3} \\
= -2(2+a)^{-3} [2 + 3a + (a-1)a^2] < 0,
\]

Thus, the derivative \( d\bar{\epsilon}/da \) is positive on the rectangle \( \Omega = [0, 2-\Delta] \times [1, 2-\Delta] \). Since at \( a = 1, \bar{\epsilon} < 1 \) and \( \lim_{a \to 2} \bar{\epsilon}(a) = \infty \), we complete the proof of the lemma. \( \Box \)

By using Lemma 1 and inverting the increasing function \( \bar{\epsilon}(\alpha) \), we obtain the increasing function \( \bar{\alpha}(\epsilon) = \bar{\epsilon}^{-1}(\alpha) \), which completes the proof of the proposition. \( \Box \)

**Proof of Proposition 3.1:** The differentiation with respect to taxes rates yields:
\[
\frac{\partial (R^c + R^p)}{\partial t^c} = x^c f(g^c) - f(g^c) \left( \frac{t^c f(g^c) - t^p f(g^p)}{2\delta} \right) \\
\frac{\partial (R^c + R^p)}{\partial t^p} = (1-x^c) f(g^p) - f(g^p) \left( \frac{t^p f(g^p) - t^c f(g^c)}{2\delta} \right).
\]

By using the fact that \( x^c \in [0, 1] \) for each of the possible corner solutions, it is straightforward to show that no other \((t^c, t^p) \in \{0, 1\}^2\), rather than \( t^c = t^p = 1 \) can maximize (5). Indeed,
under the full tax coordination, the fiscal externality is fully internalized and so there is no restriction in tax choices.

Given optimal taxes, \( x^c = \frac{1+\epsilon}{2} \), and the optimal public investment levels satisfy

\[
\phi(g^c) = \frac{1+\epsilon}{2} \quad \text{and} \quad \phi(g^p) = \frac{1-\epsilon}{2}.
\]

The optimal levels involve more investment in the core region \( g^c > g^p \). Furthermore, those levels satisfy \( \phi(g^c) + \phi(g^p) = 1 \). □

First order conditions of the tax subgame

The first order conditions are

\[
\begin{align*}
\frac{\partial R^c}{\partial t^c} &= (1 - \alpha)x^c f(g^c) + [\alpha t^p f(g^p) - (1 - \alpha)t^c f(g^c)] \frac{f(g^c)}{2\delta}, \\
\frac{\partial R^p}{\partial t^p} &= (1 - \alpha)(1 - x^c) f(g^p) + [\alpha t^c f(g^c) - (1 - \alpha)t^p f(g^p)] \frac{f(g^p)}{2\delta},
\end{align*}
\]

which are equivalent to

\[
\begin{align*}
(2 - a)t^c f(g^c) &= \alpha + (1 - 2\alpha)x^c, \\
(2 - a)t^p f(g^p) &= \alpha + (1 - 2\alpha)(1 - x^c).
\end{align*}
\]

By solving, we get

\[
\begin{align*}
t^c &= \left(3 + \epsilon + \frac{f(g^c) - f(g^p)}{\delta} \right) \frac{1}{3f(g^c)} \\
t^p &= \left(3 - \epsilon + \frac{f(g^p) - f(g^c)}{\delta} \right) \frac{1}{3f(g^p)}.
\end{align*}
\]

First order conditions of the investment subgame

By using (7) and straightforward algebra, we obtain

\[
\begin{align*}
R^c &= \frac{2\delta}{a} \left( \frac{\alpha}{2 - a} + \bar{x}(\bar{x} - \alpha) \right) - I(g^c), \\
R^p &= \frac{2\delta}{a} \left( \frac{\alpha}{2 - a} + (1 - \bar{x})(1 - \bar{x} - \alpha) \right) - I(g^p).
\end{align*}
\]

The Nash equilibrium in investment solves the first order conditions:

\[
\begin{align*}
\frac{\partial R^c}{\partial g^c} &= \frac{(2\bar{x} - \alpha)f'(g^c)}{2 + a} - I'(g^c) = 0, \\
\frac{\partial R^p}{\partial g^p} &= \frac{(2(1 - \bar{x}) - \alpha)f'(g^p)}{2 + a} - I'(g^p) = 0.
\end{align*}
\]
which can be rewritten as:

\[
\frac{2\bar{x} - \alpha}{2 + a} = \phi(g^c) \tag{12}
\]

\[
\frac{2(1 - \bar{x}) - \alpha}{2 + a} = \phi(g^p).
\]

