Testing for Asymmetry in the Inflation-Unemployment Trade-off:

Some Evidence for the USA

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Abstract Two sources of asymmetry in the Phillips curve are considered: the "capacity constraint hypothesis" and downward rigidity on wages and/or prices. The short run trade-off between inflation changes and the unemployment gap is modeled in a state-space framework that allows for time variation in both the NAIRU and the trade-off parameter. Empirical evidence for the US using the Kalman filter favors convexity of the Phillips curve, the trade-off depending positively on the unemployment gap and on inflation changes. The two sources of asymmetry produce almost equivalent observational models, so it is not possible to distinguish one from the other.

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1. Introduction

One concept of "equilibrium unemployment rate" is the (unobserved) unemployment rate that implies the constancy of inflation. It is therefore what is usually but slightly wrongly called in literature the NAIRU (non-accelerating inflation rate of unemployment). As several authors have noted, it would be better to call it the NIIRU (non-increasing inflation rate of unemployment). In this empirical work, the NAIRU/NIIRU concept is taken seriously: if the observed unemployment rate were permanently below the NAIRU, we would observe an ever-increasing inflation rate. Also, serious desinflation is only possible if observed unemployment is above the NAIRU at least for some time. In this textbook approach, the Phillips curve is vertical in the long run. Its slope increases with time, so that there is a short run trade-off between inflation and unemployment that vanishes in the long run.

The short run trade-off between unemployment and inflation is well documented for the US economy, even if there is an ongoing debate on the NAIRU concept and the reliability of NAIRU estimates[2]. It is not the purpose of this paper to provide more reliable figures for the NAIRU, although NAIRU estimates are provided as a by-product. Neither it is to model and explain the time path of the NAIRU or the natural rate, as done by Jaeger and Parkinson (1994) and Karamé (1998, 1999), although there is some methodological resemblance with their work. Here, the matter of concern is different. A recent strand of literature, both theoretical and empirical, is devoted to the possible existence of non-linearity or asymmetries in the inflation-unemployment trade-off:

- These asymmetries arise, for instance, in a "capacity constraint" model of the Phillips curve[3]. When unemployment is very low, pressure on firms under capacity constraints impinge on price changes rather than on output changes. With high unemployment and spare capacity, output changes in absolute terms are higher compared to price changes.

- "Menu costs" may imply that firms find it easier, because less costly, to increase than to decrease relative prices in a positive inflation environment. Ball and Mankiw (1995) show that a menu cost model implies an observed asymmetry in the inflation-output relationship.
- Nominal wage rigidity could imply a worsening of the inflation output trade-off for low levels of inflation, as in Akerlof, Dickens and Perry (1996). These authors propose a model that even implies a long run Phillips curve that is not vertical.

In this paper, and after a discussion of previous results by other authors in section 2 and a presentation of the empirical implementation of the Phillips curve and of data in section 3, some new evidence on asymmetries in the Phillips curve for the US is presented in section 4. Section 5 is the conclusion.

2. Previous results

Several approaches to the Phillips curve suggest there could be important non-linearities or asymmetries in the relationship between inflation or inflation changes and unemployment. Dupasquier and Rickets (1998) briefly survey some of them. In what follows, the emphasis is on two sources of asymmetry that may lead to a convex Phillips curve. These are the *capacity constraint* and the *asymmetry in price adjustment* models.

The capacity constraint idea has been under scrutiny for some time especially by researchers affiliated to research departments of two international organizations, the OECD and the IMF. This idea suggests the Phillips relationship is “really a curve” and not a straight line. In the words of Debelle and Laxton (1997): “As the unemployment rate falls below the NAIRU, bottlenecks start to develop that cause further increases in demand to have even larger inflationary consequences. Once the unemployment rate reaches some lower bound, inflation will increase at an almost infinite rate”. The basic idea, geometrically translated in a convex curve in the unemployment-inflation space, is that “excess demand conditions are much more inflationary than excess supply conditions are desinflationary”, as stated by Clark, Laxton and Rose (1996).

This kind of convexity in the short-run inflation-output or inflation-unemployment trade-off is not devoid of consequences in what concerns policy implications. In a non-linear world, a difference arises between the stochastic NAIRU, or natural rate of unemployment (NRU), and the deterministic NAIRU, or D-NAIRU. Defining the D-NAIRU as the rate of unemployment that would assure consistency between inflation and inflation expectations in the absence of stochastic shocks and the NRU as “the expected value over time of the unemployment rate that would be consistent with non-accelerating inflation, given the stochastic distribution of shocks”, Debelle and Laxton (1997) show that the NRU is greater than the D-NAIRU. Moreover, a decrease of the variance of unemployment implies a reduction in the NRU, which gets closer to the D-NAIRU. The conclusion that successful stabilization policies lead to less average unemployment comes as a corollary. Demand management policies are therefore more valuable when there is asymmetry in the inflation-unemployment trade-off. Note that if this relationship is linear, there is no influence between the range of variability of unemployment and the NRU, which coincides with the D-NAIRU.

Another important policy implication from an asymmetric Phillips curve, emphasized by Laxton, Meredith and Rose (1995) and Clark, Laxton and Rose (1996) has to do with the need for a prompt policy response to demand induced inflationary pressures. It becomes less costly in terms of output to offset them immediately than to delay the problem. Later, the underlying asymmetry will imply greater output losses if inflation is to be reduced.

The capacity constraint model and its basic implication that the Phillips curve is steeper when unemployment is below the NAIRU has been subject to different kinds of tests and empirical procedures. Turner (1995) computed output gap series for the US, Japan, Germany, France, Italy, the United Kingdom and Canada and tested the hypothesis that positive output gaps (output above trend) had a stronger impact on inflation changes than negative gaps. He found evidence of asymmetry for the US, Japan and Canada. Laxton, Meredith and Rose (1995) tested specific non-linear functional forms against the null hypothesis of a linear Phillips curve, using pooled data for the G-7 countries, and concluded in favor of asymmetry, using output gap series computed with different smoothing filters. Building on Turner (1995), Clark, Laxton and Rose (1996) again found evidence of asymmetry for the US output-inflation nexus.

