On the Behavior of Some Estimators for the Index of Stability

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ABSTRACT

Heavy-tailed distributions have been used to model phenomena in which extreme events occur with high probability. In these type of occurrences, it is likely that extreme events are not observable after a certain threshold. Appropriate estimators are needed to deal with this type of truncated data. By means of simulation, it is shown that the well-known Hill-Hall estimator yields highly biased estimates in the presence of truncated data. An unbiased modified maximum likelihood estimator and the tail regression estimator are studied. The expected value and variance of the estimators is assessed in the cases of stable- and Pareto-distributed data.

1. INTRODUCTION

Heavy-tailed distributions such as the Pareto and the non-Gaussian stable distributions have been extensively used to model situations in which extreme values are observed with a relatively high probability.

The interest on these type of probability laws can be traced back to the work of Vilfredo Pareto on income distribution and to the work of Paul Lévy on the properties of stable random variables. However, Mandelbrot's work on fractal behavior and self-similar laws was instrumental in establishing the use of heavy-tailed distributions for modeling real-world phenomena. In two influential papers, Mandelbrot (1960, 1963) introduced stable laws for modeling the stock market price changes. Since then, heavy-tailed distributions have been used to model phenomena in areas as diverse as statistical physics, weather forecasts, earthquake prediction, economics, and risk theory (e.g., Mandelbrot 1983, Embrechts, Klüppelberg, and Mikosh 1997). More recently, they have been applied for modeling time delays on the World Wide Web (Willinger *et al.* 1994) and computing costs of random algorithms (Gomes, Selman, and Crato 1997).

Various methods have been proposed for estimating the parameters of stable and Pareto distributions. Huhey (1991) provides a survey and a comparison of a number of methods. R. Adler, R. Feldman, and M.S. Taqqu (1998) provide a recent and thorough overview.

Many times, the single most important parameter to estimate on heavy-tailed data is the characteristic exponent α of the assumed distribution. This parameter determines the rate of decay of the tail and so the probability of occurrence of extreme events and the existence of moments for the distribution. In order to estimate this parameter α, it is natural to pay a particular attention to extreme events. In

[1]Part of this work was done by Leslie Dowling-DaCosta as a final project for her masters in Applied Statistics.

many cases, the available methods are semiparametric: they estimate α and are not otherwise concerned with the characteristics of the distribution. The maximum-likelihood estimator introduced by Hill (1975) and studied by Hall (1982), henceforth the Hill-Hall estimator, is a parametric estimator for observations generated from a Pareto distribution. This estimator, however, has been used as a semiparametric estimator for other heavy-tail distributions. Once truncated for non-extreme observations, it provides an asymptotically unbiased estimator for the tail behavior. The Hill-Hall estimator is arguably the most natural and the most used of such estimators (see Embrechts, Klüppelberg, and Mikosh, 1997, p. 331).

It is possible, however, to face censored observations and to be unable to observe the most extreme data.

Consider, for instance, the measurement of physical phenomena, such as wind speed or earthquake intensity. Non-Gaussian stable distributions have been deemed appropriate for these type of phenomena. However, in many extreme situations no measurements are available, since extreme hurricanes or very destructive earthquakes can damage the gauges. Consider also financial data, such as stock market returns for which non-Gaussian stable models have been used (De Lima 1977). In moments of high volatility, exactly when extreme data appear, many stock exchange markets have rules for limiting the transactions or even for closing the market in order to avoid extreme oscillations. Consider, finally, the study of random algorithms by means of simulation. In many cases the computing costs of some instances are so high that the simulations have to stop and run with different starting points for the same algorithm (Gomes, Selman, and Crato 1997).

An upper truncation constitutes a problem for the available estimators of stable and Paretian distributions. To the best of our knowledge, no estimator for the α parameter has been developed for dealing with this problem and it is worthwhile to develop appropriate methods.

We have studied two methods. First, we have applied a crude regression method to the log empirical quantiles, using only observations resulting from a lower and upper truncation. Second, we have used a simple variant of the Hill-Hall maximum likelihood estimator, which is a most natural and most used estimator. By means of simulation we show that these estimators deal appropriately with this upper truncation, while the original Hill-Hall estimator is seriously biased.