Note that

\[
\frac{d^2 R^c}{d(g^c)^2} = \frac{f''(g^c)(2\bar{x} - \alpha)}{2 + a} + \frac{a(f'(g^c))^2}{\delta(2 + a)^2} - I''(g^c), \tag{13}
\]

which, when the first order condition is satisfied, can be stated as

\[
f''(g^c)\phi(g^c) + \frac{a}{\delta(2 + a)^2} (f'(g^c))^2 - I''(g^c).
\]

(14)

Then the sufficient conditions for quasi-concavity of the payoff function (hence, existence of an equilibrium in investment levels) are: (i) the value in (14) is non-positive; (ii) the value of (11) is nonnegative when \(g^c = 0\), and (iii) the expression in (11) is continuous in \(g^c\).

**Proof of Proposition 3.3**: First let us rule out the possibility \(g^c = g^p\). Indeed, from (6), if \(g^c = g^p\), the capital allocation is \(\hat{x} = \frac{1}{2} + \frac{\epsilon}{2(2 + a)} > \frac{1}{2}\), which, by (12), implies that \(g^c > g^p\), a contradiction.

Taking the ratio of the two expressions in (12), we have

\[
\frac{2\bar{x} - \alpha}{2(1 - \bar{x}) - \alpha} = \frac{\phi(g^c)}{\phi(g^p)},
\]

implying that \(\phi(g^c) > \phi(g^p)\) if and only if \(\bar{x} > 1/2\). Finally, multiplying both sides of (7) by \(\hat{x}\) and \(1 - \hat{x}\), respectively, and taking the ratio, we have

\[
\frac{t^c f(g^c)x^c}{t^p f(g^p)(1 - x^c)} = \frac{x^c(\alpha + (1 - 2\alpha)x^c)}{1 - x^c(\alpha + (1 - 2\alpha)(1 - x^c))}.
\]

Since the value \(x^c(\alpha + (1 - 2\alpha)x^c)\) is increasing in \(x^c\), we conclude that the region with a higher fiscal revenue also achieves a higher share of capital. \(\Box\)

**Proof of Proposition 4.1** In the symmetric case where \(\epsilon = 0\), there is a symmetric outcome in taxes and public investments. The first order conditions for taxes (7) and public investments (12), evaluated at \(t = t^p, g = g^p, \bar{x} = \hat{x} = 1/2\) yield:

\[
t f(g) = \frac{\delta}{2 - a} \quad \text{and} \quad \phi(g) = \frac{1}{a(2 + a)}.
\]

Since \(\phi\) is increasing, the last expression guarantees that \(g\) is a decreasing function of \(a\), and, as expected, revenue sharing reduces the equilibrium public investment level in each region. Also, note that, due to fiscal internalization, equilibrium taxes are increasing with revenue sharing:

\[
\frac{dt}{da} = \delta \left[ \frac{f(g) - (2 - a)f'(g)\frac{dg}{da}}{f(g)^2(2 - a)^2} \right] > 0.
\]
We now show that revenue sharing is desirable for both regions regardless of the elasticity of public investment to revenue sharing (i.e. for all \( \phi > 0 \) and the degree of mobility (i.e., for all \( \delta > 0 \)). At the symmetric Nash equilibrium, tax sharing cancels out and regional payoff is

\[
R(\alpha) = \frac{tf(g)}{2} - I(g) = \frac{\delta}{2(2 - a)} - I(g).
\]

The result now follows by differentiating this expression with respect to \( a \):

\[
\frac{\partial R(\alpha)}{\partial a} = \frac{\delta}{2(2 - a)^2} - I'(g)\frac{dg}{da} = \frac{\delta}{2(2 - a)^2} + \frac{aI'(g)}{\phi'} > 0
\]

for all values of mobility and elasticity \( \delta, \phi > 0 \). \( \square \)

**Example of Section 4**

We extend the example in Section 4 to allow for revenue sharing. First note that from (13) we have

\[
\frac{\partial^2 R^c}{\partial (g^c)^2} = \frac{1 - \alpha}{\delta(3 - 2a)^2} - 1
\]

which is negative for \( \delta > \delta(\alpha) = \frac{1 - \alpha}{(3 - 2a)^2} = \frac{1}{a(2 + a)} \). Thus, the revenue functions are globally concave if the value of \( \delta \) is bounded away from \( \delta(\alpha) \).

