International mergers: Incentives and welfare

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Received 12 September 2003; received in revised form 15 May 2004; accepted 21 December 2004

Abstract

Information asymmetry creates incentives for firms from different countries to merge. To demonstrate this point, we develop a model of international oligopolistic competition under demand uncertainty and asymmetric information. We show that when domestic firms but not foreign firms are completely informed of local market demands, information sharing enhances the profitability of a merger between a domestic firm and a foreign firm. We also examine how such a merger affects the non-merging firms’ profits, consumer surplus and social welfare.

Keywords: International mergers; Cross-border mergers; Merger incentives; Welfare; Information sharing; Output coordination

JEL classification: F12; D82; L49

1. Introduction

International mergers (or cross-border mergers) have recently become profuse.¹ DaimlerChrysler is the most notable example in the auto industry.² What are the benefits of international mergers over domestic mergers? Why and when do firms from different

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1 According to UNCTAD (2000), the value of cross-border mergers and acquisitions rose from less than $100 billion in 1987 to $720 billion in 1999.

2 Other examples include the one between Ford and Mazda, the one between Renault and Nissan, and the one between GM and Saab.
countries have incentives to merge? There is a sizable literature in industrial organization studies examining the profitability of mergers under various conditions. However, these studies, except a few (to be discussed later), provide no particular explanations for international mergers. The purpose of this paper is to provide a new explanation for international mergers in terms of information sharing. We develop an asymmetric information model to analyze the incentives and welfare of international mergers. The results have important implications for both businesses’ strategic positions and governments’ anti-trust policies.

Recent literature on horizontal mergers began with Salant et al.’s (1983) seminal paper, in which they show that, in an oligopolistic industry with homogenous goods, linear demand, constant marginal costs and Cournot competition, a merger is not profitable unless the merger includes more than 80 per cent of the firms. This result does not reflect the realities of merger activity. In subsequent research, Salant et al.’s assumptions are relaxed in various ways to show that mergers are profitable. In the same spirit of these studies, this paper shows that a merger in a Cournot oligopoly is more profitable if the merging firms have asymmetric information about market demand than if they have symmetric information. In our opinion, the asymmetric information model developed in this paper is especially suitable for describing international mergers.

A firm often has better information about the local market than about foreign markets. For example, compared to a foreign firm, a domestic firm is more familiar with local consumer tastes, rules and the culture of the labor market, effective ways of advertising, the distribution network, government regulations, and market interactions between suppliers, consumers and competing firms. This information asymmetry creates incentives for firms from different countries to merge. To demonstrate this point, we develop a model of international oligopolistic competition under asymmetric information. There are domestic firms and one foreign firm. They produce differentiated products and compete in the domestic market with uncertain demand. Before production takes place, the domestic firms are fully informed of the realization of demand, but the foreign firm is not. We argue that a market/contract for information exchange does not exist and, hence, a merger with a domestic firm is the only way for the foreign firm to acquire information. In such a setting, we consider a two-stage game. First, a domestic firm and the foreign firm together decide whether or not to merge. Then, demand is realized and domestic firms are fully informed. In the second stage, all firms produce and compete à la Cournot.

In light of the above-mentioned literature, it is not surprising to show that if all firms are fully informed about demand, then the merger is profitable only when the products are

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3 Church and Ware (2001, chapter 23) and Pepall et al. (2002, chapter 8) are two sources of summaries of the merger literature.

4 For example, a merger that consists of less than 80 per cent of the firms may be profitable if marginal costs are increasing (Perry and Porter, 1985), if there are cost synergies (Farrell and Shapiro, 1990), if products are sufficiently differentiated (Lommerud and Sorgard, 1997) or if competition is Bertrand (Deneckere and Davidson, 1985).

5 We focus on horizontal mergers, i.e., mergers between firms in the same industry. According to UNCTAD (2000), about 70 per cent in terms of value, or 50 per cent in terms of number, of cross-border M&As are horizontal.
sufficiently differentiated. This allows us to investigate the role of information sharing. To this end, we emphasize two features of international mergers, i.e., output coordination and information sharing. In the presence of asymmetric information, a merger enables the two merging firms to share the information about market demand. If they simply share information but do not coordinate output, we find that the (uninformed) foreign firm’s profit increases while all the (informed) domestic firms’, including the merging one’s, profits decrease. We also show that the foreign firm gains more than its merging partner loses, so the merger is profitable overall. Thus, information sharing always facilitates mergers. If the merging firms share information and also coordinate output, then the merger is again profitable only when products are sufficiently differentiated, but the range of product differentiation within which the merger is profitable under asymmetric information is strictly larger than that in the case when all firms are completely informed. This is because there are always gains from information sharing and, for a certain range of product differentiation, within which a pure output coordination merger is not profitable, the information sharing gains overwhelm the losses that are generated by the output coordination.6

This paper also explores how mergers affect the non-merging firms’ profits, consumer surplus, and domestic and global welfare. We show that a merger reduces non-merging firms’ profits when products are sufficiently differentiated, but it always increases the whole industry’s profits. A merger raises consumer surplus and social welfare if and only if products are sufficiently differentiated. Implications for anti-trust policies can be drawn from this part of the analysis. Specifically, we find that when demand uncertainty is large and market competition is intense, international mergers should be encouraged because they are privately unprofitable but socially desirable. Under the opposite conditions, international mergers should be discouraged because firms have incentives to merge, but such mergers reduce social welfare.7

The explanation for international mergers given in this paper is new and different from explanations given by other researchers. Long and Voussden (1995) investigate the profitability of cross-border mergers in the presence of trade liberalization. They show that the result depends on whether the trade liberalization is unilateral or bilateral and on how large the cost savings generated from the mergers can be. Horn and Persson (2001) use the coalition formation approach to analyze international mergers. They

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6 We build our model on the literature on information sharing in oligopoly. Important contributions to this literature are made by Novshek and Sonnenschein (1982), Clarke (1983), Vives (1984), Gal-Or (1985), Li (1985), Shapiro (1986) and Raith (1996). These papers concentrate on a firm’s incentives to share its private information with competing firms, but they do not consider mergers. In particular, they show that firms competing in quantities are not willing to reveal their private information about market demand. Hence, it is interesting to know whether and how mergers affect firms’ willingness to reveal information. We show that a merger makes a firm willing to share its private information about demand with its merging partner even under Cournot competition.

7 Horn and Persson (2001) are also interested in the conflict between the private and social incentives for mergers. They find that private and social incentives for mergers may differ with weak merger synergy, but converge if the synergy is strong. Head and Ries (1997) are mostly concerned about the welfare implication of mergers. By focusing on mergers that raise prices and reduce world welfare, they show that a national government can be relied on to block a world welfare-reducing merger if the merger does not generate cost savings.
show that international mergers may arise due to lower trade costs, contrary to the “tariff jumping” argument. Lommerud et al. (2005, in press) explain international mergers as a result of oligopolistic competition in the presence of plant specific unions. They argue that unions are plant specific in the international setting and, hence, international mergers are profitable because wages decrease after the mergers. More recently, Neary (2004) uses a general equilibrium model to show that international differences in technology generate incentives for cross-border mergers in which low-cost firms from one country take over high-cost firms from another country. Such mergers serve as instruments of comparative advantage.8

The present paper is also related to two other studies on mergers with asymmetric information. Gal-Or (1988) shows that mergers may create informational disadvantages to the merging firms under Cournot competition. Our model differs from Gal-Or (1988) in two important aspects. First, while she considers the case when every firm has partial private information about demand, we consider the case when all domestic firms are fully informed while the foreign firm is not. Her case better describes the information structure among domestic firms, but our case is more suitable to characterize the information asymmetry between domestic and foreign firms. Because of this difference, we obtain a different result: the merging firms as a whole always benefit from information sharing even under Cournot competition. Second, although firms produce differentiated products, Gal-Or (1988) assumes that after the merger, only one product is produced. In contrast, the merging firms in our model continue to produce two differentiated products after the merger.

