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Tribes under Threat – The Collective Behavior of Firms During the Stock Market Crisis

WP 28/2008/DE/UECE
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- The Collective Behavior of Firms
During Stock Market Crises

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May 5, 2008

Abstract

Due to their unpredictable behavior, stock markets are examples of complex systems. Yet, the dominant analysis of these markets assumes simple stochastic variations, eventually tainted by short-lived memory. This paper proposes an alternative strategy, based on a stochastic geometry defining a robust index of the structural dynamics of the markets and based on notions of topology defining a new coefficient that identifies the structural changes occurring on the S&P500 set of stocks. The results demonstrate the consistency of the random hypothesis as applied to normal periods but they also show its inadequacy as to the analysis of periods of turbulence, for which the emergence of collective behavior of sectoral clusters of firms is measured. This behavior is identified as a meta-routine.

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†Supported by FCT Portugal, Project PDCT/EGE/60193/2004
1 Introduction

The evolution of stock markets has been eluding many established models, which interpret its dynamics according to the hypotheses that such markets behave as a simile of Brownian motion. We follow an alternative strategy, assuming a complex system: first, we look for evidence of structure as a generator of perturbations, and not of white noise shocks devoid of any information; second, we argue that an empirically based interpretation of this dynamics is possible and effective in the detection of patterns of change, and, third, we identify this formation of patterns as a meta-routine. The analytical method, based on empirically oriented and computationally highly demanding techniques, is guided by these choices.

This paper develops and applies a stochastic geometry approach designed to describe the dynamics of the object emerging from the collective behavior of the complex system. In the current case, the market is analyzed according to the evolution of the population of two examples: the first one considers 253 stocks of the S&P500 index including all the surviving firms for the whole period from August 1988 to January 2008. In the second example, we consider the 424 stocks of the S&P500 including all the surviving firms for the more recent period from January 1998 to March 2008.

We define a metric and use a properly defined distance, computed from the correlation coefficients between daily returns of all these firms, and then proceed to the identification of the geometric object formed by the set of distances among such firms. The results prove that the resulting ellipsoid is a cloud of points, which is uniformly distributed along its first leading directions, whenever business-as-usual predominates, but that it is suffers severe distortions along several dimensions whenever a crisis occurs. The geometric and topological properties of the dynamics of this market can be measured and guide an empirically oriented interpretation of its evolution ([1], [2]).

The approach we suggest allows for:

1. the identification of the minimum number of relevant dimensions describing the evolution of the market;
2. the identification of the dimensions along which the distortion occurs;
3. the measurement of the effect of that distortion;
4. the identification of the patterns of change in the market and, namely,
5. of its sectoral dynamics.

The last point is the theme for this paper, in which we argue for a more
general concept of sectoral routines based on these patterns of change.

2 The behavior of tribes under threat

The existence of a large number of degrees of freedom in the action of spec-
ulative markets, as well as in others, was discussed in economic theory from
two opposed points of view. One is reminiscent of the early social statist-
tics of Adolphe Quetelet: although there are many individuals, order reigns
supreme given the averaging out of their characteristics and actions. The
”homme moyen” of Quetelet and the ”representative agent” of Alfred Mar-
shall and neoclassical economics meet in the forest of intentions and actions
in the market. In that framework, determinism is recapitulated as com-
manding laws of behavior and economic theory postulates the dominance of
an ordered structure, eventually with simple random shocks impinging upon
the course of the system and whose distribution follows a Gaussian law, the
language of perturbations in Nature.

Another view was proposed by Trygve Haavelmo as he proposed the re-
construction of economics according to the probability approach. In this
case, all economic variables are redefined as stochastic processes: order itself
is randomness. The degrees of freedom describe the uncertainty of agents
in their actions, of the scientist in her measurement and of the model it-
self in its omissions. The currently dominating models for the analysis of
stock markets are distant inheritors of the generalized probability approach.
In spite of evidence of stable distributions and heteroskedasticity, they were
able to define sophisticated models of analysis and prediction based on these
premises.