The plan for the rest of this paper is as follows. Section 2 presents heavy-tail distributions and discusses estimators for the characteristic exponent α. Section 3 presents the modified Hill-Hall estimator for dealing with doubly truncated data. Section 4 presents a simulation study comparing the non-corrected Hill-Hall, the modified maximum likelihood, and the regression estimators. Serious biases on the non-corrected Hill-Hall estimator are detected and the finite-sample properties of the new estimator are investigated.

2. Estimating the characteristic exponent α

We will consider probability laws, such as the stable and the Pareto laws, which asymptotically have tails of the Pareto-Lévy form, *viz.*

$$\Pr\{|X| > x\} \sim C\,x^{-\alpha}, \tag{1}$$

where α is a constant in the interval $(0, 2)$. One or both tails of these distributions have a *hyperbolic decay*. Without loss of generality, we will discuss the right tail behavior and assume that the distribution has support on the positive half line only.

The constant α is called the *characteristic exponent* or *index of stability* of the distribution. Since the existence or nonexistence of moments is completely determined by the tail behavior, α is also called the *maximal moment exponent* of the distribution. Moments of X of order less than α are finite while all higher order moments are infinite, i.e., $\alpha = \sup\{a > 0 : \mathrm{E}|X|^a < \infty\}$.

As in many real-life applications, we will consider only the asymptotic tail behavior of the distributions and the estimation of the parameter α.

Stable distributions with $\alpha < 2$—also called *non-Gaussian stable* distributions—are an important example of probability laws with heavy tails of the Pareto-Lévy type. It should be noted that distributions with tails of the form (1) are in the *domain of attraction* of stable distributions, *i.e.*, properly normalized sums of variables with tails of the Pareto-Lévy type converge in distribution to an α-stable random variable.

Stable laws

Stable laws are defined either through the form of the characteristic function or the closure to normalized convolution. More precisely, we say the random variable X follows a *stable law* if there is an $\alpha \in 90, 2]$ and a real number D_n such that

$$X_1 + X_2 + \ldots + X_n \stackrel{\mathcal{D}}{=} n^{1/\alpha} X + D_n \tag{2}$$

where $X_1, X_2, \ldots, X_n$ are independent copies of X and $\stackrel{\mathcal{D}}{=}$ means equality in distribution.

Only in a few special cases the form of a stable distribution is known in analytic form. These cases include the Gaussian distribution, which corresponds to a stable distribution with $\alpha = 2$. For $\alpha < 2$, this parameter is the maximal moment exponent of the stable law.

A modern treatment of stable distributions can be found in Samorodnitsky and Taqqu (1994).

Pareto laws

A random variable X is said to be of the Pareto type if its probability density function is of the form

$$f(x) = \alpha C^{\alpha} x^{-\alpha-1}, \quad \text{for } 0 < C \leq x < \infty,\ 0 < \alpha. \tag{3}$$

This is a Pareto distribution of the first of three kinds. Although the estimators we are going to discuss could be applied to Pareto laws without this restriction, we will consider the case $\alpha \leq 2$.

The Hill-Hall maximum likelihood estimator

Let $X_{n1} \leq X_{n2} \leq \ldots \leq X_{nn}$ be the order statistics, i.e., the ordered values of the sample $X_1, X_2 \ldots, X_n$. Set $r < n$ as a truncation value which allows to consider only the extreme observations. The Hill estimator is

$$\hat{\alpha}_r = \left(r^{-1} \sum_{j=1}^{r} \ln X_{n,n-j+1} - \ln X_{n,n-r} \right)^{-1} . \tag{4}$$

Hall (1982) has established the asymptotic normality of this estimator and determined the optimal choice of the truncation parameter r. However, since this parameter is a function of the *unknown* parameters of the distribution, it is common to use a set of different truncation values.

The maximum likelihood estimator for censored data

If the most extreme data is not observed, then we have to modify the Hill-Hall maximum likelihood estimator, by conditioning both on the lower (arbitrary) truncation and the upper censoring value.