Das and Sengupta (2001) consider private information about both demand and costs. They argue that asymmetric information is always a barrier to mergers. In sharp contrast, we show that asymmetric information is always conducive to mergers. The reason for the different conclusions lies in the assumptions on how information is used in their model and ours. In their model, two firms bargain on a merger deal and each uses its private information to affect the bargaining outcome, but in our model, two firms share information when they merge. In their model, firms receive respective market information before they decide on a merger, but in our model, the reversed sequence is assumed.9

The rest of the paper is organized as follows. In Section 2, we present the basic model of international trade under oligopolistic competition and asymmetric information. In Section 3, we focus on output coordination mergers by assuming symmetric information. In Section 4, we bring asymmetric information back to the model in order to examine the implications of asymmetric information on mergers. In Section 5, we explore mergers’

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8 In the international trade literature, most studies are concerned with trade and competition policies in the presence of mergers. In particular, researchers in this area are interested in questions such as how trade policies and/or competition policies should respond to mergers, and what the effects of mergers under various policy regimes are. Examples include Ross (1988), Levinsohn (1996), Richardson (1999), Horn and Levinsohn (2001) and Collie (2003). Unlike these studies, we focus on the incentives for international mergers and the welfare effects of such mergers.

9 Banal-Estanol (2002) investigates incentives to merge when firms have private information about costs, but not about demand.
welfare effects. In Section 6, we discuss the robustness of the main results. Section 7 concludes the paper. All proofs are contained in the Appendix A.

2. The model

In this section, we describe the model under a set of assumptions. In Section 6, we explore implications of relaxing some of these restrictive assumptions. We consider an industry that consists of \( n \) domestic firms and one foreign firm.\(^\text{10}\) The foreign firm competes against all the domestic firms in the domestic market by exporting its product to the market. The foreign firm is indexed by 0 and the domestic firms are indexed by \( i \in \mathbb{N} = \{1, 2, \ldots, n\} \). Hence, \( \mathbb{N} \) is the set of all domestic firms, and \( M = \{0\} \cup \mathbb{N} \) is the set of all firms. Assume that firms produce differentiated products and the market demand is given as

\[
 p_i = a + \theta - q_i - bQ - i, \quad i \in M,
\]

where

\[
 p_i \text{ is the price of product } i,
\]

\[
 q_i \text{ is the output of product } i,
\]

\[
 a \text{ is a constant, which is assumed to be sufficiently large so that all firms produce positive amounts in equilibrium,}
\]

\[
 b \in (0,1) \text{ is a constant capturing the extent of product differentiation,}
\]

\[
 Q \text{ is the total output of all firms other than } i,
\]

\[
 \theta \text{ is a random variable with zero mean and variance } \sigma^2 = \text{Var}(\theta) = E(\theta^2).
\]

Hence, \( \sigma^2 \) captures demand fluctuations.

We consider a two-stage game as follows. In the first stage, one domestic firm, say firm 1, denoted as \( F_1 \), and the foreign firm, denoted as \( F_0 \), together decide whether or not to merge. In the second stage, all firms produce and compete in the market à la Cournot (see Section 6 for justifications for our focus on Cournot competition). During the transition from stage 1 to stage 2, uncertainty about \( \theta \) is realized and all domestic firms learn the exact value of \( \theta \). However, the foreign firm continues to be ignorant unless it merges with a domestic firm in the first stage. In Section 6, we argue that, in this model, there exists no market for information transaction and, hence, a merger is the only channel for information acquisition.

We derive and analyze the subgame perfect Nash equilibrium (SPNE) of the above-described game. To abstract away from merger incentives arising from cost synergies, we assume that all firms have zero marginal cost of production and that there is no trade and transportation cost.\(^\text{12}\) Without a cost differential, we define a merger between \( F_1 \) and \( F_0 \) as sharing information and coordinating their output to maximize joint profits. A merger is profitable if and only if the sum of the profits of \( F_1 \) and \( F_0 \) under the merger is greater than that under separate firms. Whenever it is profitable, a monetary transfer can be arranged between \( F_0 \) and \( F_1 \) such that both benefit from the merger. Having assumed that, we focus only on merger incentives and pay no attention to the amount of transfer that is necessary to make the merger a reality.

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\(^\text{10}\) Since this study focuses on the incentive to merge between an uninformed foreign firm and an informed domestic firm in an oligopolistic market, it should be clear that our analysis and results should not be altered qualitatively if we allow more than one foreign firm to exist in the model.

\(^\text{11}\) Implicitly, we also assume that \( \theta \) has finite support, say \([\theta_L, \theta_U]\), and \( a \) is large enough that, even at \( \theta = \theta_L \), all firms have positive output. In this particular model, it turns out that we need to assume \( \theta_L > -\frac{(2+bn-b)a}{2+bn} \).

\(^\text{12}\) It is worth pointing out that our model of product differentiation with constant marginal costs is analytically equivalent to an alternative model of homogeneous goods with increasing marginal costs.
3. Mergers under symmetric (complete) information

In this section, we assume that all firms (including the foreign firm) have complete information about \( \theta \) before production takes place. This allows us to focus on mergers for output coordination, called an output coordination merger. When \( F_0 \) and \( F_1 \) merge in the first stage, they make output decisions to maximize their expected joint profits.

Suppose there is no merger in the first stage. Then, in the Cournot game all firms have the same equilibrium output and profit:

\[
q^* = \frac{a + \theta}{2 + bn} \quad \text{and} \quad \pi^* = \frac{(a + \theta)^2}{(2 + bn)^2}.
\]

(1)

Suppose now that \( F_0 \) and \( F_1 \) merge in the first stage. Then, in the second stage, the merged entity maintains the two separate product lines but chooses \( q_0^c \) and \( q_1^c \) to maximize the joint profits, \( (p_0^c q_0 + p_1^c q_1) \). The market equilibrium is

\[
q_c^i = \frac{a + \theta}{2 + bn - b^2}, \quad \pi_c^i = \left( \frac{q_c^i}{2 + bn - b^2} \right)^2, \quad i \in \{2, \ldots, n\}.
\]

(2)

Direct comparison based on (1)–(3) yields the difference in total profits of the merged entity before and after the merger:

\[
\Delta \pi^c = (\pi_0^c + \pi_1^c) - (\pi^* + \pi^*) = \frac{b^2(a + \theta)^2 Y(n, b)}{2(2 + bn)^2(2 + bn - b^2)^2},
\]

where \( Y(n, b) = n^2b^3 - (3n^2 - 4n + 4)b^2 - 4(n - 1)b + 4 \). We establish the following result.

