The effects of market linkages and the natural rate of discoveries on market structure*

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abstract

The traditional approach makes investment in innovation constrained by market structure. This paper explores the causality from innovation to market structure. Omitting this causality direction on empirical models may explain empirical problems and contradictions on these models.

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1. Introduction

The traditional approach makes investment in innovation constrained by market structure. In fact, probably the causality runs both ways. This paper explores the causality from innovation to market structure and shows that this in fact probable. Omitting this causality direction on empirical models may explain empirical problems and contradictions on these models. For example, Liberman (1987) finds a positive correlation between market concentration and innovation, and Scherer (1984) finds that in some cases, small firms are relatively more likely to make major innovations. Other studies report results that seem contradictory: Cohen, Levin and Mowery (1994) conclude that R&D intensity varies with firm size in some industries and not others, and where it does vary, it may be negatively or positively related to size. This paper presents some answers to these puzzles.

This is paper presents a different approach from the standard paradigm that relates innovation to market structure. It considers inventions exogenous, (see Kato 2005 for a different approach) and investigates the effects of simultaneous investment in both horizontal expansion (product innovation) as in Dixit-Stiglitz (1977) and Romer (1990), and vertical expansion (process innovation), a.k.a. quality ladders, as in Aghion and Howit (1992).

2. The model

Firms and markets

There are $s$ markets, and each has $v_{k,t}$ endogenously determined firms in moment $t$, $k=1,...,s$. For now let's define market as a strong break in the chain of substitution on the demand side. A more formal definition will be given later in this paper. Each firm is assumed to use a production function of the form:

\[
x_{i,t} = A_{i,t}
\]

That is, the quantity produced of each variety $i$ in each period $t$, $x_{i,t}$ is equal to the technological level in production, $A_{i,t}$, of that variety. All varieties are heterogeneous and each variety is produced by a single firm. The number of varieties produced by firm $j$ are $n_{j,t}$.

The profit function for any firm $j$ takes the following form, for each period $t$:

\[
\Pi_{j,t} = \sum_{i=1}^{n_{j,t}} x_{i,t} p_{i,t} - \Psi c_{j,t} - \Psi \sum_{i=1}^{n_{j,t}} d_{i,t}
\]

Where $x_{i,t}$ and $p_{i,t}$ are respectively the quantity and the price of variety $i$. There are only two types of costs: $c_{j,t}$ is spending on product innovation and $\sum_{i=1}^{n_{j,t}} d_{i,t}$ total spending on process innovation. Marginal cost of both types of innovation is assumed to be constant and equal to $\Psi$.

Assumption 1. The total number of product innovations in each period $t$ made by each individual firm is never very large. This means that when optimizing, each firm will take as null the impact of an additional variety on its profits from the varieties that existed in previous periods.
This assumption is necessary for the sake of the analytical simplicity of the model. (this can be easily seen in the appendix).

Invention and innovation

Process innovation follows the process:

\[
A_{i,t+1} - A_{i,t} = \lambda_k \cdot A_{i,t} \cdot d_{i,t}
\]

and product innovation follows the process:

\[
n_{i,t+1} - n_{i,t} = \lambda_k \cdot n_{i,t} \cdot c_{j,t}
\]

Where \(\lambda_k\) is the (exogenous) invention rate which is assumed to be the same for both kinds of innovation processes. Note that this is a constant that would correspond to a Poisson arrival rate in a model with uncertainty. New varieties are created with initial technology \(A_0\).

**Assumption 2.** There is a potentially different invention rate \(\lambda_k\) for each one of the \(k = 1, \ldots, s\) markets.

What is assumed is that each market has its own invention rate because it corresponds to some specific scientific area.