In our view, the existence of a large number of degrees of freedom can
be assessed from still another perspective. We assign the variability of these
markets to the functioning of its inner structure, including to the rules that
define the complexity of its dynamics. Consequently, even if they are not
describable under the authority of a deterministic law, some of the systematic
characteristics of these systems may be measured. In this case, we are not
looking for shocks but for the evolution of the structure itself and, since this
view is also inspired by the evolutionary and institutional reconsideration of
economic theory, we intend to highlight both identifiable factors of mutation
and the role of routines in the functioning of the markets.

One of such institutional factors is the sector itself. Of course, for a
purely stochastic interpretation, the returns of stocks in highly competitive
environment would be expected to evolve under the influence of so numerous
factors that a random trajectory could be supposed to result. We prove that
this is not the case in the periods of turbulence, although in normal periods
the random hypothesis holds satisfactorily. In other words, our thesis is that
the tribes are constituted under threat.

3 The stochastic geometry approach

Unless the proportion of systematic information present in correlations be-
tween stocks in a complex system is relatively small, the corresponding man-
ifold is a not a low-dimensional entity and therefore its understanding is
virtually unreachable. But the evidence of alternate states of apparent ran-
doness and the emergence of structured collective dynamics suggests it may
correspond to a low-dimensional object. The rationale for this intuition is as
follows: considering the existence of competition among multiple agencies,
firms, information and strategies, the response by the agents to the multiple
signals can create an object comparable to that obtained from random pro-
cesses, which is a powerful analogy for such markets except when collective
behavior emerges and dominates its dynamics. If this is the case, the relevant
point is to capture this evolution and we are reduced to an embedding ques-
tion, the definition of the smallest manifold that contains the set of points
describing the market. In that case, its dynamics can be observed as its form
is shaped by the occurrence of bubbles and crises.

The stochastic geometry strategy is simply stated in the following terms.

3.1 The metric and the definition of a distance

From the set of returns of the stocks \(^1\) and their historical data of returns
over the time interval, and using an appropriate metric ([3],[4]), we compute
\(^1\)\(N=424\) stocks as the population of surviving firms in the S&P500 for January 1998 -
March 2008
the matrix of distances between the stocks. Considering the returns for each stock,

\[ r(k) = \log(p_t(k)) - \log(p_{t-1}(k)) \] (1)

a normalized vector

\[ \rho^*(k) = \frac{\bar{r}(k) - \langle \bar{r}(k) \rangle}{\sqrt{n\langle (r^2(k)) - (r(k))^2 \rangle}} \] (2)

is defined, where \( n \) is the number of components (number of time labels) in the vector \( \bar{r} \). With this vector the distance between the stocks \( k \) and \( l \) is defined by the Euclidian distance of the normalized vectors.

\[ d_{ij} = \sqrt{2(1 - C_{ij})} = \| \rho^*(k) - \rho^*(l) \| \] (3)
as proposed in [3] and [4], with \( C_{ij} \) being the correlation coefficient of the returns \( r(i), r(j) \).

### 3.2 Identification of the relevant directions and the index \( S \)

As the distance is properly defined, it is possible to obtain, from the matrix of distances, the coordinates for the stocks in a Euclidean space of dimension smaller than \( N \). Then the standard analysis of reduction of the coordinates is applied to the center of mass and the eigenvectors of the inertial tensor are computed.

The same technique is applied to surrogate data, namely to data obtained by independent time permutation for each stock and these eigenvalues are compared with those obtained in from the data, in order to identify the directions for which the eigenvalues are significantly different.

For both surrogate and actual data, the sorted eigenvalues, from large to small, decrease with their order. In the surrogate case, the uniform decrease of the eigenvalues shows that the directions are being extracted from a spherical configuration, corresponding to the randomness of the configuration. The display of a uniform and smooth decrease in the values of the sorted eigenvalues is characteristic of random cases and is also experimentally observed when the market space is built from historical data corresponding to a period of business as usual.
The procedure is straightforward. After the distances \( d_{ij} \) are calculated for the set of \( N \) stocks, they are embedded in \( R^D \), where \( D < n \), with coordinates \( \vec{x}(k) \). The center of mass \( \vec{R} \) is computed and coordinates reduced to the center of mass.

\[
\vec{R} = \frac{\Sigma \vec{x}(k)}{k}
\]

(4)

\[
\vec{y}(k) = \vec{x}(k) - \vec{R}
\]

(5)

and the inertial tensor

\[
T_{ij} = \Sigma_k \vec{y}_i(k) \vec{y}_j(k)
\]