**Proposition 1.** Suppose there is symmetric (complete) information among all firms.

(i) For any given \( n \), there exists a unique \( b_0(n) \in (0, 1) \) such that, for \( b < b_0 \), the SPNE is that the merger occurs in the first stage with the second-stage market outcomes \( \{q_0^c, q_1^c, \ldots, q_n^c\} \), and, for \( b \geq b_0 \), the SPNE is that the merger does not occur in the first stage and all firms produce \( q^* \) in the second stage. Moreover, \( b_0(n) \) decreases with \( n \).

(ii) In comparison, \( q_0^c < q_1^c < q^* \), \( q_i^c > q^* \), and \( \pi_i^c > \pi^* \) for \( i \in \{2, \ldots, n\} \).

The above proposition says that a merger is more likely to be profitable if products are more differentiated and the number of firms in the market is fewer. Moreover, after the merger, the two merging firms produce less than before, while the non-merging firms produce more and have higher profits than before.

\( F_0 \) and \( F_1 \) will merge if the merger can increase their joint profits. Without the merger, all firms behave just like in a usual Cournot Nash game in which they compete aggressively. Intensive competition creates negative externalities among the firms. When \( F_0 \) and \( F_1 \) engage in an output coordination merger, they reduce or eliminate the negative
externalities between themselves by producing less. Due to strategic substitution, non-merging firms will raise their output and benefit from the reduced competition. Although $F_0$ and $F_1$ benefit from internalizing the negative externalities between themselves, they suffer a loss because non-merging firms increase their output. Hence, output coordination mergers do not guarantee larger profits for the merged entity. Proposition 1 shows that output coordination mergers bring the merged entity more benefit than harm if and only if the products are sufficiently different. The conventional result that mergers are not profitable under Cournot competition (Salant et al., 1983) is a special case of Proposition 1 for $b = 1$.\(^{13}\)

4. Mergers under asymmetric information

We now return to the asymmetric information case. In order to understand the role of information sharing in international mergers, we assume in Subsection 4.1 that when a merger occurs in the first stage, $F_1$ shares its information with $F_0$, but, in the second stage, they compete in the market as if they were still independent firms. We call this type of merger an information sharing merger. In Subsection 4.2, we investigate an individual firm’s incentives for information revelation and acquisition without mergers. Finally, in Subsection 4.3, we analyze full-fledged mergers in which $F_0$ and $F_1$ share information and coordinate output.

4.1. Merger for information sharing

4.1.1. Second-stage analysis

Suppose there is no merger in the first stage. Then, we have the usual Cournot game with $F_0$ having incomplete information in the second stage. It is easy to obtain the solution to the game:

$$q^u_0 = \frac{a}{2 + bn} \quad \text{and} \quad q^u = \frac{a}{2 + bn} + \frac{\theta}{2 + bn - b}.$$  \(4\)

$F_0$’s and $F_1$’s realized profits are

$$\pi^u_0 = (q^u_0)^2 + \frac{(2 - b)a\theta}{(2 + bn)(2 + bn - b)} \quad \text{and} \quad \pi^u = (q^u)^2.$$  \(5\)

We next suppose that $F_0$ and $F_1$ engage in an information sharing merger in the first stage, in which $F_1$ reveals information to $F_0$. Then, the second stage game becomes the usual Cournot game with complete information, i.e., all firms (including $F_0$) know the realization of $\theta$. This has been derived in (1) and can be rewritten as:

$$q^s_0 = q^s = \frac{a + \theta}{2 + bn} \quad \text{and} \quad \pi^s_0 = \pi^s = \frac{(a + \theta)^2}{(2 + bn)^2}.$$  \(6\)

\(^{13}\) In fact, Lommerud and Sorgard (1997) have also reached the same result.
4.1.2. Information sharing and the first-stage analysis

In the first stage, \( F_0 \) and \( F_1 \) decide whether or not to merge in order to share information. The necessary and sufficient condition for a merger is that the merged entity’s expected profits must be greater than the sum of \( F_0 \)’s and \( F_1 \)’s expected profits without the merger. Using (5) and (6), the comparison is reduced to

\[
E[(\pi_0^s + \pi^s) - (\pi_0^u + \pi^u)] = \frac{\sigma^2 Z(n, b)}{(2 + bn)^2(2 + bn - b)^2} > 0, \tag{7}
\]

where \( Z(n, b) = (2 + bn - 2b)^2 - 2b^2 \). Note \( \partial Z(n, b)/\partial n > 0 \) and \( Z(2, b) = 4 - 2b^2 > 0 \) except at \( b = 1 \). We have \( n \geq 2 \) and so \( Z(n, b) > 0 \). The collective profit of the merged entity is always higher than the sum of the two firms without the information sharing merger. Provided that there is a mechanism for appropriate inter-firm profit transfer, \( F_0 \) and \( F_1 \) always choose to merge.

**Proposition 2.** Suppose that the merging firms (\( F_0 \) and \( F_1 \)) only share information but do not coordinate output.

(i) The SPNE is characterized as below: \( F_0 \) and \( F_1 \) merge in the first stage; \( F_0 \) produces \( q_0^s \); and every domestic firm produces \( q^s \). The merged entity’s profit is \( (\pi_0^s + \pi^s) \), and every other domestic firm’s profit is \( \pi^s \).

(ii) For a larger \( \sigma^2 \), a smaller \( n \) (except when \( n = 2 \)), or a smaller \( b \), the net profit gains from the merger are larger. More precisely,

\[
\frac{\partial E[(\pi_0^s + \pi^s) - (\pi_0^u + \pi^u)]}{\partial \sigma^2} > 0;
\]

\[
\frac{\partial E[(\pi_0^s + \pi^s) - (\pi_0^u + \pi^u)]}{\partial n} < 0 \quad \text{ (for } n \geq 3 \text{)};
\]

\[
\frac{\partial E[(\pi_0^s + \pi^s) - (\pi_0^u + \pi^u)]}{\partial b} < 0.
\]

We explain the intuition for Proposition 2 at the end of Subsection 4.3.

4.2. Incentives for information revelation and acquisition

Even without a merger, will any informed domestic firm voluntarily reveal its private information to the uninformed \( F_0 \)? Does the uninformed \( F_0 \) benefit from getting more information? We search for answers to these questions in this subsection. Let us compare (5) and (6).

\[
E(\pi_0^s - \pi_0^u) = \frac{\sigma^2}{(2 + bn)^2} > 0, \tag{8}
\]
\[ E(\pi^s - \pi^u) = -\frac{b(4 + 2bn - b)s^2}{(2 + bn)^3(2 + bn - b)^2} < 0. \] (9)

Hence, we establish the following result.

**Proposition 3.**

(i) In the model with one uninformed foreign firm and \( n \) informed domestic firms, the foreign firm always wants to acquire information about the demand, but in the absence of a merger, the domestic firms are not willing to reveal the information.