The resulting estimator, used first in Gomes, Selman, and Crato (1998), is

$$\hat{\alpha}_{r,u} = \left(\frac{1}{r} \sum_{j=1}^{r-1} \ln X_{n,n-r+j} + \frac{u+1}{r} \ln X_{n,n} - \frac{u+r}{r} \ln X_{n,n-r} \right)^{-1} , \tag{5}$$

and its variance is given by

$$\widehat{\mathrm{Var}}(\hat{\alpha}) = \frac{\hat{\alpha}^2 (r+1)^2}{r^2 (r-1)} . \tag{6}$$

Remark. With this notation, n is the number of observed variables, $r+1$ is the number of selected upper order statistics, and u is the number of unobserved extreme values. If all variables are observable and there is no extreme truncation, then $u = 0$ and (5) is just the usual Hill-Hall estimator.

The regression estimator

If a Pareto-Lévy tail is observed, then the rate of decrease of the estimated density is hyperbolic — *i.e.*, slower than the exponential rate. The complement to one of the cumulative distribution, $\overline{F}(\cdot)$, also displays a hyperbolic decay

$$\overline{F}(x) = 1 - F(x) = \Pr\{X > x\} \sim C\, x^{-\alpha} . \tag{7}$$

Then, for a heavy-tailed random variable, a log-log plot of the frequency of observed values after x should show an approximate linear decrease at the tail. Moreover, the slope of the observed linear decrease provides an estimate of the index α. In contrast, for a distribution with an exponentially decreasing tail, the log-log plot should show a faster-than-linear decrease of the tail.

A natural regression estimator stemming from (7) is the ordinary least squares (OLS) estimator. This estimator can be readily expressed in terms of a selected number of extreme statistics. The censored data case is handled without any special problems. The only practical point to stress is the need to record the number u of unobserved data.

Assume a sample of $k = n + u$ iid random variables is drawn. Assume also that only the n smallest values of the random variable X are observed and we constitute the order statistics $X_{n1} \leq X_{n2} \leq \ldots \leq$

X_{nn}. Assume that, for $X_{n,n-r} \leq X \leq X_{nn}$, the tail distribution is of the Pareto-Lévy type. Then, the OLS regression estimator for the maximal moment exponent α is

$$\hat{\alpha} = -\frac{\sum(\log X_{ni} l_i) - (\sum \log X_{ni})(\sum l_i)/(r+1)}{\sum(\log X_{ni})^2 - \left(\sum \log X_{ni}\right)^2/(r+1)} \tag{8}$$

where $l_i = \log \frac{n+u-i}{n+u}$ and the sums range from $i = n-r$ to $i = n$. If all $k = n+u$ values of the sample are observed, then $u = 0$ and $k = n$.

3. Finite sample properties of the estimators

In order to study the finite sample performance of the estimators a large size simulation experiment was conducted.

We performed 1,000 replications for each case resulting from the following options.

Distribution. We generated random samples from both Pareto laws of the first kind (equation 3) and Stable laws with maximal positive skewness. Pareto variates X were simply generated by a direct transformation of pseudo-random uniform variates U: $X = U^{-1/\alpha}$. Stable variates were generated by the method of Chambers, Mallows and Stuck (1976).

Maximal Moment Exponent. We used the values α = 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, and 2.00. This last case does not correspond to a Pareto-Lévy tail and was studied for reference.

Sample Size We used sample sizes k = 500, 1,000, 5,000, and 10,000.

Unobserved Data Points. We censored the u most extreme observations on the range u = 1, 5, 10, 50, 100, and 500. However, we never censored more than 10% of the data since this is not a realistic situation.

Truncation. The data points actually used for the estimations are the $r+1$ most extreme observed values. This parameter was set as a fraction of the number of observed values: 1%, 2.5%, 5%, 10%, 15%, and 20

Estimators. We used (i) the uncorrected Hill-Hall estimator (Hill), as displayed in equation (4); (ii) the corrected Hill-Hall estimator (T. Hill), as displayed in equation (5); and (iii) the regression estimator (Regression), as displayed in equation (8). In all cases the standard deviation of the obtained estimates was computed. In the T. Hill case we also show the theoretical standard deviation as computed from equation (6).

The results of the simulations are reported on the 48 tables in Appendix 2.

Firstly, we note a fact that persists across all studied cases: *the Pareto distributed data allows for better estimates of* α *than the stable data*. As McCulloch (1997) has noted in the context of non-censured samples, only asymptotically does the stable distribution display a hyperbolic tail. However, the Hill-Hall estimator (4) assumes a perfectly hyperbolic tail. As a result, estimates for α are positively biased for this distribution. In contrast, the Pareto distribution has a tail which displays a hyperbolic decay across its complete support. As a result, the corresponding estimates are less biased.