In an approach that is closer to the one adopted in the empirical work presented later, Debelle and Laxton (1997) estimate and compare a Phillips *line* and a Phillips *curve* for the US. The non-linear version of the Phillips relationship is of the form:

$$p_t = p_t^e + g(u^* - u)/u + e_t, \qquad (1)$$

where p_t, p_t^e, u^* and u stand for the inflation rate, expected inflation, the NAIRU and the observed unemployment rate. The NAIRU is allowed to vary in time, with a random walk transition equation, the model being estimated by the Kalman filter. Comparison of the two models led the authors to conclude in favor of the non-linear version, after imposing “plausible restrictions on the variability in the NAIRU”. It is important to note that in this approach the non-linear model does not encompass the linear one and that an econometric test for convexity is not provided.

Dupasquier and Rickets (1998) explicitly test for the dependency of the trade-off parameter on the output gap in the Canadian economy. In their state-space formulation, a Phillips curve in terms of an output gap previously obtained is treated as a measurement equation. The trade-off parameter follows an autoregressive process with the general form:

$$g = a + r\, g_{-1} + kX_{t-1} + m_t, \qquad (2)$$

where X_{t-1} is the output gap, when the capacity constraint model is considered. The authors find some evidence in favor of capacity constraint non-linearity, but this is not robust to the inclusion of other types of asymmetry. Their tests resemble the ones presented in this paper in what concerns the trade-off parameter time variation, although a specific random perturbation to it is not considered here. On the other hand, the Phillips curve considered in the next section is specified in terms of the unemployment gap, which is estimated, so that results do not depend on an output gap obtained from other sources.

Ball and Mankiw (1994, 1995) show that the existence of menu costs may imply an *asymmetry in price adjustment* when there is positive trend inflation. Cost adjustments imply that relative price decreases by firms are more easily done by not adjusting the nominal price, thus avoiding to pay the menu cost and letting the inflation do the work. In contrast, the only way to increase relative prices is by increasing the nominal price and thus by paying the menu cost. In the latter case, price changes are done quicker. This reasoning could lead to a non-linear aggregate relationship between inflation changes and unemployment where the trade-off depends on the sign of inflation changes, as prices are more flexible upwards than downwards. Note that this asymmetry disappears with zero inflation and is reversed with negative inflation.

Akerlof, Dickens and Perry (1996) provide an alternative view of the inflation asymmetry. They start from the hypothesis that nominal wages are rigid downwards. When inflation is lower, some firms could find they could not reduce real wages, and reduce employment instead. The trade-off between inflation and unemployment could be a long run one, so the long run Phillips curve could be non vertical. In the following empirical work, the not so strong implication that the short run sacrifice ratio could be greater in times of desinflation is retained.

Dupasquier and Rickets (1998) address this idea by including dummy variables for periods of low nominal wage growth in vector X, in equation (2), for the Canadian economy. They do not find consistent evidence in favor of nominal wage rigidity induced asymmetry. In a comparable but slightly different approach, the trade-off in the following empirical work for the US is allowed to depend on actual price changes.

Both the capacity constraint model and the inflation asymmetry model imply a convex Phillips curve, as opposed to a linear one. Figure 1 depicts a linear Phillips curve. The trade-off parameter, g, is constant. In Figure 2, $g$increases with the unemployment gap, in a capacity constraint interpretation[4]. From an inflation asymmetry point of view, however, it can also be said that g is bigger, or that the curve becomes more vertical, when inflation increases. In the empirical work that follows, convexity of the US Phillips curve is tested against a linear version. These tests are based on a dependency of g on the unemployment gap and/or inflation changes.

3. The empirical Phillips curve

The expectations-augmented Phillips curve

Most studies of the inflation-unemployment trade-off start from an expectations-augmented Phillips curve similar to the following one:

$$p_t = p_t^e + g(u_t^* - u_t) + e_t , \qquad (3)$$

where p_t, p_t^e, u^* and u are the inflation rate, expected inflation, the NAIRU and the observed unemployment rate, respectively. Some authors include other regressors to control for supply shocks. Some have considered that the NAIRU is constant while others admit that it may vary with time.[5] The NAIRU is implicitly defined as the unemployment rate that makes inflation expectations consistent with observed inflation. In equation (3), there are two unobserved variables, p_t^e and u^*. Estimation of u^* and of g is only possible if assumptions are made on inflation expectations or if an inflation expectations series is available. Here, and following an approach similar to Staiger, Stock and Watson (1996), it is assumed that expectations are rational and that inflation is a unit root process.

Assume that inflation is generated by the following unit root process[6]:

$$p_t = p_{t-1} + A(L)\Delta p_{t-1} + n_t, \qquad (4)$$

where $A(L)$ is a lag polynomial representing some persistence in inflation changes and n_t is an innovation in inflation in period *t*. Rational expectations imply that:

$$p_t^e = p_{t-1} + A(L)\Delta p_{t-1}, \qquad (5)$$

Substitution of (5) in (3) leads to the following expression for the Phillips curve:

$$\Delta p_t = A(L)\Delta p_{t-1} + g(u_t^* - u_t) + e_t \qquad (6)$$

where g is the short run unemployment-inflation trade-off (the inflation increase due to one point increase in the unemployment gap). Note that both the trade-off and the NAIRU are potentially time varying. In the next section it will be assumed first that the trade-off and the NAIRU are constant. Then, the constancy assumption of the NAIRU will be dropped. These two benchmark cases will be contrasted to two asymmetry tests, namely that g depends on the unemployment past levels or on inflation past changes. Before, data for inflation and unemployment are presented and some of their properties are established.