(ii) For a larger \( \sigma^2 \) or a smaller \( n \), the uninformed foreign firm’s gain from acquiring information becomes larger and the loss to each informed domestic firm from revealing information, if it does, also becomes larger. For a smaller \( b \), the foreign firm’s gain is larger, but the domestic firms’ loss may be larger or smaller. More precisely,

\[
\frac{\partial E(\pi^u_0 - \pi^u)}{\partial \sigma^2} > 0, \quad \frac{\partial E(\pi^s_0 - \pi^u_0)}{\partial n} < 0, \quad \frac{\partial E(\pi^s - \pi^u)}{\partial b} < 0;
\]

\[
\frac{\partial E(\pi^s - \pi^u)}{\partial \sigma^2} < 0, \quad \frac{\partial E(\pi^s - \pi^u)}{\partial n} > 0,
\]

\[
\frac{\partial E(\pi^s - \pi^u)}{\partial b} < 0 \quad (\text{for small } b), > 0 \quad (\text{for large } b).
\]

Hence, as indicated by part (i) of the proposition, information sharing benefits the uninformed firm, but hurts all informed firms. Without the information, \( F_0 \) under produces when actual demand is high, but over produces when actual demand is low. With the information, however, it is able to produce more accurately according to the demand, which creates a positive value for \( F_0 \). In contrast, without revealing information, the informed domestic firms benefit from the foreign firm’s underproduction (when demand is high), but lose from its overproduction (when demand is low). The gain from not revealing information more than compensates for the loss. Hence, in the absence of an information sharing merger in the first stage, no domestic firm will reveal information to \( F_0 \) and the equilibrium is given by (4) and (5).

To understand the effect of information sharing on profit changes, note that \( \pi_0 = p_0q_0 \) for \( F_0 \) and \( \pi_i = p_iq_i \) for the domestic firms, where the price functions are \( p_0 = a + \theta - q_0 - bnq \) and \( p_i = a + \theta - q_i - b(n - 1)q_j - bq_0 \). Let us examine \( F_0 \)’s profit change first. With demand fluctuation, \( F_0 \)’s price also fluctuates but its output does not in the absence of information sharing. However, when it receives the information, \( F_0 \) produces output according to the realized demand and so its output and price moves accordingly. Since \( q_0^s \) and \( p_0^s \) move in the same direction, the ability to move creates a positive value for \( F_0 \). \( F_0 \)’s gain from information acquisition is positively correlated with the degree of price fluctuation under information sharing. The fluctuation is captured by

\[ a + \theta - bnq^s = 2(a + \theta)/(2 + bn) \] from \( F_0 \)’s price function.
In contrast, both the output and price of a domestic firm fluctuate as demand changes, with or without information sharing. However, due to F0’s ability to adjust its output in the case of information sharing, a domestic firm’s fluctuation of output and price is smaller with information sharing than without. This reduction in fluctuation lowers a domestic firm’s expected profits. A domestic firm’s loss from information revelation is positively correlated with the degree of the reduction in its price fluctuation. Basically, if demand fluctuates more, the private information for the informed domestic firms also becomes more valuable and it is also more desirable for F0 to acquire it.

With the above understanding, the intuition behind part (ii) of Proposition 3 becomes clear.

A well-established literature on information sharing under oligopoly has shown that firms have no incentives to reveal their private information about the market demand if they compete in quantities (see, for example, Gal-Or, 1985). Our Proposition 3 confirms this result and goes further to show that the uninformed firm has incentives to acquire the information. Moreover, it shows how various parameters (the degree of demand fluctuation, the market structure and product differentiation) affect the incentives. Our Proposition 2 adds to the literature by showing that the uninformed firm’s gain from information sharing outweighs the loss to an informed firm, which provides incentives for them to engage in an information sharing merger.

The intuition behind such a result in Proposition 2 is as follows. Output fluctuates because of \( \theta \), and informed firms benefit from the fluctuation. Before the merger, however, F0 does not gain from the fluctuation. F1’s gain is proportional to the degree of the fluctuation, by a factor of \( 1/(2 + b(n - 1))^2 \). After the information sharing merger, each firm including F0 gains from the fluctuation by a factor of \( 1/(2 + bn)^2 \). Compared with the case without the merger, F1’s gain is smaller, but F0’s gain is larger with the merger. The final comparison rests on that between \( 2/(2 + bn)^2 \) (with a merger) and \( 1/[2 + b(n - 1)]^2 \) (with no merger), which is equivalent to the sign of \( Z(n,b) \). We have shown that \( Z(n,b) > 0 \) except at \( b = 1 \) and \( n = 2 \), but \( Z(2,1) = 0 \). That is, the total gain to the merging firms from the output fluctuation is greater than F1’s gain in the absence of a merger.

### 4.3. Merger for information sharing and output coordination

In this subsection, we examine the full-fledged merger under asymmetric information. We have already obtained the expressions of all the equilibrium quantities and profits before a merger (in Subsection 4.1) and after a merger (in Section 3). Thus, letting \( \Delta \pi^{a} = (\pi^{a}_{0} + \pi^{a}_{1}) - (\pi^{a}_{0} + \pi^{a}_{1}) \) denote the profit differential for the merged entity, we obtain

\[
E(\Delta \pi^{a}) = \frac{1}{(2 + bn)^2} \left[ \frac{\sigma^2 Z(n,b)}{(2 + bn - b)^2} + \frac{b^2(a^2 + \sigma^2) Y(n,b)}{2(2 + bn - b^2)^2} \right].
\]

\[\text{14 However, both Kirby (1988) and Hwang (1994) show that firms may have a mutual incentive to share their information, depending on the properties of their cost and demand functions.}\]
We can show that for any given $n(\geq 2)$, there exists a unique $b_1(n) \in (0,1)$ such that

$$
\begin{align*}
E(\Delta \pi^n) &> 0 \quad \text{for all } b \in [0, b_1) \\
E(\Delta \pi^n) &= 0 \quad \text{at } b = b_1 \\
E(\Delta \pi^n) &< 0 \quad \text{for all } b \in (b_1, 1].
\end{align*}
$$

(10)

Moreover, $b_1(n) > b_0(n)$. Thus, we establish the following proposition.

**Proposition 4.** In the SPNE under asymmetric information, for any given $n$, there exists a unique $b_1(n) \in (b_0(n), 1)$. If $b < b_1$, then $F_0$ and $F_1$ merge in the first stage, with the second-stage market outcomes $\{q^0_0, q^1_1, \ldots, q^n_n\}$ as given in (2) and (3). If $b \geq b_1$, then $F_0$ and $F_1$ do not merge in the first stage, with the second-stage market outcomes $\{q^u_0, q^u_1, \ldots, q^u_n\}$ as given in (4).

Proposition 4 says that a merger is profitable if and only if products are sufficiently differentiated. Since $b_1 > b_0$, a merger occurs more often under asymmetric information than under symmetric information.

5. Welfare analysis

We have so far examined firms’ incentives for mergers and now we investigate the welfare implications of mergers under asymmetric information. In particular, we want to know how mergers affect total industrial profits, consumer surplus, and social welfare. The results are summarized in Section 5.4 where the policy implications are also discussed.

5.1. Industrial profits

In previous sections, we have shown that under certain conditions, the joint profit of the merging firms is increased after the merger. The non-merging firms, however, can be affected differently.