(6)

is diagonalized to obtain the set of normalized eigenvectors \( \{\lambda_i, \vec{e}_i\} \). The eigenvectors \( \vec{e}_i \) define the characteristic directions of the set of stocks. The characteristic directions correspond to the eigenvalues \( \lambda_i \) that are clearly different from those obtained from surrogate data. They define a reduced subspace of dimension \( f \), which carries the systematic information related to the market correlation structure. In order to improve the decision criterion on how many eigenvalues are clearly different from those obtained from surrogate data, a normalized difference \( \tau \) is computed:

\[
\tau(i) = \lambda(i) + 1 - \lambda'(i)
\]

(7)

and the number of significantly different eigenvalues is given by the highest value of \( i \) to which \( (\tau(i) - \tau(i - 1)) > 3(\tau(i + 1) - \tau(i)) \).

It was empirically found that markets of different sizes, ranging from 70 to 424 stocks, across different time windows (from one year to 35 years) and also from different market indexes\(^2\) have only six effective dimensions ([5], [1], [2]).

This corresponds to the identification of empirically constructed variables that drive the market and, in this framework, the number of surviving eigenvalues is the effective characteristic dimension of this economic space. Taking the eigenvalues of order smaller or equal than the number of characteristic dimensions, the difference between eigenvalues from data and those obtained from surrogate data are computed. The normalized sum of those differences is the index \( S \), which measures the evolution of the distortion effect in the shape of the market space.

\(^2\)stocks from the S&P500 and Dow Jones indexes were considered
\begin{equation}
S_t = \sum_{i=1}^{6} \frac{\lambda_t(i) - \lambda'_t(i)}{\lambda'_t(i)} = \sum_{i=1}^{6} \frac{\lambda_t(i)}{\lambda'_t(i)} - 1
\end{equation}

where \(\lambda_t(1), \lambda_t(2), ..., \lambda_t(6)\) are the six largest eigenvalues of the market space and \(\lambda'_t(1), \lambda'_t(2), ..., \lambda'_t(6)\) are the largest six eigenvalues obtained from surrogate data. In computing \(S\), at a given time \(t\), both \(\lambda_t\) and \(\lambda'_t\) are obtained over the same time window and for the same set of stocks.

## 4 The 1998-2008 History

The proposed approach is applied to the history of 424 stocks of the S&P500 index that includes all the surviving firms for the period under consideration. We presume this population to be representative of the dynamics of the stock market and its behavior to be a symptom of the evolution of the economy of the US.

The changing patterns for this long period are notorious. In Figure 1, we show the object describing the evolution of the market as replicated in the three dominant directions, as obtained following the method indicated in the last section, and the object of a period of business-as-usual is compared to another formed in a period of crash. The differences are imposing, since in the latter type of situation the clustering of firms (colored accordingly to the sector they belong) and the deformation of the market space are obvious.

The object has a characteristic dimension, which allows for a description projecting its typical shape and the identification of the patterns of its evolution. The index \(S\) is useful for this identification of shapes and patterns and the results of its computation for the whole period are indicated in Figure 2, in which the impacts of the shock waves of crashes are evident. Looking for relevant distortions in the shape of the S&P500 market space through the last years, we found that amongst the highest values of the index are those computed for the moments of crash, such as the 27th October 1997 and 11th September 2001, as expected (Fig. 2). For the recent years, the 2000-2001 crash and the subprime crisis (August 2007 to the Winter 2008) attain the highest values. These crashes can be classified according to a seismography measuring their impact and characteristics, as we previously proposed ([2]).

The index provides information on the evolution of the object describing the dynamics of the markets. It indicates the moments of perturbations, proving that the dynamics is driven both by shocks and by structural change.
This is graphically evident in Figure 2 and is confirmed by the rigorous measurement of the distortion of the shape of the object describing the market. In Figure 2, the seismography of crashes is depicted, registering those crashes attaining $S > 5$.

As compared to the previous periods, the results suggest that a new regime emerges after 1997 and it is a regime of frequent storms. In this sense, this period combines some of the features of the 1920s and some of those characteristics of the manias for canals and railways, which accompanied earlier waves of technical change in the nineteenth-century ([6]).