This can have serious consequences when assessing the existence of moments is crucial for later data modeling (see, e.g., de Lima 1997). Consider, for instance, the case of $k = 1000$ data points, noncensored data, $u = 0$ (Hill and the T. Hill estimators coincide), and $r = 0.05(k - u) = 50$ points used for the estimation. This corresponds to the first blocks of the tables X and XXXIV. In the case $\alpha = 1.75$, the estimates for the Pareto data correctly indicate that data was generated from a distribution with no finite variance ($\alpha < 2$). The estimates for the corresponding stable data, however, indicate that data was generated from a distribution with finite variance ($\alpha \geq 2$). This type of positive bias for the stable data can again be very problematic on a neighborhood of 1, making data distributed from a stable law with infinite mean ($\alpha \leq 1$) look as if generated by a distribution with a first moment ($\alpha > 1$).

Secondly, we note that the *all estimators display marked biases*. These biases are systematically positive for the Hill-Hall estimator and systematically negative for the regression estimator. As tables X and XXXIV show, the positive bias of the Hill-Hall estimator is more serious for the stable data, while the negative bias of the regression estimator is less serious for the same data. This fact is related with the bias above noted: the stable tail shape forces the values of the estimates to increase; consequently, the Hill-Hall positive bias is worsened and the regression negative bias is weakened.

Thirdly, restricting ourselves to the situation to which the estimators were designed, i.e., noncensored data from a perfect hyperbolic tail, we note that *the Hill-Hall estimator behaves better than the regression estimator*. This is not surprising, since the Hill-Hall is the maximum likelihood estimator.If we analyze the first block of Table X, we note that both the bias and the variance of the Hill-Hall estimator are smaller than the corresponding values on the regression estimator.

Finally, we analyze the behavior of the estimators in the presence of censored data. As tables X and XXXIV perfectly show, *even a very small truncation, say $u = 5$ or even $u = 1$, is enough to seriously bias the Hill-Hall estimator*. Consider, for instance, the Pareto distributed data for sample size $k = 1000$, a 5% truncation r, and 5 censored data points. The positive bias attains values of 0.3 and 0.5, which can seriously mislead the practitioner. The magnitude of the bias increases with the actual values of α.

For the stable distributed data, the bias due to censoring adds to the existing positive bias. With the parameters considered above, data generated by a stable distribution with $\alpha = 1.5$ is likely to be mistaken by a distribution with finite moments.

The bias increases as the sample size decreases, as expected. The bias also increases as the number of censored data points increases. However, even for very large samples, say $k = 10000$, and a very extreme truncation, say $r = 0.01(k - u)$, the biases are still apparent.

These results show how unreliable the Hill-Hall estimator is in the presence of censoring. We now analyze how the new corrected estimator and the regression method are able to cope with censoring.

Firstly, we notice that *the biases do not deteriorate with censored data*, provided censoring is taken into account. The truncated maximum likelihood estimator (T. Hill) reveals a positive bias, which does not change with censoring, and the regression estimator reveals a negative bias, which does not change either. This means that, whatever drawbacks these estimators have, these drawbacks are not exacerbated by censoring, provided this fact is correctly taken into account.

Secondly, we note that *the biases given by the corrected estimator decrease with the sample size and do not increase with censoring*. We do not see any bias pattern across censoring sizes. In many cases, the bias even decreases as censoring increases, but we cannot detect any rule over the natural simulation uncertainty.

Thirdly, we note the *reasonable performance of the regression estimator*. The resulting biases are of the same order of magnitude as the ones resulting from the modified Hill-Hall estimator.

Finally, we note the *very reasonable behavior of the corrected Hill-Hall estimator*. This estimator works as well with censored as with noncensored data. The magnitude of the biases are within the limits expected for this type of parameter estimation—the problem is well-known for its difficulty.

4. Conclusions and future work

We have assessed the bias and variance of different estimators for the maximal moment exponent of Pareto and stable distributions. Specifically, we have considered the well-known Hill-Hall estimator, a modified Hill-Hall estimator able to deal with censored extreme data, and a regression method.