Look at the information sharing merger first. Eq. (9) indicates that every non-merging firm’s profit drops after $F_1$ reveals the information to $F_0$. We can show that $F_0$’s gain is larger (smaller) than the total loss to all informed firms if $b$ is small (large). Next, in the case of an output coordination merger under symmetric information, the market competition is reduced after the merger. Hence, total industrial profits increase. Finally, we

<table>
<thead>
<tr>
<th>Type of merger</th>
<th>Industrial profit</th>
<th>Consumer surplus</th>
<th>Global welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output coordination</td>
<td>+</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Information sharing</td>
<td>+ for small $b$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Full-fledged</td>
<td>+</td>
<td>+ for small $b$</td>
<td>+ for small $b$</td>
</tr>
</tbody>
</table>
examine the net effect of the merger under asymmetric information and summarize the comparison in the following proposition.

**Proposition 5.** In a market with one uninformed foreign firm and \( n \) informed domestic firms, if a domestic firm and the foreign firm merge, total industrial profits increase when the market is not too competitive (more precisely, \( n < 20 \)).

5.2. Consumer surplus

Next, we look at the changes in consumer surplus due to a merger. In the beginning of Section 2, we specified the demand functions, which can be derived from a representative consumer’s utility function as given below:

\[
U = (a + \theta) \sum_i q_i - \frac{1}{2} \sum_i q_i^2 - \frac{b}{2} \sum_i \sum_{j \neq i} q_i q_j
\]

\[
= (a + \theta) \sum_i q_i - \frac{1}{2} \sum_i q_i^2 - \frac{b}{2} \left[ \left( \sum_i q_i \right)^2 - \sum_i q_i^2 \right].
\]

Consumer surplus is defined as the net benefit from consumption: \( CS = U - \sum_{i=0}^n p_i q_i \).

By comparing the consumer surplus without any merger (\( CS_N \)) to the consumer surplus under the information sharing merger (\( CS_S \)), we obtain

\[
E(CS_S - CS_N) = \frac{b^2(3 - b)n^2 + b(8 - 5b + b^2)n + (2 - b)^2}{2(2 + bn)^2(2 + bn - b)^2} \sigma^2 > 0.
\]

Hence, information sharing between the firms unambiguously benefits consumers. By comparing \( CS_S \) to the consumer surplus under the full-fledged merger (\( CS_M \)), we also have

\[
E(CS_M - CS_S) = b(a^2 + \sigma^2) \frac{F}{4(2 + bn)^2(2 + bn - b)^2} < 0, \quad \text{where} \quad F = -b^2(8 - 5b + b^2)n^2 - 2b(3 + b)(2 - b)^2n - 2(8 - 6b - 2b^2 + b^3) < 0.
\]

The reason is simple: output coordination reduces market competition, which hurts consumers.

The combined effect of the full-fledged merger under asymmetric information, \( E(CS_M - CS_N) \), is the result of the two conflicting forces above. In simulations,\(^{16}\) we can see that the pattern of consumer surplus changes with the merger: Given \( \sigma^2/a^2 \) and \( n \), there exists a critical level of \( b \) such that the consumer surplus is higher (lower) after the merger if \( b \) is smaller (larger) than the critical level.

5.3. Global welfare

Global welfare consists of consumer surplus and all producers’ profits. Since we have assumed that production costs are zero, global welfare is simply equal to \( U \).

\(^{15}\) We can also prove that “demand fluctuation is not too severe (more precisely \( \sigma^2/a^2 < 0.44 \)) and market is not too competitive (more precisely \( n < 36 \))” is another sufficient condition for industrial profits to increase.

\(^{16}\) Details of the simulation results are available from the authors upon request.
In the case of no merger, global welfare is
\[ U_N = \frac{(n + 1)(3 + bn)a^2}{2(2 + bn)^2} + \frac{(3n + bn^2 + 2 - b)a\theta}{(2 + bn)(2 + bn - b)} + \frac{n(3 + bn - b)}{2(2 + bn - b)^2} \theta^2. \]

In the case of an information sharing merger between \( F_0 \) and \( F_1 \), global welfare is
\[ U_S = \frac{(n + 1)(3 + bn)(a + \theta)^2}{2(2 + bn)^2}. \]
Under the full-fledged merger, global welfare is
\[ U_M = \frac{2bn^2 + (6 + 2b - 4b^2)n + 6 - 4b - 5b^2 + 3b^3(a + \theta)^2}{4(2 + bn - b^2)^2}. \]

Based on the above results, we can examine the welfare changes from the merger. First, \( E(U_S - U_N) > 0 \). That is, information sharing increases global welfare. Second, \( E(U_M - U_S) < 0 \). Hence, output coordination reduces global welfare because it lowers market competition. The net effect of the full-fledged merger under asymmetric information depends on the relative degree of these two conflicting effects. We can show that \( E(U_M - U_N) < 0 \) if and only if \( \sigma^2 \) is sufficiently low.

In simulations, we can see another pattern of the global welfare changes brought about by a merger: Given \( \sigma^2/a^2 \) and \( n \), there exists a critical level of \( b \) such that the global welfare is higher (lower) after the merger if \( b \) is smaller (larger) than the critical level.

5.4. Summary and policy implications

We summarize the results of this section in Table 1, where + indicates an increase and – a decrease.

Let us next investigate whether private incentives to merge are compatible with social incentives. It suffices to give three numerical examples to illustrate the basic points. First, suppose \( \sigma^2/a^2 = 0.6 \) and \( n = 15 \) (uncertainty is large and the market is very competitive). By calculation, we obtain that \( \Delta\pi^a > 0 \) if and only if \( b \leq 0.61 \) and \( E(U_M - U_N) > 0 \) if and only if \( b \leq 0.66 \). This indicates that (a) whenever the two firms decide to merge, the merger increases the global welfare, and (b) it is possible that a merger raises the global welfare but the firms do not have incentives to merge, which is the case when \( b \in (0.61, 0.66] \).

Second, suppose \( \sigma^2/a^2 = 0.3 \) and \( n = 15 \) (uncertainty is small and the market is very competitive). Then, we have \( \Delta\pi^a > 0 \) if and only if \( b \leq 0.48 \) and \( E(U_M - U_N) > 0 \) if and only if \( b \leq 0.46 \). That is, when the two firms merge, global welfare increases in some cases but decreases in others. Hence, sometimes a merger should be encouraged and sometimes it should be discouraged. The same qualitative conclusion holds in a third example in which \( \sigma^2/a^2 = 0.6 \) and \( n = 8 \) (uncertainty is large and the market is not very competitive). In this case, we have \( \Delta\pi^a > 0 \) if and only if \( b \leq 0.64 \) and \( E(U_M - U_N) > 0 \) if and only if \( b \leq 0.61 \).

We can draw a hypothesis from the above analysis: When demand uncertainty is large and market competition is intense, international mergers should be encouraged (because mergers are socially desirable but some are not taken up by firms); however, when demand uncertainty is very small and market competition is very weak, international mergers should be discouraged (because mergers occur but are not socially beneficial).