5 Networks under Threat

The previous results suggest that, as the markets suffer a crash, there is a distortion in the dominant directions representing its leading variables. But our data prove as well that such distortion follows a sectoral pattern. Consequently, we discuss in this section the form of collective dynamics emerging under threat, using a graph representation of the network of stocks.

To characterize the additional information on the structure of the market spaces, we define the coefficient $R$, which quantifies the distribution of the intensity of correlation among stocks present in the S&P500 market space along the last 10 years.

From the matrix of distances between stocks (equation 1) computed in the reduced six dimensional space $(D^6)$ over a time window of 22 days, we apply the hierarchical clustering process to construct the minimal spanning tree (MST) that connects the $N$ securities. Then the boolean graph $B_{D^6}$ is defined by setting $b(i,j) = 1$ if $d^6(i,j) \leq L_{D^6}/2$ and $b(i,j) = 0$ if $d^6(i,j) > L_{D^6}/2$, where $L_{D^6}$ is the smallest threshold distance value $d^6(i,j)$ that assures connectivity of the whole network in the hierarchical clustering process.

Of course, this network behaves very differently when business-as-usual dominates and whenever a crash occurs, as revealed by Figures 3 to 5. These figures show, in their first sub-plots the whole network of companies, while the other seven sub-plots, the sectoral networks of Energy, Industry&Materials, Consumer, Health Care, Finance, Information Technology and Utilities stocks. The sectoral networks built from August 2000 data (Fig. 3) are sparse and sparseness predominates even in the cases of Financial and Energy sectors. Conversely, the majority of the networks for March 2008 exhibits a very high degree of connectivity, which is particularly high.
in the Industry, Utilities, Financial and Energy sectors. It is also obvious that the nature of the shock and the evolution of the market produce different sectoral dynamics. As Fig.4 highlights, for the Winter 2008 crisis, the dominant impacts were in the Financial and Utilities sectors, whereas for the case of the 2001 crisis the impacts were more intense on Technologies, Industry and Energy, as Fig.5 shows.

The figure shows the structure of each crisis, as measured according to the density of relations among sectors; the subprime crisis is concentrated in the financial and utilities sectors, in contradistinction to other episodes of turbulence, such as the crash after 9.11, concentrated in the energy and industry sectors. The profile of each of the crisis can consequently be described and measured following the indications of this topology.

Improving our investigation on the topological aspects of the stocks behaviour, leads to the definition of the coefficient \( R \), which captures the relative distribution of the distance values below and above the smallest threshold distance value \( L_{D^6} \) that insures connectivity of the whole network of companies.

\[
R_t = \frac{\sum_{d^6_{t}(i,j) \leq L_{D^6}} d^6_{t}(i,j)}{\sum_{d^6_{t}(i,j) > L_{D^6}} d^6_{t}(i,j)}
\]  

(9)

Results, as indicated in Fig.6, show that the amount of highly correlated (short-distant) fluctuations whenever a crash occurs is very large. These networks display a large amount of distances whose values are below the endogenous threshold value. This is due to the emergence of a relevant set of highly correlated fluctuations of the stock returns during market shocks forcing several weak correlated fluctuations to leave this category. Although the values of the overall network distances decrease with crashes, the emergence of highly correlated groups of stocks occupying the prominences in the market distorted shape leads to an increase of the value of the endogenous threshold \( L_{D^6} \). As a consequence, the number of distances below \( L_{D^6} \) tend to be much higher than the number of those that remain above the endogenous threshold, leading to a significant increase of the values of \( R \).

During the Subprime Crisis, \( R \) reaches 1.4, while the same coefficient computed for normal periods rests below 0.5 (computing \( R \) from surrogate data yields typical values around 0.025). The evolution of \( R \) confirms our previous results, identifying the major crashes in the period and detecting how peculiar it is the Winter 2008 crisis. The Subprime Crisis constitutes one of the highest peaks in the evolution of \( R \) for the period under consideration.
Furthermore, using the $L_{D6}/2$ threshold to filter the distances, we describe a network of companies whose stocks are required to be close enough in order to be connected. The notion of 'state' is then defined according to the connectedness of different companies, with those sharing the same state displaying synchronous behavior.