The data

Year on year inflation was computed from quarterly data on the US consumer price index, collected from the OECD Main Economic Indicators, seasonally adjusted by national source. Unemployment is seasonally adjusted unemployment from the same source. 110 observations from 1970 to the second quarter of 1997 were considered.

Inflation and unemployment are pictured in figure 3. Visual inspection suggests that unemployment is a stationary process while inflation displays a more erratic behavior. This is confirmed by Augmented Dickey Fuller (ADF) stationarity tests presented in Table 1, where the regression included a constant but not a trend[7]. A sufficient number of lags to yield a white noise residual were included in all ADF regressions, starting from 12 lags and reducing their number according to the significance of the t-statistic of the last lag and to the Breusch-Godfrey autocorrelation in the residuals test. The hypothesis of non-stationarity for inflation cannot be dismissed, but it is clearly rejected for inflation changes. The null hypothesis of a unit root in the unemployment series is dismissed at approximately 1 percent confidence level.

These results vindicate equation (6) as a relationship between two stationary variables. Moreover, they are consistent with equation (4), which assumes that inflation is a I(1) process.

4. Inflation-unemployment trade-off, the unemployment gap and inflation changes

Constant NAIRU

If one does not allow for any kind of time variation, equation (6) becomes:

$$\Delta \pi_t = \gamma u^* + A(L)\Delta \pi_{t-1} - \gamma u_t + \varepsilon_t . \qquad (7)$$

which can be estimated by ordinary least squares. Results are presented in Table 2, where eight lagged values of the dependent variable were included (two years)[8,9].

Note that the parameter that corresponds to unemployment *u* is - *g* and that the constant is the product of two parameters of interest: constant = gu^*. Therefore, a point estimate for *u** is the following:

$$\hat{u}^* = \frac{\hat{c}}{\hat{g}} \cong 0.0647\,, \qquad (8)$$

where the "^" sign stands for an estimated parameter and *c* for "constant". An estimate for the variance of this estimator is given by[10]:

$$\overset{\wedge}{Var}(\hat{u}^*) = \begin{bmatrix} \frac{\partial u^*}{\partial c} & \frac{\partial u^*}{\partial g} \end{bmatrix} \overset{\wedge}{Cov}(\hat{c}, \hat{g}) \begin{bmatrix} \frac{\partial u^*}{\partial c} \\ \frac{\partial u^*}{\partial g} \end{bmatrix}, \qquad (9)$$

where the derivatives are evaluated at point $(\hat{c}, \hat{g})$ and $\overset{\wedge}{Cov}(\hat{c}, \hat{g})$ is the estimated variance-covariance matrix of $(\hat{c}, \hat{g})$.

Expression (9) allows the computation of a t-statistic for the estimated NAIRU:

$$t_{\hat{u}*} = -\frac{\hat{c} / \hat{g}}{\sqrt{\overset{\wedge}{Var}(\hat{u}^*)}} = 26.71\,,$$

with a p-value of 1.

These results are useful as a baseline for further work. It is remarkable that estimates for the equilibrium unemployment rate and for the trade-off are very significant. If the unemployment rate is one point below the NAIRU, i. e. if *u*=0.0547 percent, inflation increases by more 0.183 percent points in the short run (this quarter).

A state-space representation (NAIRU is time-varying)

In a state-space representation, equation (6) is taken as the measurement equation. It can be written as:

$$\Delta p_t = z_t . a_t + d_t + e_t , \qquad (10)$$

with

$$z_t = \left[\Delta p_{t-1} \quad \cdots \quad \Delta p_{t-8} \quad g_t \right], \qquad a_t = \begin{bmatrix} a_{1,t} \\ \cdots \\ a_{8,t} \\ u_t^* \end{bmatrix},$$

$$d_t = - g_t u_t , \quad \text{and } e_t \sim N(0, s_e^2).$$

Note that a is a vector of unobserved parameters while z contains observed variables (lagged values of the inflation change) and the short-run inflation-unemployment trade-off parameter g, which may vary with time and has to be estimated (more on this below).

Transition equations for the unobserved parameters can be presented in matrix notation as follows:

$$a_t = T a_{t-1} + h_t , \qquad (11)$$

with

$$T = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} \qquad \text{and } h = \begin{bmatrix} h_1 \\ \vdots \\ h_8 \\ h_9 \end{bmatrix}.$$

It is assumed that h is a normally distributed vector with a variance-covariance matrix Q:

$$h \sim N(0, Q).$$

The nullity of all elements of *Q* but the last diagonal one was imposed, so that the only parameter that really varies with time is the NAIRU, u_t^*. In fact, the NAIRU is modeled as a random walk[11].

$$u_t^* = u_{t-1}^* + h_{9,t} \, . \qquad (12)$$

Equations (10) and (11) are a state space representation of the Phillips curve and the NAIRU. If all elements of matrices *T*, *Q*, *z*, and *d*, and the value of s_e^2 were known, and initial values ("guesses") for parameters a and their standard errors were provided, the time path of a, and especially of u^*, could be estimated using the Kalman filter (see the appendix). Here one does not know the values assumed by g, and therefore by elements of the matrices *z* and *d*, so they have to be estimated. Indeed, the focus here is on g, as one is particularly interested in testing the following:

i) The "capacity utilization hypothesis"; the inflation-unemployment trade-off g depends positively on the unemployment gap ($u_t^* - u_t$). When the unemployment gap is positive and the economy is booming, prices go up very rapidly.

ii) The "desinflation asymmetry hypothesis"; there is the recurrent macroeconomic idea that the short to medium run unemployment-inflation trade-off is higher when inflation is declining than when inflation is increasing. This idea is translated in g depending negatively on inflation changes.