The above should be viewed as a policy recommendation by economists who are concerned with total efficiency. However, we can show that even if anti-trust authorities care just about consumer welfare, the same policy recommendation can still be made.
We should always be cautious when drawing welfare implications based on a specific model. The above welfare analysis is conducted based on the situation when an international merger occurs due to the benefit of information sharing. If the international merger also generates other synergies, such as cost advantages as emphasized by Perry and Porter (1985) and Farrell and Shapiro (1990), the welfare implications might be different from those discussed/suggested above.

6. Robustness

How robust are the main results derived in this paper? In this section, we explore the robustness in a number of contexts.

6.1. Information exchange through markets and contracts

In the model described in Section 2, we assumed that merger is the only way for F0 to acquire information. We now argue that due to the specific nature of information, contracts and markets generally do not exist for information exchange. To demonstrate this point, we now modify the game so that firms have more options for information acquisition. In addition to mergers, we assume that firms may also sign a contract or sell information. If a contract is signed in the first stage between F0 and F1, F1 promises to tell F0 the true $h$ once the information is available. In return, F0 will pay a fixed amount, $T$, to F1. If neither a merger nor a contract is chosen, then after all domestic firms receive the information about $h$, a market for information is opened: F0 demands the information and all domestic firms supply the information.

Because the contract or market transaction is about information, the verifiability of information becomes crucial. To this end, we add some realistic elements to our model. (Note that we did not do so in the previous sections because such elements do not affect the qualitative results but make the expressions messier.) First, a firm can only observe the market price of each good but not its competitors’ outputs. Second, suppose that demand is given by $p_i = a + \varepsilon_i + \theta - q_i - b Q_i - C_0 q_i$, where $\varepsilon_i$ are random variables with zero mean. Moreover, $\varepsilon_i$ and $\theta$ are independent. The information structure about $\theta$ is the same as described before, but no firm realizes the values of $\varepsilon_i$ at any time of the game. This specification of uncertainty and information asymmetry reflects the stylized fact that no firm has complete information about the market, but the domestic firms have more information than the foreign firm has.

Suppose $F0$ and $F1$ sign a contract in the first stage and then $\hat{\theta}$ is realized by $F1$ and other domestic firms. It is easy to show that $F1$’s profit is higher if $F1$ reports a smaller $\theta$ and $F0$ believes this report. Thus, $F1$ has incentives to underreport. This behavior cannot be deterred unless the information can be verified by $F0$ or by a court. However, even after all firms have produced their products, based on the market information (prices), the court is not able to infer the true $\hat{\theta}$ because $F1$’s output is not observable due to uncertainty, $\varepsilon_i$. Hence, the court cannot verify the information. Having anticipated this misreporting incentive, $F0$ will not believe $F1$. As a result, no contract with a positive payment, $T$, will be signed.
Let us now examine the case of the market for information, assuming that no merger or contract has been reached in the first stage. Suppose $F_0$ offers $F_1$ a price for the information. Again, because information is not verifiable, $F_1$ always has incentives to underreport. Hence, $F_0$ is not willing to pay any positive price. All other domestic firms have the same incentives as $F_1$. Therefore, the market for information does not exist.\(^{17}\)

### 6.2. Uncertainty about the slope of the demand curve

In the information sharing literature on uncertainty and asymmetric information about demand, all papers, except Malueg and Tsutsui (1996), assume that only the intercept of the linear demand curve is uncertain. Maleug and Tsutsui analyze the model in which the slope of the demand function is uncertain and find that, under certain conditions, two firms earn greater profits when they share their information than when they keep the information private. If such a result also holds in our model, then a merger is not necessary because firms will voluntarily share their private information. However, their result does not apply to our model because of the difference about information distribution in the two models: both firms receive private information in their model while the domestic firms have more information than the foreign firm has in our model. Hence, introducing uncertainty about the slope of the demand to our model does not eliminate or even reduce the firms’ incentives for international mergers. We prove this now.

Let us consider the simplest case in which demand is given as:

\[
\pi_i = \frac{a}{C_0}h(q_i + bQ_i)/C_0, \\
\text{where } h \text{ can take one of the two values, } h_L \text{ or } h_H, \text{ with equal probability. The probabilistic distribution is common knowledge, but when uncertainty is resolved, only the domestic firms know the realization of } h.
\]

Suppose a merger does not occur between $F_1$ and $F_0$ in the first stage. Then, in the second stage, firms play the usual Cournot game under asymmetric information. Denoting $\hat{\theta} = (\theta_L + \theta_H)/2$, the equilibrium expected profits ($E\pi^u_0$ for $F_0$ and $E\pi^u$ for each domestic firm) are (they are “expected” at the beginning of the game):

\[
E\pi^u_0 = \frac{a^2}{\theta(2 + bn)^2}, \quad E\pi^u = \frac{8a^2}{(2 + bn)^2(2 + bn - b)^2} \sum_{x=L,H} \left( \frac{(2 + bn)\hat{\theta} - b\theta_x}{\theta_x} \right)^2.
\]

Suppose now that a merger occurs between $F_1$ and $F_0$ in the first stage so that information is shared. Let us focus on the information sharing merger. In the second stage, all $n + 1$ firms play the usual Cournot game with complete information. All firms receive the same expected profit $E\pi^s = a^2(1/\theta_L + 1/\theta_H)/2(2 + bn)^2$.

By comparison, we have $E\pi^s < E\pi^u$. $F_1$’s expected profit is lowered if it shares information with $F_0$. Hence, $F_1$ is not willing to share information without any compensation from $F_0$. This result is in contradiction to that of Malueg and Tsutsui (1996).

\(^{17}\) Will a merger still be advantageous over pure information-sharing if firms compete repeatedly in the market? In other words, are our results robust in a dynamic model? The answer is yes after the just-mentioned two assumptions are added to the model. In that case, the foreign firm will not be able to learn about the true demand because it cannot observe the true behavior of the domestic firms.
because, in our model, $F_1$ does not get any information from $F_0$ in return. However, the gain to $F_0$ always outweighs the loss to $F_1$ and, hence, the information sharing merger is always profitable. This is because

$$2E\pi^a - (E\pi_0^a + E\pi^u) = \frac{a^2(\theta_H - \theta_L)^2}{4(2 + bn - b)^2(2 + bn)^2}Z(n,b)>0.$$  

With the above result, it is straightforward to anticipate that the full-fledged merger is profitable in a larger range of product differentiation than in the benchmark case when there is no uncertainty in demand and no room for information sharing through a merger. In fact, we can show (but we omit it here to save space) that the critical values of $b$ for profitable mergers are the same as those in the case of the uncertain demand intercept. Hence, the major (qualitative and quantitative) results derived in this paper are robust when the uncertainty is about the slope of the demand curve.

6.3. Non-linear demand

When a firm holds private information about market demand, there is a trade-off between keeping the information private and sharing it with its competitors. The traditional result, which is derived in models with linear demand structures, is that, under Cournot competition, information is valuable to a firm. Einy et al. (1995) confirms this conclusion in a model with a general demand function, showing that, with homogenous goods and constant marginal costs, a firm with superior information about market demand receive higher profits.\textsuperscript{18}

To examine the implications of non-linear demand on our results, we adopt a specific non-linear demand function (it is also used by others, e.g., Fauli-Oller, 2000):

$$p_i = a + \theta - (q_i + bQ_{-i})^{\gamma+1}/(\gamma+1),$$

where $\gamma>-1$ is a measure of demand concavity. The random variable $\theta$ can take one of two values, $\theta_L$ and $\theta_H$, with equal probability.