From the boolean graph $B_{D6}$ we define $s(i) = 1$ if $\exists j | b(i, j) = 1$ and $s(i) = 0$ otherwise. In so doing, we are able to identify, along different periods of observation, those companies that are connected (being closer than the threshold distance) to at least one other companies in the whole network of companies.

The space of the synchronous companies in our population is described in Figures 7 to 9. Stocks are disposed on seven rings, whose color is determined accordingly to each specific sector (Energy, Industry&Materials, Consumer, Health Care, Finance, Information Technology and Utilities). Over the rings, the position of each stock is determined by its order number. However, those evidencing a non-synchronous ($s(i) = 0$) evolution are missing, otherwise ($s(i) = 1$) they are indicated with colors accordingly to the corresponding sector. Figure 7 to 9 exhibit monthly observations.

The plots in Fig.7 show that in March 1998, a period of 'business as usual' there are few synchronous companies, in almost every sectors, except in the Energy one. Conversely, in September 2001, almost every sector displays a highly synchronous set of stocks. The last plot in Figure 8 orders the synchronous movements evidencing the sectoral dynamics as showed in March 2008, where synchronization prevails in Industry, Utilities, Financial and Energy sectors. On the contrary, in August 2000, a period of 'business as usual' there are few synchronous companies, in almost every sectors.

The three plots in Fig.9 show the strong synchronization pattern among stocks in the last 3 months of the Winter 2008 crisis. As they clearly demonstrate for the case of the subprime crisis, synchronization is related to the occurrence of market shocks and, furthermore, the dynamics of the stocks of firms tend to follow sectoral patterns.

Synchronization in the market plays is related to the occurrence of bubbles and crashes. Synchronization it at the root of the disproportionate impact of public events relative to their intrinsic information content. This applies to unanticipated public events but also to pre-scheduled news announcements. Connectivity patterns as those dictated by sectorial dynamics and the relative distributions of 'weak' and 'strong' connections provide useful insight on synchronization and market shocks.
6 Conclusion

The previous results can be interpreted from an evolutionary perspective, namely the two emergent properties here discussed.

Collective dynamics is the first emergent property in these markets. It is a property of the structure itself and it allows for the compatibility between the randomness hypothesis, which provides a fair description of the market in normal periods, and the highly structured response to the major crashes as the market space is distorted, which reveals the working of its fundamentals. Indeed, the market exhibits a large number of degrees of freedom under normal circumstances but tends to a very reduced number of degrees of freedom under crises.

Synchronization is the second emergent property of the stock markets. The results show that synchronization is a rule of behavior and that it follows sectoral patterns. Therefore, routines apply at the firm level but also as part of the configuration of the space of the market itself. This is an intuition going back to the path-breaking work of Nelson and Winter ([7]), who pointed out the organizational and institutional role of routines. Our method and results prove the emergence of meta-routines of social behavior of populations of agents in the speculative market, as a response to signals of turbulence: the meta-routine organizes the routines of agents and firms. Consequently, not only the structure of the firm but also the structure of the market itself generates patterns of behavior, through the formation of expectations and the choice of actions, that define the collective dynamics.

Competition creates routines that create bonds, and the collective dynamics of the reactions of firms and the markets to the crashes is a case in point. Tribes tend to reunite whenever the bell tolls.

References


Figure 1: Market space described along the three dominant directions, for a period of business-as-usual and for a turbulent period.
Figure 2: The evolution of the index $S$ measuring the evolution of the S&P500 structure for the surviving firms for 1988-2008 and the Seismography of crashes for the same period, considering those attaining $S > 5$

Figure 3: The sparse sectoral networks of stocks obtained for August 2000
Figure 4: The networks of stocks obtained for January 2008, when connectivity inside each sector is the dominant behaviour.

Figure 5: The highly connected (and generalized) networks in September 2001.
Figure 6: The evolution of the coefficient $R$ captures the emergence of highly correlated groups of stocks and detect how peculiar it is the Winter 2008 crisis.
Figure 7: The calm period of March 1998 contrasting to September 2001, when almost every sector displays a highly synchronous set of stocks
Figure 8: The August 2000 period of 'business as usual' displays few synchronous companies, in almost every sector contrasting to strong synchronization among stocks in the last month of the Winter 2008 crisis.
Figure 9: Strong synchronization among stocks in the last months of the Winter 2008 crisis