Given revenue sharing, the interior equilibrium outcome is given by

\[
\begin{align*}
\hat{\kappa} &= \delta a_2 + a \delta(2 + a)(2 + a + (2 - a)\epsilon) - 2a \\
\hat{p} &= \delta a_2 + a \delta(2 + a)(2 + a - (2 - a)\epsilon) - 2a \\
\hat{x} &= \frac{1}{2} + \frac{1}{2(2 + a)^2}\delta - 2a' \\
\hat{g} &= \frac{1}{a(2 + a)} + \frac{a\delta\epsilon}{(2 + a)^2}\delta - 2a' \\
\hat{g} &= \frac{1}{a(2 + a)} - \frac{a\delta\epsilon}{(2 + a)^2}\delta - 2a'.
\end{align*}
\]

Plugging the equilibrium values in the payoff functions, we get

\[
\begin{align*}
R^c(\alpha) &= \frac{1}{2} \left( \frac{\delta}{2 - a} - \frac{1}{a^2(2 + a)^2} \right) - a\delta^2 \frac{a - (2 + a)^2\delta}{2} \frac{(2 + a)^2\delta - 2a}{(2 + a)^2\delta - 2a} \epsilon^2 + \frac{\delta}{a(2 + a)} \frac{(2 + a)^2\delta - 2a}{(2 + a)^2\delta - 2a} \epsilon, \\
R^c(\alpha) &= \frac{1}{2} \left( \frac{\delta}{2 - a} - \frac{1}{a^2(2 + a)^2} \right) - a\delta^2 \frac{a - (2 + a)^2\delta}{2} \frac{(2 + a)^2\delta - 2a}{(2 + a)^2\delta - 2a} \epsilon^2 - \frac{\delta}{a(2 + a)} \frac{(2 + a)^2\delta - 2a}{(2 + a)^2\delta - 2a} \epsilon
\end{align*}
\]

Total fiscal revenue is

\[
R^c(\alpha) + R^p(\alpha) = \frac{\delta}{2 - a} - \frac{1}{a^2(2 + a)^2} - a\delta^2 \frac{a - (2 + a)^2\delta}{(2 + a)^2\delta - 2a} \epsilon^2.
\]
and the gap is

\[ R^c(\alpha) - R^p(\alpha) = \frac{2\delta}{a(2 + a)} \left( \frac{2 + a}{(2 + a)^2} - a \epsilon \right). \]  

(16)

Given the above expressions, it is straightforward to derive how revenue sharing influences fiscal revenue levels in each of the regions. First of all, for \( \epsilon = 0 \) we have

\[ \frac{dR^c}{d\alpha} = \frac{dR^p}{d\alpha} = \frac{2(1 + a)}{a^3(2 + a)^3} + \frac{\delta}{2(2 - a)^2} > 0. \]

Hence, when regional asymmetry is not too large, both regions benefit from revenue sharing.

We may also show that total fiscal revenue increases with \( \alpha \). To do so, we use, on the one hand, the fact that \( dR/d\alpha \) and \( dR^p/d\alpha \) are quadratic in \( \epsilon \). On the other hand, to get interior solutions for taxes and capital allocation and non-negative investment levels we must limit the range of admissible \( \epsilon \) (computations available upon request).

Figures 6 and 7 display equilibrium taxes, investment levels and capital allocation for the examples presented in section 4. The dashed (solid) curves refer to the periphery (core). Note that, in most cases, revenue sharing increases taxes and decreases investment levels (the only exception being the left panel of Figure 6).

References


Figure 6: Equilibrium when core invests more

\[ \epsilon = 0.01, \delta = 0.23 \]

\[ \epsilon = 0.3, \delta = 0.31 \]


Figure 7: Equilibrium when periphery invest more

$\epsilon = 0.7, \delta = 0.12$

$\epsilon = 0.2, \delta = 0.2$