We define three scenarios and compare the firms’ expected profits. Scenario 1 is the case when no merger occurs in the first stage; scenario 2 is the case when $F_0$ and $F_1$ engage in the information sharing merger in the first stage; and scenario 3 is the case when $F_0$ and $F_1$ engage in the full-fledge merger. We are able to derive closed-form solutions only for scenario 2. However, by numerical simulation, we are able to show that (i) the information sharing merger (i.e., moving from scenario 1 to scenario 2) always raises $F_0$’s expected profit, reduces $F_1$’s expected profit, and raises the joint profits of $F_0$ and $F_1$; (ii) the output coordination merger (i.e., moving from scenario 2 to scenario 3) is profitable for the merged entity when and only when $b<b_0^*$, where $b_0^* \in (0,1)$; and (iii) the full-fledged merger (i.e., moving from scenario 1 to scenario 3) is profitable for the merged entity when and only when $b<b_1^*$, where $b_1^*>b_0^*$ and $b_1^* \in (0,1)$. Therefore, the major results obtained under linear demand also hold under non-linear demand.

\textsuperscript{18} However, the opposite result may occur if the marginal cost is an increasing function of output. Einy et al. (1995) show this based on a numerical example.
6.4. Cournot vs. Bertrand competition

It is well known in the merger literature that the Cournot model (e.g., Salant et al., 1983) tends to predict that merging firms lose while non-merging firms gain, known as the “Cournot merger paradox”. Our new explanation offers a solution to this paradox: domestic firms do not merge because they have symmetric information but international mergers are profitable because the firms have asymmetric information. It is clear that the present study is in line with the literature (e.g., Perry and Porter, 1985; Farrell and Shapiro, 1990) seeking to modify the Cournot model such that a merger may benefit insiders and hurt outsiders. Information sharing is a source of synergy, as are cost savings and capital pooling.

What if the Bertrand model is chosen? We can show that (i) the pure information sharing merger benefits both parties of the merger even without any monetary transfer, and (ii) the full-fledged merger is beneficial with any degree of product differentiation. These results are not surprising at all. The first result is consistent with the well-known conclusion from the information sharing literature that a Bertrand firm is willing to reveal its private information about demand to its rivals (Vives, 1984). In addition, we show that the uninformed party also benefits from receiving the information. Then, given this information sharing result, the second result can be easily understood by recalling Deneckere and Davidson’s (1985) finding that, in the absence of incomplete information, a merger is always profitable under Bertrand competition. Hence, the Bertrand model does not allow us to highlight the importance of information sharing in international mergers. That is why we chose the Cournot model.

7. Concluding remarks

We have investigated international mergers under asymmetric information by concentrating on two features of a merger, i.e., output coordination and information sharing. We show that the foreign firm and a domestic firm always want to share information, but output coordination is not always profitable, depending on the extent of product differentiation. We have also examined how the full-fledged merger affects the non-merging firms’ profits, consumer surplus, domestic welfare and global welfare. The extent of product differentiation plays a critical role.

Firms from different countries have different incentives to merge as opposed to firms in the same country. Because a foreign firm is less likely to be as well informed as a domestic firm about the local market, we have emphasized the incentives to share information about the market demand in this paper. Firms from different countries also have different corporate cultures, management styles, technologies and market shares. It would be interesting to investigate how these differences affect incentives for international mergers.

Acknowledgments

We benefited from presentations at the Second Chinese Economics Annual Meeting, the Second Biennial Conference of the Hong Kong Economic Association, the
International Conferences held in Hitotsubashi University and Kobe University, Midwest International Economics Meetings and seminars at HKUST and the University of Hong Kong. We thank Steven Chiu, Esther Gal-Or, Hiroshi Ohta and especially Jonathan Eaton (the editor) and the referee for their comments and suggestions. We are grateful for financial support from the Research Grants Council of Hong Kong (HKUST6214/00H).

Appendix A

**Proof of Proposition 1.** Hereafter, let a function with a subscript represent partial differentiation, e.g., $Y_b = \partial Y(n,b)/\partial b$. Since $Y_b = -[3n^2b + 4(n - 1)](1 - b) - nb(3n - 4) - 4b < 0$, $Y(n,0) = 4 > 0$ and $Y(n,1) = 4 - 2n^2 < 0$, there is a unique $b_0(n) \in (0,1)$ such that $Y(n,b) > 0$ if and only if $b < b_0(n)$. Total differentiation of $Y(n,b_0) = 0$ yields $d b_0(n) / dn = -2b[nb(3 - b) + 2(1 - b)]/(3n^2b + 4n - 4)(1 - b) + b(3n^2 - 4n + 4) < 0$. Note that $\Delta \pi^c$ and $Y(n,b)$ have the same sign. This completes the proof for part (i).

The proof for part (ii) is straightforward. \hfill \Box

**Proof of Proposition 2.** Part (i). This part is in the text preceding the proposition, particularly (7).

Part (ii). Differentiating (7), denoting $H = E[\pi^s - \pi^u] - (\pi^u + \pi^u)$, yields

$$
\frac{\partial H}{\partial b} = -2\sigma^2 n(n - 1)b[(n^2 - 4n + 2)b^2 + 6(n - 2)b + 12] + 8(n + 1)
\quad < 0,
$$

$$
\frac{\partial H}{\partial n} = -2\sigma^2 \left[2(n - 1)^3 - n^3\right]b^3 + 6(n^2 - 4n + 2)b^2 + 12(n - 2)b + 8
\quad < 0 \text{ for } n \geq 3.
$$

When $n = 2$, $\partial H/\partial n$ is increasing in $b$ and the root is $b = 0.7$. \hfill \Box

**Proof of Proposition 3.** Part (i). This part is proven by (9).

Part (ii). The following results are immediately obtained by inspecting (8) and (9):

$\partial E(\pi^s - \pi^u)/\partial \sigma^2 > 0$, $\partial E(\pi_0^s - \pi_0^u)/\partial n < 0$, $\partial E(\pi_0^s - \pi_0^u)/\partial b < 0$, and $\partial E(\pi^s - \pi^u)/\partial \sigma^2 < 0$.

Differentiating (9) with respect to $n$ and $b$, respectively, yields

$$
\frac{\partial E(\pi^s - \pi^u)}{\partial n} = 2\sigma^2 b^2 \left[3b^2n(n - 1) + (b - 3)^2 + 3 + 12bn\right]
\quad > 0,
$$

$$
\frac{\partial E(\pi^s - \pi^u)}{\partial b} = \frac{[2n(n - 1)(6 + 2bn - b)b^2 - 16]\sigma^2}{(2 + bn)^3(2 + bn - b)^3}.
$$
The numerator of the last equation is increasing in $b \in [0,1]$ and has a unique root in $(0,1)$.

**Proof of Proposition 4.** It is clear that the sign of $E(\Delta \pi^b)$ is the same as $X(n,b)$, where $X(n,b) = 2(2+bn-b^2)^2Z(n,b)+(1+a^2/\sigma^2)b^2(2+bn-b)^2Y(n,b)$. Recall that $Z_n>0$ and $Z(n,b)>0$ for all $n \geq 2$. Also, $Z_n=2(n-2)^2b+4(n-2-b)$. So, $Z_b<0$ for $n \in \{2, 3\}$ but $Z_b>0$ for all $n>3$.