In the following part of this section, the state-space model (11)-(12) is estimated with different formulations for g. The simplest one is to consider that the trade-off is constant. The dependency of g on the unemployment gap and on inflation changes is tested first separately and then together.

The trade-off is constant

This more simple case is useful as a benchmark. Here q is constant:

$$q = k_1 . \qquad (13)$$

As explained in the appendix, the likelihood function depends on a_0 and P_0, the initial values for the time-varying parameters and their standard errors, and on the parameters to be estimated, k_1, s_e and s_h, which stand for the trade-off and the standard deviations of the disturbances in the measurement and transition equations[12].

Initial values for the time-varying parameters and corresponding standard errors were set to “sensible values” (see the Appendix). Estimates of k_1, s_e and s_h were obtained by numerically maximizing the likelihood function[13]. Their standard errors were estimated from the inverse of the Hessian matrix. Results are presented in Table 3.

The short run trade-off is estimated at 0.405, which is higher than the earlier estimate (0.183), when constancy of the NAIRU was imposed. The NAIRU itself varies less than the observed unemployment rate. This is apparent from figure 4, where “NAIRU 1” is the estimate from this model. Figure 5 shows the estimated unemployment gap and its close relationship with inflation changes.

The trade-off depends on the unemployment gap

The capacity constraint hypothesis was tested in several ways: q was made a function either of lagged unemployment gaps or of the mean of past unemployment gaps, and different time spans were tried. Here, g_t, and therefore the matrices *d* and *z*, are functions of past values of u^*, one of the unobserved variables. u^*_{t-k} was replaced by $\hat{u}^*_{t-k}$, its updated estimate.

One of the formulations was the following:

$$g_t = k_1 + 100k_2(\bar{u}^*_{t-1} - \bar{u}_{t-1}), \qquad (14)$$

with

$$\bar{u}_{t-1} = \sum_{i=t-4}^{t-1} \frac{u_i}{4} \text{ and } \bar{u}^*_{t-1} = \sum_{i=t-4}^{t-1} \frac{\hat{u}^*_i}{4}.$$

One would expect a significantly positive value of $\hat{k}_2$ under the capacity constraint hypothesis. The trade-off is allowed to be different when the observed unemployment rate is consistently below the NAIRU. Main results are shown in Table 4. $\hat{k}_2$ has the expected sign, and it is significant at the 5 percent level. In figure 6, NAIRU 2 is the NAIRU estimate from this model, very close to the previous one.

The trade-off depends on inflation changes

When unemployment equals the NAIRU, equation (6) implies that present inflation change is determined by past changes and innovation e:

$$\Delta p_t = A(L)\Delta p_{t-1} + e_t \qquad (15)$$

In the short-run, a one point decrease in inflation from the time path implied by (15) implies, *ceteris paribus*, an increase of the unemployment rate equal to $\frac{0.01}{g_t}$, sometimes called the “sacrifice ratio”. As stated before, the desinflation asymmetry hypothesis suggests that the sacrifice ratio is higher in periods of desinflation, implying thus that g_t is higher when inflation increases.

As before, several alternatives were tried. g_t was made a linear function of past values of inflation changes or of their sign, allowing for different lag lengths. g_t was in most formulations a statistically significant positive function of inflation changes. In the following, the trade-off decreases (i. e., the sacrifice ratio increases) when inflation last quarter was smaller than a year before:

$$g_t = k_1 + 100 k_3 (p_{t-1} - p_{t-5}) \qquad (16)$$

Results read from Table 5 tell that year on year desinflation equal to one percent point $(p_{t-1} - p_{t-5} = -0.01)$ implies a short-run increase in the sacrifice ratio from $0.0286 = \frac{0.01}{0.35}$ to $0.0311 = \frac{0.01}{0.35 - 0.0282}$.

These results suggest a considerable degree of variation in the unemployment-inflation trade-off through time. Figure 7 displays the time path of g_t, under the name of “gamma 3”, where troughs correspond to desinflation periods. Note that “gamma 3” is not very different from “gamma 2”, which is the time-varying trade off parameter from the “capacity constraint” model. “Gamma 1” is the benchmark constant trade-off estimate. NAIRU estimates, represented in figure 6 under the legend “NAIRU 3” are virtually indistinguishable from the other two estimated series.

<u>Combined model – trade-off depends on the unemployment gap and on inflation changes</u>

As there is no apparent reason why g could not depend both on the unemployment gap and on inflation changes a combination of the capacity constraint and inflation asymmetry models was envisaged. The following process for the trade-off was taken:

$$g_t = k_1 + 100k_2(\bar{u}^*_{t-1} - \bar{u}_{t-1}) + 100k_3(p_{t-1} - p_{t-5}) \qquad (17)$$

It seems from results expressed in Table 6 that asymmetry in the Phillips curve is better described as the trade-off parameter depending on the unemployment gap, as the estimate of k_2 is significant while k_3 is not. One sensible interpretation derives from the collinearity between inflation changes and the unemployment gap[14]. This explains why the linear version of the Phillips curve is rejected against both the “capacity constraint” and the “inflation asymmetry” models, taken separately. These models are, in what concerns data considered here, nearly observational equivalents. This point is completely consistent with the closeness of the trade-off parameter estimates from both models (figure 7).

The strong correlation between inflation changes and unemployment is the source of the Phillips relationship. In figure 8, standardized annual inflation changes and unemployment differences from the NAIRU are plotted. These are the two series that are supposed to have an influence on the time path of g in equation (17). The clearly move together, so that any convex linear combination of them would do almost as well as taking one of them alone.

5. Conclusion

Different approaches to the short run Phillips relationship suggest that this could indeed be a “curve”, meaning that asymmetric effects would be at work. Two of them were considered here: the “capacity constraint hypothesis” and downward rigidity on wages and/or prices.