All methods worked better for the data generated by Pareto laws than for the data generated by stable laws, confirming recent findings of McCulloch (1997). The stable laws only asymptotically have tails of the Pareto-Lévy type, while Pareto laws have this type of tail behavior throughout all their support.

When data are uncensored and the only truncation is the one chosen by the analyst in order to select the extreme observations, the original and the modified Hill-Hall estimators coincide and the results are very similar to the ones obtained by the regression method. As it has been noted by other researchers, all methods showed some finite-sample bias, even for samples of large size. However, the maximum likelihood estimator performed generally better than the regression method.

When data are censored for the most extreme observations, we found that the original Hill-Hall estimator was very seriously biased upwards, even when a very small number of data points were truncated.

Both the regression method and the modified Hill-Hall estimator performed relatively well. The biases obtained were very small in both cases, although the regression methods has shown, in general, a larger variance.

These findings prompt the reevaluation of the empirical work that has used truncated data for estimating the tail exponent.

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APPENDIX 1. Derivation of the truncated Hall estimator

For the sake of completeness, we reproduce here the derivation of the modified Hill-Hall estimator.

Theorem 1 *Assume a sample of $k = n + u$ iid random variables is drawn. Let the order statistics for the smallest n ($n \leq k$) values on the sample be $X_{n1} \leq X_{n2} \leq \ldots \leq X_{nn}$. Assume that, for $X_{n,n-r} \leq X \leq X_{nn}$, the tail distribution is of the Pareto-Lévy type. Then, the conditional maximum-likelihood estimator for the maximal moment exponent α is*

$$\hat{\alpha}_{r,u} = \left(\frac{1}{r} \sum_{j=1}^{r-1} \ln X_{n,n-r+j} + \frac{u+1}{r} \ln X_{n,n} - \frac{u+r}{r} \ln X_{n,n-r} \right)^{-1} \tag{9}$$

and its variance is given by

$$\mathrm{Var}(\hat{\alpha}) = \frac{\hat{\alpha}^2 (r+1)^2}{r^2 (r-1)}.$$

Proof. In order to simplify the computations we will change momentarily the notation and derive the estimator in the context of a lower tail of the Pareto-Lévy form. The modification of the estimator for dealing with upper-tail values is trivial. We will follow closely the approach of Hill (1975).

$$Z^{(k)} \leq Z^{(k-1)} \leq \ldots \leq Z^{(l+s)} \leq \ldots \leq Z^{(l+1)} \leq Z^{(l)} \leq \ldots \leq Z^{(2)} \leq Z^{(1)}.$$

We have a total of . We assume that for $Z \leq Z^{(l)}$, at least, these are of the Pareto-Lévy type. However, we only observe the $l+s+1$ variables $Z^{(l+s)} \leq \ldots \leq Z^{(l+1)} \leq Z^{(l)}$, and not the $k-l-s$ variables $Z^{(k)} \leq \ldots \leq Z^{(l+s+1)}$. Thus, only the $s+1$ variables $Z^{(l+s)} \leq Z^{(l+s-1)} \leq \ldots \leq Z^{(l)}$ are included in the maximum likelihood estimator.

As in Hill (1975), the derivation is based on the Renyi (1953) representation theorem. Assume that the cumulative distribution function for Z, F, is continuous and strictly increasing. Then,

$$Z^{(i)} = F^{-1}\left(\exp\left\{-\frac{e_1}{k} - \frac{e_2}{k-1} - \ldots - \frac{e_i}{k-i+1}\right\}\right),$$

where the e_i are independent exponential random variables with mean 1. Restricting our attention to the $s+1$ variables $Z^{(l+s)} \leq \ldots \leq Z^{(l+1)} \leq Z^{(l)}$, we have

$$\ln F(Z^{(l+i)}) = -\frac{e_1}{k} - \frac{e_2}{k-1} - \ldots - \frac{e_{l+i}}{k-l-i+1}$$

and so

$$(k-l-i+1)\left(\ln F(Z^{(l+i)}) - \ln F(Z^{(l+i+1)})\right) = e_i, \quad \text{for } i = 0, 1, \ldots, s.$$