From the analysis of the three functions, $X(n,b)$, $Y(n,b)$ and $Z(n,b)$, we immediately obtain the first result: $X(n,b)>0$ for all $b \leq b_0$. Let us now focus on $b \in (b_0, 1]$. Recall that $Y(n,b)<0$ for $b \in (b_0, 1]$. Because $X(n,b_0)=2(2+bn-b^2)^2z>0$, $X(n,1)=-2(k-1)(n^2-2)(n+1)^2<0$, and $X(n,b)$ is continuous in $b$, there exists a $b_1 \in (b_0,1)$ such that $X(n,b_1)=0$. We argue that $b_1$ is the only solution to $X(n,b)=0$. We will prove this by contradiction.

Suppose there are multiple solutions to $X(n,b)=0$. We let $b_1$ denote the smallest one. Then, $X(n,b)$ must be decreasing at $b=b_1$, i.e., $X_b(n,b_1)<0$. Moreover, there is at least another solution (the second smallest one) called $b_2 \in (b_1, 1)$ such that $X(n,b_2)=0$ and $X(n,b)$ is increasing at $b=b_2$, i.e., $X_b(n,b_2)>0$.

Denoting $f(n,b)=2(2+bn-b^2)^2Z(n,b)$, and $g(n,b)=b^2(2+bn-b)^2Y(n,b)$, we have $X(n,b)=f(n,b)+(1+a^2/\sigma^2)g(n,b)$ and $X_b(n,b_2)=2(2+b_2n-b_2^2)(2+b_2n)/b_2(2+b_2n-b_2)(h(n,b_2)/Y(n,b_2)-32)$, where $h(n,b)=\sum_{i=0}^{7} w_ib^i$, whereas $w_0=64$, $w_1=32(5-3n)$, $w_2=16(6n^2-16n+5)$, $w_3=8(20n^3-71n^2+90n-42)/2$, $w_4=52n^4-308n^3+576n^2-504n+224$ if $n \geq 4$, $w_5=6n^5-60n^4+164n^3-192n^2+132n-64$ if and only if $n \geq 7$, $w_6=-3n^5+15n^4-14n^3+4$ if $n=2,3$, and finally, $w_7=n(n-1)(n^2-4n+2)$ for $n \geq 4$.

Now let $h_1=(1/16)\sum_{i=0}^{7} w_ib^i = 6b^2n^2-2b(3+8b)n+(5b^2+10b+4)$. It is clear that $h_1>0$.

Let $h_2=\sum_{i=3}^{7} w_ib^i$. For $n \geq 7$, since all coefficients of $b$th except $w_6$ are positive, and $w_4+w_5+w_6>0$, we know that $h_2>\sum_{i=4}^{6} w_ib^i=(w_4+w_5+w_6)b^6>0$. For $4 \leq n < 7$, we know $w_5<0$ and $w_6<0$, but $w_3+w_4+w_5+w_6>0$. For $n=3$, $w_4<0$, $w_5<0$ and $w_7<0$, but $w_3+w_4+w_5+w_7=448>0$. Therefore, $h_2>0$ for all $n \geq 3$.

The above two paragraphs together show that $h=16h_1+h_2>0$ for $n \geq 3$ and $b \in (b_0, 1]$. Because $Y(n,b_2)<0$, $h(n,b_2)/Y(n,b_2)-32<0$, which implies that $X_b(n,b_2)<0$ for $n \geq 3$. At $n=2$, we have $h(n,b)-32Y(n,b)=-4b^7+36b^6-24b^5-112b^4-16b^2+208b^2+96b-64>0$ for $b>b_0=0.555$. Hence, $X_b(n,b_2)<0$ at $n=2$.

Thus, we have shown $X_b(n,b_2)<0$ for all $n \geq 2$, which contradicts the supposition of having $b_2$ as the second smallest value to $X(n,b)=0$, with $X_b(n,b_2)<0$. This proves (10) and Proposition 4.

**Proof of Proposition 5.** From (8) and (9), we obtain the difference between total industrial profits under the information sharing merger, denoted as $\Pi_S$, and total industrial profits before the merger, denoted as $\Pi_N$, $E(\Pi_S-\Pi_N)=[4(1-b)-(n^2+n-1)b^2]/(2+bn)^2(2+bn-b)^2$. Note that $E(\Pi_S-\Pi_N)$ decreases in $b$ and is positive at $b=0$ but negative at $b=1$. Next, the difference in total profits under output coordination, denoted as $\Pi_M$, and total profits under symmetric information but without output coordination, i.e., $\Pi_S$, is $E(\Pi_M-\Pi_S)=b^2(a^2+\sigma^2)[b(4-3b+b^2)n^2+2(2-b)^2n-4+4b-2b^2]/(2+bn)^2(2+bn-b)^2>0$ for $b>0$. 

\[\square\]
From the above, we obtain $E(\Pi_M - \Pi_N) = a^2[g_1 + k b^2 g_2]/(2 + bn)^2(2 + bn - b)^2$, where $g_1 = (2 - b)^2 - (n + 2)b^2$, $g_2 = (2 - b)^2(bn^2 + 2n - 1) + b^2(n^2 - 1)$, and $k = [(a^2 + a^2)/2a^2][(2 + bn - b)/(2 + bn - b)^2]_2^2$.

Since $a^2 < a^2$, $(a^2 + a^2)/2a^2 > 1$. The function $(2 + bn - b)^2/(2 + bn - b)^2$ is increasing in $n$, but has a U-shape in $b$. Noting this, we can then easily get $(2 + bn - b)^2/(2 + bn - b)^2 > 0.825$ for all $n \geq 2$ and $b \in [0, 1]$. Thus, $k > 0.825 > 33/40$.

Because $g_2 > 0$, we know that $E(\Pi_M - \Pi_N) > 0$. If $3g_1 + 4b^2 g_2 > 0$. Letting $x(n) = x_2 n x_2 n + x_0 = 40 g_1 + 33 b^2 g_2$, we have $b^2 (-40 + 132 b - 9b^2 + 33 b^3), x_1 = 2 b^2 (112 - 132 b + 33 b^2), x_0 = 160 - 160 b - 92 b^2 + 132 b^3 - 66 b^4$. We find that $x_2$ is positive when $b > 0.417$, $x_2 = 0$ if $b = 0.417$, $x_1$ is always positive, and $x_0$ is positive when $b < 0.871$. Hence, for $b \in [0.417, 1]$, $x$ is increasing in $n$ and $x_2 = 160 - 160 b + 196 b^2 + 132 b^3 - 330 b^4 + 132 b^6 > 0$. This shows that $x > 0$ for $b \in (0.417, 1]$.

For $b \in [0, 0.417)$, $x > 0$ if and only if $n < n^* = \left(-x_1 + \sqrt{x_1^2 - 4x_0 x_2}\right)/2x_2$. Computer calculation can verify that $n^* \geq 19.4$. This shows that $x > 0$ for $b \in [0, 0.417]$ and $n < 20$. □

References