In this empirical work on US data, and after a modeling exercise of the inflation-unemployment gap in a state-space framework that allowed for time variation in the trade-off parameter and in the NAIRU, evidence on convexity of the Phillips curve was apparent. The trade-off depends positively on inflation changes. This implies a sacrifice ratio (increase in unemployment per unit decline in inflation) that is higher in periods of desinflation, as suggested by menu costs and nominal wage rigidity approaches.

In what amounts to almost the same result from the observational point of view, the trade-off also depends on the unemployment gap, as suggested by the capacity constraint approach. The trade-off is higher when unemployment is below the NAIRU. This is so because desinflation periods tend to coincide with negative unemployment gaps.

Evidence presented in this paper clearly favors the asymmetry hypothesis. In this respect they are consistent to results presented by Debelle and Laxton (1997) in what concerns the US and also other industrialized countries, with the advantage of providing a proper econometric test of asymmetry. They also bear close resemblance to evidence from the Canadian economy provided by Dupasquier and Ricketts (1998), but here the unemployment gap is also estimated. Nevertheless, doubts on the source of asymmetry arise. It was shown here that it is possible either to consider that the trade-off depends on the unemployment gap or on inflation changes. The economics behind the two hypothesis is very different and so are probably the policy implications of each of them.

Clarifying the more detailed economic phenomena behind the non-linearity of the Phillips curve, for which evidence was presented in this paper, and establishing and possibly disentangle the policy implications of different types of asymmetry in the Phillips curve become important items in the research agenda.

Appendix[15]

i) The Kalman filter

As stated in the main text, the general state-space model considered has the following measurement equation:

$$\Delta p_t = z_t . a_t + d_t + e_t , \qquad \text{(A1)}$$

with

$$z_t = \begin{bmatrix} \Delta p_{t-1} & \cdots & \Delta p_{t-8} & g_t \end{bmatrix}, \qquad a_t = \begin{bmatrix} a_{1,t} \\ \cdots \\ a_{8,t} \\ u_t^* \end{bmatrix}, \; d_t = -g_t u_t , \qquad e_t \sim N(0, s_e^2).$$

Transition equations are:

$$a_t = T a_{t-1} + h_t , \qquad \text{(A2)}$$

with:

$$T = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix}, \qquad h = \begin{bmatrix} h_1 \\ \vdots \\ h_8 \\ h_9 \end{bmatrix}, \qquad h \sim N(0, Q).$$

Assume that a_{t-1}, the value for the state vector at time *t-1*, has an optimal estimator a_{t-1} with the covariance matrix of the estimation error given by P_{t-1}. At time *t-1*, the optimal prediction for a_t is:

$$a_{t|t-1} = T a_{t-1} \qquad \text{(A3)}$$

with the covariance of the estimation error given by:

$$P_{t|t-1} = TP_{t-1}T' + Q \qquad \text{(A4)}$$

One more observation is available at time *t*. Consequently, the estimator of a_t and corresponding estimation error are updated to:

$$a_t = a_{t|t-1} + v_t f_t^{-1} P_{t|t-1} z'_t, \qquad \text{(A5)}$$

$$P_t = P_{t|t-1} - f_t^{-1} P_{t|t-1} z'_t\, z_t P_{t|t-1} \qquad \text{(A6)}$$

with

$$v_t = \Delta p_t - z a_{t|t-1} - d_t \qquad \text{(A7)}$$

and

$$f_t = z_t P_{t|t-1} z'_t + s_e^2. \qquad \text{(A8)}$$

If initial values a_0 and P_0 were available *and* if the matrices *T*, *z*, *d*, *Q* and scalar s_e^2 did not contain any unknown parameter, a time series of optimal predictors a for a would be readily available from equations (A3) to (A6). In fact, the interest here is not so much in a but more on estimating the unknown parameters, s_e^2, s_h^2 and particularly g.

<u>Maximum likelihood estimation</u>

Maximization of the log likelihood function of the model in prediction error decomposition form is equivalent to minimize *L*:

$$L(k_3, s_e, s_n, g(k_1, k_2, k_3)) = \frac{1}{2}\sum\left(\log f_t + f_t^{-1} v_t^2\right), \qquad \text{(A9)}$$

which was done numerically. The covariance matrix was estimated by the inverse of the Hessian matrix, also numerically evaluated. Note that q may depend on a number of other parameters, according to the model in consideration.

Initial parameter values

The state vector includes a set of 9 parameters related to lagged values of Δp and the NAIRU. The initial prediction for the first 8 parameters was set to zero, with an associated standard error equal to 0.5. A 95 percent confidence interval for each of these parameters goes approximately from –1 to 1, which is a considerable degree of uncertainty. The initial NAIRU was predicted as being equal to the average within sample unemployment rate (6,37 percent). The corresponding standard error was made equal to 2. A 95 percent prediction interval contains almost all unemployment observed rates in the sample.

Notes

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[2] See Staiger, Stock and Watson (1996), Blanchard and Katz (1997) and Gordon (1997).

[3] See Debelle and Laxton (1997).

[4] Note that a "positive unemployment gap" is defined here as unemployment being below u^*, the NAIRU.

[5] See Staiger, Stock and Watson (1996).

[6] Statonarity tests presented below do not allow the rejection of a unit root in the inflation series.

[7] See MacKinnon (1996). P-values were computed with the associated program available from the *Journal of Applied Econometrics* archive.

[8] The number of lags minimized Shwarz B.I.C. There was no evidence of the errors not being white noise according to the Breusch/Godfrey LM test (1 to 12 lags).

[9] Estimation was performed using TSP. A more extended output will be provided by the author on request.

[10] See Breusch and Wickens (1988).

[11] The random walk formulation for the NAIRU is a fairly general formulation, in the sense that it allows for different types of behavior. A "return to normality model" produced similar results, so the random walk formulation was retained as a parsimonious representation.