Unlike Hill (1975) we assume directly that $F(z) = Cz^{\alpha}$ for the range above, which rewritten as $\ln F(z) = C + \alpha \ln z$ and introduced in the previous equation shows that the random variables T_j

$$T_{k-l-i+1} = (k-l-i+1)\left(\ln Z^{(l+i)} - \ln Z^{(l+i+1)}\right) = e_i/\alpha, \quad \text{for } i = 0, 1, \ldots, s,$$

are independent and exponentially distributed with parameter α. Therefore, conditioning on the boundary variables $Z^{(l)}$ and $Z^{(l+s)}$, the maximum likelihood estimator for α is $\hat{\alpha} = s/\sum T_i$. It is easy to verify that the sum $\sum T_i$ collapses to

$$(k-l)\ln Z^{(l)} - \sum_{i=1}^{s-1} \ln Z^{(l+i)} - (k-l-s+1)\ln Z^{(l+s)}.$$

Therefore, the estimator becomes

$$\hat{\alpha} = \left(\frac{k-l}{s}\ln Z^{(l)} - \frac{1}{s}\sum_{i=1}^{s-1}\ln Z^{(l+i)} - \frac{k-l-s+1}{s}\ln Z^{(l+s)}\right)^{-1}.$$

In order to modify this estimator for working with upper tails of the Pareto-Lévy type, we just note that letting $Z = X^{-1}$ we have $\Pr\{Z \leq x\} = \Pr\left\{X \geq x^{-1}\right\} = Cx^{-\alpha}$. The changes lead directly to equation (5).

An estimator of the estimator's variance can be obtained by noticing that $\hat{\alpha} = s/\sum T_i$ has an inverted gamma distribution. Therefore

$$\widehat{\text{Var}}(\hat{\alpha}) = \frac{\hat{\alpha}^2(r+1)^2}{r^2(r-1)}. \tag{10}$$

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APPENDIX 2. Simulation results

TABLE I: Pareto Distribution, $k = 500$, $s = .01(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.638	0.945	1.234	1.547	1.897	2.222	2.464
(s.d.)	(0.372)	(0.557)	(0.681)	(0.828)	(1.017)	(1.239)	(1.259)
T. Hill	0.638	0.945	1.234	1.547	1.897	2.222	2.464
(s.d)	(0.372)	(0.557)	(0.681)	(0.828)	(1.017)	(1.239)	(1.259)
(Theor. s.d.)	(0.300)	(0.450)	(0.600)	(0.750)	(0.900)	(1.050)	(1.200)
Regression	0.504	0.746	0.979	1.220	1.504	1.789	1.971
(s.d.)	(0.352)	(0.537)	(0.687)	(0.905)	(1.045)	(1.341)	(1.438)
$u = 1$							
Hill	0.998	1.487	1.973	2.415	2.938	3.462	3.970
(s.d.)	(0.666)	(1.054)	(1.347)	(1.589)	(2.003)	(2.286)	(2.961)
T. Hill	0.673	1.011	1.340	1.635	1.987	2.335	2.681
(s.d)	(0.449)	(0.724)	(0.926)	(1.068)	(1.333)	(1.528)	(2.005)
(Theor. s.d.)	(0.361)	(0.541)	(0.722)	(0.902)	(1.083)	(1.263)	(1.443)
Regression	0.532	0.810	1.083	1.303	1.572	1.852	2.134
(s.d.)	(0.365)	(0.606)	(0.805)	(0.898)	(1.095)	(1.274)	(1.731)
$u = 5$							
Hill	2.140	3.172	4.197	5.463	6.546	7.709	8.453
(s.d.)	(1.445)	(2.559)	(2.826)	(4.536)	(4.734)	(5.551)	(5.708)
T. Hill	0.663	0.978	1.303	1.712	2.042	2.393	2.643
(s.d)	(0.420)	(0.719)	(0.854)	(1.487)	(1.450)	(1.685)	(1.802)
(Theor. s.d.)	(0.361)	(0.541)	(0.722)	(0.902)	(1.083)	(1.263)	(1.443)
Regression	0.562	0.831	1.112	1.454	1.742	2.034	2.257
(s.d.)	(0.360)	(0.606)	(0.757)	(1.280)	(1.260)	(1.423)	(1.578)
$u = 10$							
Hill	3.559	5.480	7.194	8.834	10.710	12.482	13.942
(s.d.)	(2.487)	(4.297)	(6.111)	(6.354)	(7.636)	(9.110)	(9.366)
T. Hill	0.666	1.028	1.362	1.666	2.022	2.348	2.658
(s.d)	(0.450)	(0.800)	(1.137)	(1.158)	(1.391)	(1.540)	(1.783)
(Theor. s.d.)	(0.361)	(0.541)	(0.722)	(0.902)	(1.083)	(1.263)	(1.443)
Regression	0.579	0.894	1.184	1.458	1.763	2.045	2.325
(s.d.)	(0.397)	(0.719)	(0.995)	(1.039)	(1.216)	(1.356)	(1.573)
$u = 50$							
Hill	15.298	21.464	30.075	38.185	43.263	50.583	60.343
(s.d.)	(11.708)	(15.273)	(24.519)	(34.554)	(29.675)	(31.395)	(45.479)
T. Hill	0.696	0.969	1.361	1.723	1.933	2.313	2.705
(s.d)	(0.543)	(0.647)	(0.990)	(1.398)	(1.197)	(1.400)	(1.797)
(Theor. s.d.)	(0.361)	(0.541)	(0.722)	(0.902)	(1.083)	(1.263)	(1.443)
Regression	0.626	0.863	1.227	1.542	1.735	2.066	2.409
(s.d.)	(0.490)	(0.575)	(0.903)	(1.248)	(1.092)	(1.262)	(1.587)