Demand

Following Dixit-Stiglitz (1977), the demand for variety \(i\) in period \(t\) is:

\[
x^d_{i,t} = (p_{i,t}/ P_t)^{\sigma_k} \cdot [y_{k,t}/m_t]
\]

The demand \(y_{k,t}\) corresponds to the total demand that in period \(t\) is directed to the industry \(k\) to which variety \(i\) belongs to. As this is a partial equilibrium model, it is considered exogenous. \(P_t\) and \(p_{i,t}\) are respectively the price level and the price of variety \(i\) in period \(t\) in market \(k\). (Wherever clear from the context to which market they belong, these shall not be indexed by \(k\).) The parameter \(\sigma_k\) measures the degree of linkage between submarkets of the same market, in the Sutton (1998) fashion. Lower \(\sigma_k\) means, on the demand side, less substitutability between varieties to the consumers, and on the supply side it may represent the existence of scope economies in production. The total number of varieties in the industry in period \(t\) are \(m_t = \sum^v n_{j,t}\).

I will now follow a third assumption.

**Assumption 3.** There is independence between different markets. This means: a) there is a potentially different \(\sigma_k\) for each market. b) varieties are only substitutes between submarkets of the same market and never between markets. Any scope economies that may exist apply only inside the same market. c) there is a fixed and exogenous demand \(y_{k,t}\), potentially different for each market \(k = 1, \ldots, s\).
**Optimization**

Each firm's problem is to maximize the inter-temporal present discounted value of profits:

\[
\sup_{t=0}^{\infty} (\sum \beta^t \Pi_{j,t})
\]

The state variables for each firm are the technology level for each one of the existing varieties \(A_{i,t}, i=1,\ldots,n_{j,t}\) and the total number of varieties \(n_{j,t}\). The control variables are the amount of spending on innovation on each one of the varieties \(i\) on period \(t\) (process innovation), \(d_{i,t}, i=1,\ldots,n_{j,t}\) and the amount of spending on the development of new products, \(c_{j,t}\).

**Closed-form solution**

It turns out that a unique closed-form solution exists for the optimal spending on each type of investment on innovation for each period \(t\). (the proof can be found in the appendix).

\[
d_{i,t} = \phi \left[ (1-1/\sigma_k) A_{i,t}^{1-1/\omega_k} n_{j,t}^{\omega_k - 1} (m_{t+1}^{-1}) - \Psi \sigma_k (\lambda_k \Psi \sigma_k)^{-1} \right]
\]

and

\[
c_{j,t} = \phi \left( A_0^{1-1/\omega_k} n_{j,t}^{\omega_k - 1} - [(m_{t+1}+1)/n_{j,t}] (\lambda_k \Psi \sigma_k)^{-1} \right)
\]

where \(\phi = y_{k,t+1} (\lambda_k \beta P_{t+1} \sigma_k)^{\sigma_k} (\lambda_k \Psi \sigma_k)^{-1}\).

**The long run**

The long run condition is:

\[
\sum_{t=0}^{\infty} \beta^t \Pi_{j,t} = 0 \quad \text{for all } j.
\]

**Market structure**

It is possible to show that:

\[
(\partial d_{j,t} / \partial \sigma_k) / (c_{j,t} / \partial \sigma_k) > 1
\]

Where \(d_{j,t} = \sum_{i} d_{i,t}\) this is, total spending on process innovation. The intuition for the above result is as follows. The larger is the linkage between submarkets \(\sigma_k\) (more elasticity of substitution to the consumer), the more will firms invest in process innovation relatively to product innovation because when investing in process
innovation they will more easily "steal" demand from other trajectories (see Sutton 1998).

Regarding the long run market structure:

(11) \( \frac{\partial v_k}{\partial \lambda_k} < 0 \)

This means that markets characterized by a large numbers of scientific discoveries will have, ceteris paribus, a smaller concentration of firms. The intuition is that a small number of firms are required to invest a large amount in innovation.

Also:

(12) \( \frac{\partial v_k}{\partial \sigma_k} < 0 \)

This means that a higher linkage between submarkets is positively related to market concentration. The intuition is related to the condition (10): markets with stronger market linkages are more prone to a small number of process inventing firms.