[12] Remember that matrix Q has all elements set to zero except the last diagonal one.

[13] The author used Gauss and the *optmum* optimization routine. Programs are available on request.

[14] In the deterministic and static version of figures 1 and 2, they are even coincident.

[15] This appendix is based in Harvey (1989).

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Tables

TABLE 1 . – AUGMENTED DICKEY-FULLER (ADF) STATIONARITY TESTS

	ADF t-statistics	H_o: non-stationarity p-values
Inflation (p)	-1.04	0.737
Change in inflation (Δp)	-4.42	0.00048
Unemployment (u)	-3.44	0.01159

TABLE 2. - CONSTANT NAIRU

Sample: 1970:1 to 1997:2	Variance of residuals = 0.215×10^{-4}
110 observations	Std. error of regression = 0.464×10^{-2}
Mean of dep. var. = -0.304×10^{-3}	$R^2 = 0.614$
Std. dev. of dep. var. = 0.715×10^{-2}	Adjusted R^2 =0 .579
Sum of squared residuals = 0.215×10^{-2}	Durbin's h alt. = 1.05819
	Jarque-Bera test = 1.89259

Variables	Estimated coefficients	T-values	Two sided p-values
constant	0.0118	4.24	1.000
Dp_{-1}	0.458	5.27	1.000
Dp_{-2}	-0.152	-1.58	0.883
Dp_{-3}	0.399	4.11	1.000
Dp_{-4}	-0.671	-6.75	1.000
Dp_{-5}	0.280	2.82	0.994
Dp_{-6}	-0.038	-0.40	0.310
Dp_{-7}	0.212	2.27	0.974
Dp_{-8}	-0.445	-5.62	1.000
u	-0.183	-4.35	1.000

TABLE 3. – CONSTANT TRADE-OFF MODEL

Parameters	Estimated values	T-values	One sided p-values
$k_1(= g)$	0.405	2.36	0.991
s_e	0.385×10^{-03}	5.01	1.000
s_h	0.335×10^{-03}	1.79	0.964

TABLE 4. – TRADE-OFF DEPENDENCY ON THE UNEMPLOYMENT GAP

Parameters	Estimated values	T-values	One sided p-values
$k_1(= g)$	0.410	4.71	1.000
k_2 (capacity constraint)	0.079×10^{-03}	2.71	0.997
s_e	0.396×10^{-03}	11.50	1.000
s_h	0.229	2.99	0.999

TABLE 5. – TRADE-OFF DEPENDENCY ON INFLATION CHANGES

Parameters	Estimated values	T-values	One sided p-values
$k_1(= g)$	0.350	3.74	1.000
k_3 (inflation asymmetry)	0.0282	1.98	0.976
s_e	0.411×10^{-03}	10.84	1.000
s_h	0.211×10^{-03}	2.16	0.985

TABLE 6. – TRADE-OFF DEPENCY ON INFLATION CHANGES AND ON THE UNEMPLOYMENT GAP

Parameters	Estimated values	T-values	One sided p-values
$k_1 (= q_t)$	0.418	4.74	1.000
k_2 (capacity constraint)	0.092	1.83	0.966
k_3 (inflation asymmetry)	-0.0065	-0.293	0.385
s_e	0.393×10^{-03}	11.08	1.000
s_h	0.238×10^{-03}	2.90	0.998