TABLE II: Pareto Distribution, $k = 1000$, $s = .01(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.560	0.838	1.114	1.393	1.651	1.961	2.219
(s.d.)	(0.208)	(0.296)	(0.391)	(0.496)	(0.570)	(0.718)	(0.781)
T. Hill	0.560	0.838	1.114	1.393	1.651	1.961	2.219
(s.d)	(0.208)	(0.296)	(0.391)	(0.496)	(0.570)	(0.718)	(0.781)
(Theor. s.d.)	(0.183)	(0.275)	(0.367)	(0.458)	(0.550)	(0.642)	(0.733)
Regression	0.460	0.681	0.909	1.130	1.343	1.597	1.787
(s.d.)	(0.218)	(0.310)	(0.426)	(0.517)	(0.629)	(0.722)	(0.794)
$u = 1$							
Hill	0.724	1.087	1.436	1.821	2.135	2.541	2.874
(s.d.)	(0.279)	(0.418)	(0.549)	(0.699)	(0.817)	(0.915)	(1.125)
T. Hill	0.566	0.851	1.125	1.423	1.675	1.991	2.255
(s.d)	(0.214)	(0.322)	(0.426)	(0.538)	(0.632)	(0.716)	(0.874)
(Theor. s.d.)	(0.196)	(0.295)	(0.393)	(0.491)	(0.589)	(0.687)	(0.786)
Regression	0.476	0.717	0.952	1.201	1.417	1.686	1.905
(s.d.)	(0.193)	(0.299)	(0.402)	(0.497)	(0.576)	(0.700)	(0.812)
$u = 5$							
Hill	1.209	1.833	2.467	3.082	3.666	4.385	4.904
(s.d.)	(0.519)	(0.703)	(0.928)	(1.241)	(1.432)	(1.760)	(1.920)
T. Hill	0.551	0.837	1.116	1.414	1.674	2.004	2.229
(s.d)	(0.217)	(0.302)	(0.406)	(0.558)	(0.634)	(0.767)	(0.832)
(Theor. s.d.)	(0.196)	(0.295)	(0.393)	(0.491)	(0.589)	(0.687)	(0.786)
Regression	0.492	0.747	0.993	1.267	1.503	1.805	1.977
(s.d.)	(0.199)	(0.277)	(0.390)	(0.525)	(0.622)	(0.723)	(0.781)
$u = 10$							
Hill	1.795	2.739	3.603	4.518	5.439	6.553	7.298
(s.d.)	(0.685)	(1.083)	(1.405)	(1.779)	(2.049)	(2.731)	(2.967)
T. Hill	0.553	0.842	1.107	1.395	1.666	2.016	2.256
(s.d)	(0.195)	(0.320)	(0.409)	(0.510)	(0.575)	(0.778)	(0.868)
(Theor. s.d.)	(0.196)	(0.295)	(0.393)	(0.491)	(0.589)	(0.687)	(0.786)
Regression	0.503	0.762	1.008	1.273	1.515	1.840	2.066
(s.d.)	(0.186)	(0.306)	(0.390)	(0.491)	(0.534)	(0.753)	(0.832)
$u = 50$							
Hill	6.470	9.591	12.737	16.076	20.246	22.552	26.243
(s.d.)	(2.823)	(3.672)	(5.302)	(6.632)	(8.780)	(8.748)	(10.737)
T. Hill	0.564	0.829	1.110	1.392	1.732	1.958	2.268
(s.d)	(0.221)	(0.288)	(0.424)	(0.506)	(0.657)	(0.695)	(0.838)
(Theor. s.d.)	(0.196)	(0.295)	(0.393)	(0.491)	(0.589)	(0.687)	(0.786)
Regression	0.527	0.773	1.031	1.303	1.618	1.829	2.130
(s.d.)	(0.215)	(0.280)	(0.406)	(0.490)	(0.648)	(0.678)	(0.815)
$u = 100$							
Hill	12.102	18.440	24.220	30.802	37.257	43.877	47.981
(s.d.)	(5.109)	(7.674)	(9.675)	(12.401)	(16.254)	(19.684)	(20.405)
T. Hill	0.553	0.850	1.108	1.394	1.704	2.008	2.201
(s.d)	(0.215)	(0.322)	(0.403)	(0.512)	(0.662)	(0.831)	(0.837)
(Theor. s.d.)	(0.196)	(0.295)	(0.393)	(0.491)	(0.589)	(0.687)	(0.786)
Regression	0.525	0.806	1.054	1.306	1.616	1.904	2.078
(s.d.)	(0.229)	(0.349)	(0.437)	(0.524)	(0.708)	(0.858)	(0.884)