Finally:

(13) \( \frac{\partial v_k}{\partial d_{j,t} + c_{j,t}} < 0 \)

Total investment on innovation is positively related to market concentration. The intuition is the following: Suppose that on period \( t-1 \), all firms invest a given amount on innovation. For some reason, on period \( t \) that amount increases. Note that in the next periods firms will have more capacity, i.e. will have products of superior quality and a larger number of varieties that otherwise would have existed if the investment in innovation had stayed in the original level. Because of this, if all other variables remain constant (notably demand) the market is capable of holding an inferior number of firms that otherwise would have been. Thus, concentration on that market rises.

3. Conclusion

Markets characterised by different degrees of linkage between submarkets and invention rates will have different market structures. Traditional empirical studies do not distinguish process from product innovation, and so are led to find conflicting evidence regarding market structure.
Appendix

Proof of equations (7) and (8): The solution can be obtained using dynamic programming. The objective function is (6). The constraints are (1), (2), (3), (4) and (5). The state variables are \( n_{j,t} \) and \( A_{i,t} \) for \( i=1,\ldots, n_{j,t} \). The control variables are \( c_{j,t} \) and \( d_{i,t} \) for \( i=1,\ldots, n_{j,t} \).

We are now able to set up the Bellman equation:

\[
V(A_{i,b}, n_{j,t}) = \max \left\{ \Pi_{j,t} + \beta V(A_{i,t+1}, n_{j,t+1} | A_{i,b}, n_{j,t}) \right\}
\]

The first-order conditions are:

\[
\begin{align*}
\Psi &= \beta \frac{\partial V(A_{i,t+1}, n_{j,t+1} | A_{i,b}, n_{j,t})}{\partial d_{1,t}} \\
\Psi &= \beta \frac{\partial V(A_{i,t+1}, n_{j,t+1} | A_{i,b}, n_{j,t})}{\partial d_{n_{j,t},t}} \\
\Psi &= \beta \frac{\partial V(A_{i,t+1}, n_{j,t+1} | A_{i,b}, n_{j,t})}{\partial c_{j,t}}
\end{align*}
\]

These correspond to the usual marginal cost equals marginal revenue conditions. Solving the first order conditions (A2) to (A3) is straightforward. After using the chain rule on the right hand side and substituting restriction (3), when solving in order to \( d_{i,t} \), we get equation (7).

For solving the right hand side of (A4) it is necessary to use assumption 1. Using the chain rule:

\[
\begin{align*}
\Psi &= \beta \frac{\partial V(A_{i,t+1}, n_{j,t+1} | A_{i,b}, n_{j,t})}{\partial n_{j,t}} \frac{\partial n_{j,t}}{\partial c_{j,t}} \frac{\partial c_{j,t}}{\partial n_{j,t}} \\
\frac{\partial V(A_{i,t+1}, n_{j,t+1} | A_{i,b}, n_{j,t})}{\partial n_{j,t}} &= \frac{\partial V(A_{i,t+1}, n_{j,t+1} | A_{i,b}, n_{j,t})}{\partial n_{j,t}}
\end{align*}
\]
\[
\begin{align*}
\eta_{j,t+1} &= A_0^{(t+1)} P_{t+1} \left( y_{k,t+1}/(m_{t+1}+1) \right)^{1/\sigma_k} - \left[ \sum_{i=1}^{\eta_{j,t+1}} A_{ik}^{(t+1)} P_{t+1} \left( y_{k,t+1}/m_{t+1} \right)^{1/\sigma_k} \right] - \sum_{i=1}^{\eta_{j,t+1}} A_{ik}^{(t+1)} P_{t+1} \left( y_{k,t+1}/m_{t+1}+1 \right)^{1/\sigma_k} \\
\text{equal to zero by assumption 1}
\end{align*}
\]

Substituting the above and restriction (4) on (A5), the result is equation (8).

References