Figures

FIGURE 1. – LINEAR PHILLIPS CURVE

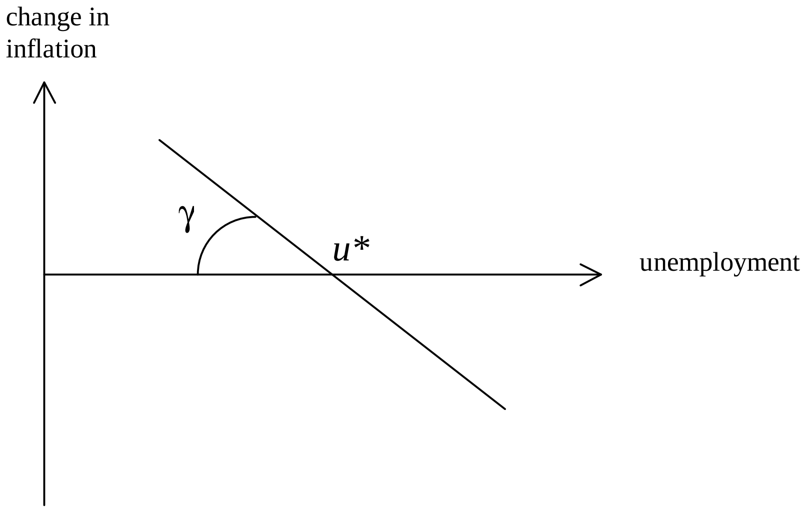

FIGURE 2. – CONVEX PHILLIPS CURVE

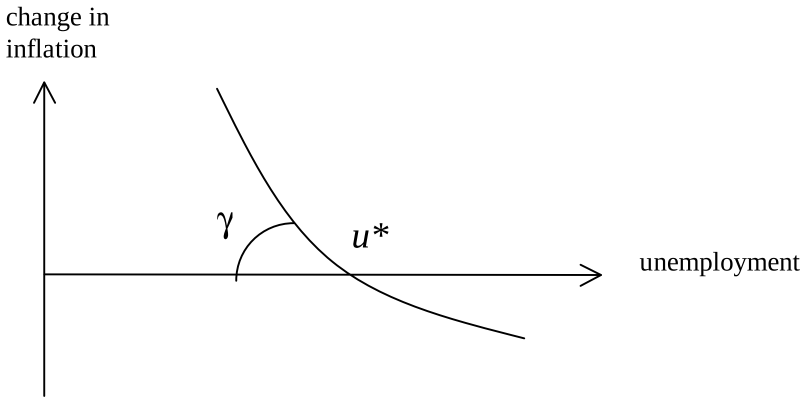

FIGURE 3. – US INFLATION AND UNEMPLOYMENT

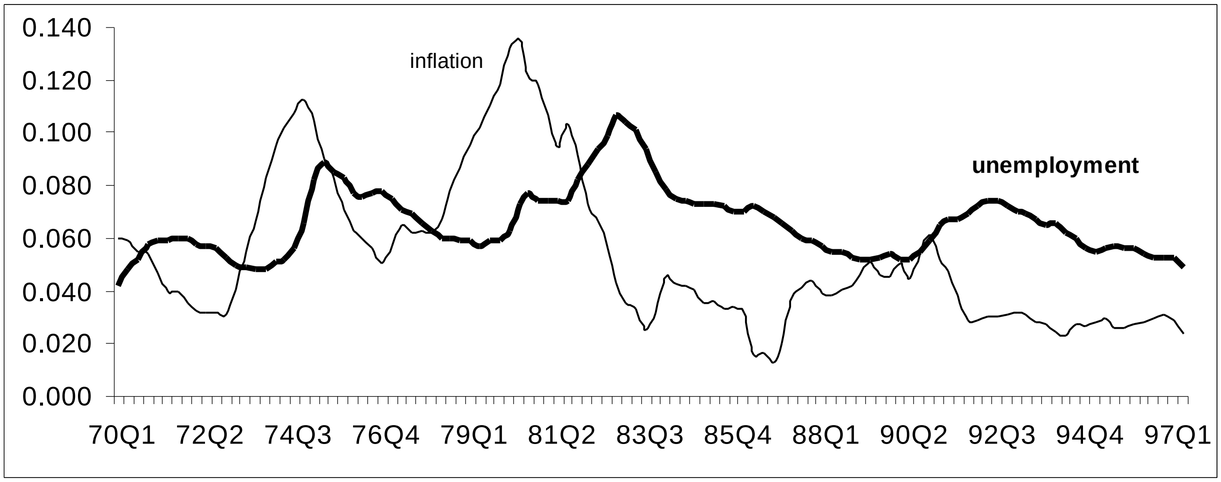

FIGURE 4. – ESTIMATED NAIRU AND OBSERVED UNEMPLOYMENT

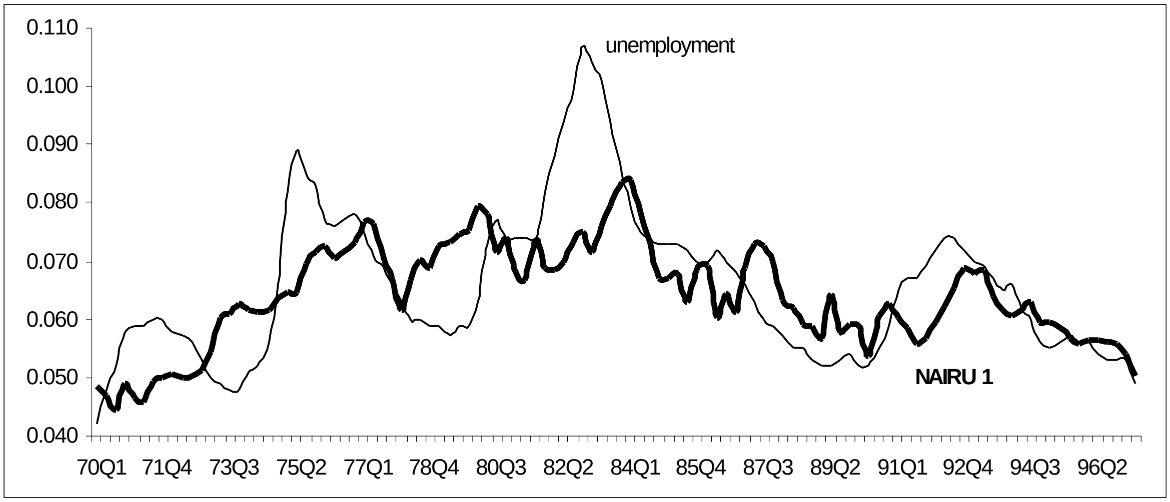

FIGURE 5. – UNEMPLOYMENT GAP AND INFLATION CHANGE

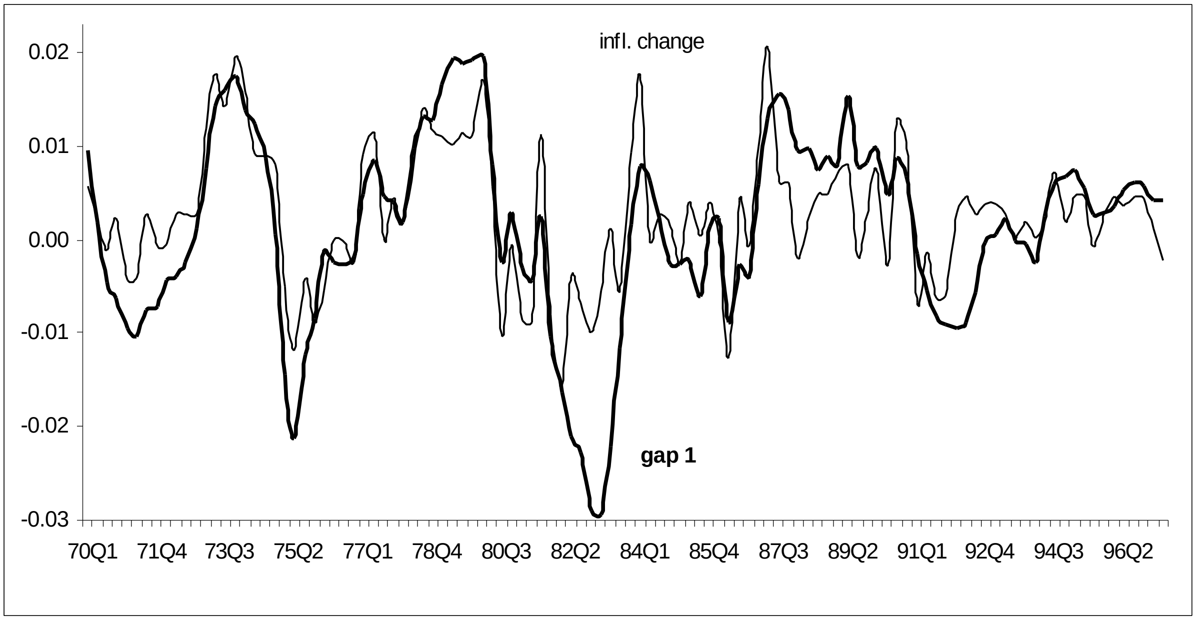

FIGURE 6. – NAIRU ESTIMATES

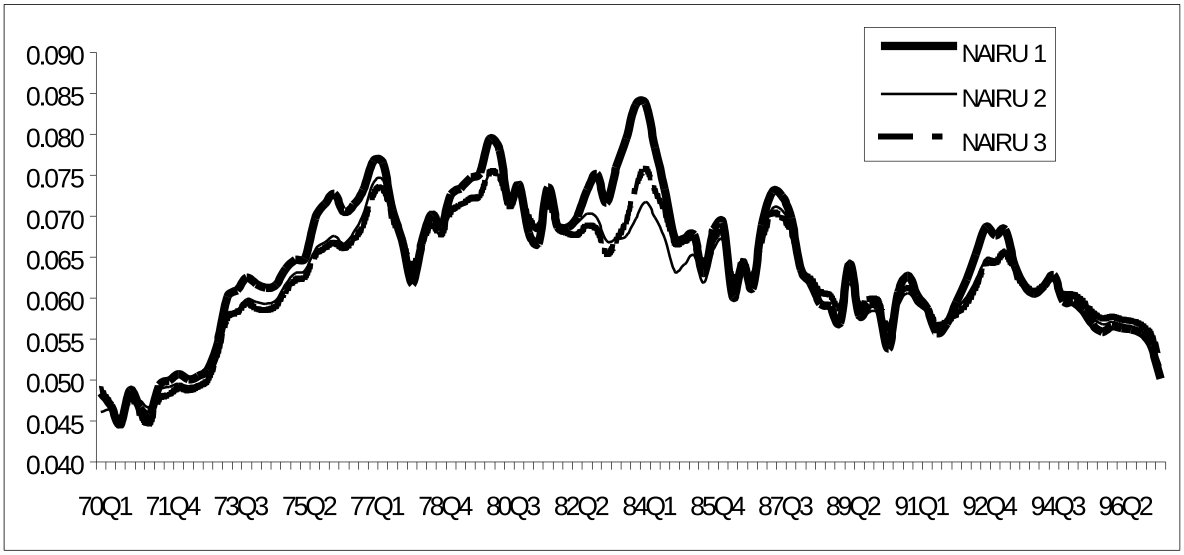

FIGURE 7. – ESTIMATED TRADE-OFF PARAMETERS

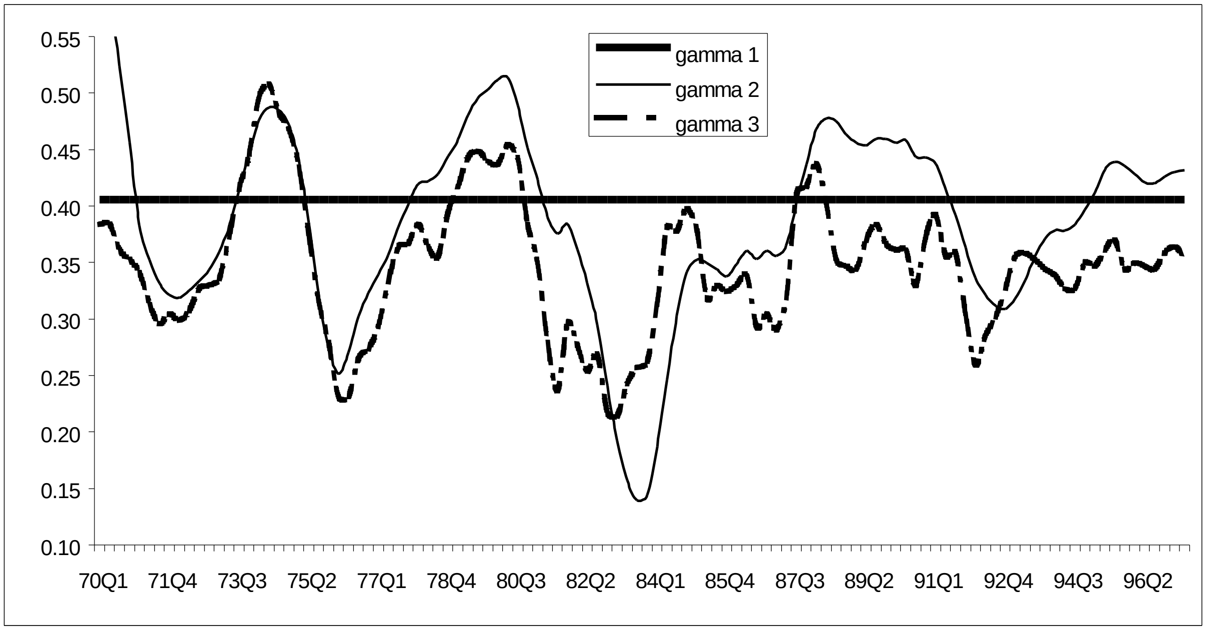

FIGURE 8. – STANDARDIZED INFLATION CHANGES AND AVERAGE UNEMPLOYMENT GAP