TABLE III: Pareto Distribution, $k = 5000$, $s = .01(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.511	0.767	1.014	1.275	1.530	1.783	2.047
(s.d.)	(0.074)	(0.112)	(0.148)	(0.183)	(0.215)	(0.253)	(0.296)
T. Hill	0.511	0.767	1.014	1.275	1.530	1.783	2.047
(s.d)	(0.074)	(0.112)	(0.148)	(0.183)	(0.215)	(0.253)	(0.296)
(Theor. s.d.)	(0.073)	(0.109)	(0.146)	(0.182)	(0.219)	(0.255)	(0.291)
Regression	0.463	0.689	0.914	1.151	1.377	1.608	1.847
(s.d.)	(0.090)	(0.143)	(0.186)	(0.226)	(0.269)	(0.313)	(0.349)
$u = 1$							
Hill	0.551	0.824	1.090	1.373	1.634	1.913	2.218
(s.d.)	(0.084)	(0.118)	(0.156)	(0.199)	(0.237)	(0.270)	(0.312)
T. Hill	0.512	0.765	1.012	1.274	1.517	1.777	2.060
(s.d)	(0.078)	(0.110)	(0.144)	(0.184)	(0.219)	(0.250)	(0.290)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.479	0.714	0.945	1.190	1.415	1.652	1.930
(s.d.)	(0.088)	(0.128)	(0.171)	(0.209)	(0.253)	(0.283)	(0.338)
$u = 5$							
Hill	0.668	1.004	1.337	1.655	1.985	2.323	2.669
(s.d.)	(0.097)	(0.152)	(0.196)	(0.244)	(0.284)	(0.357)	(0.412)
T. Hill	0.512	0.769	1.021	1.270	1.521	1.779	2.042
(s.d)	(0.072)	(0.113)	(0.147)	(0.182)	(0.212)	(0.269)	(0.307)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.492	0.739	0.976	1.226	1.462	1.711	1.959
(s.d.)	(0.082)	(0.125)	(0.163)	(0.203)	(0.227)	(0.299)	(0.327)
$u = 10$							
Hill	0.791	1.190	1.598	1.988	2.367	2.762	3.164
(s.d.)	(0.117)	(0.186)	(0.248)	(0.302)	(0.352)	(0.400)	(0.477)
T. Hill	0.510	0.767	1.030	1.279	1.529	1.783	2.041
(s.d)	(0.073)	(0.114)	(0.156)	(0.187)	(0.222)	(0.243)	(0.293)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.494	0.744	0.998	1.234	1.482	1.732	1.981
(s.d.)	(0.078)	(0.123)	(0.168)	(0.198)	(0.245)	(0.266)	(0.327)
$u = 50$							
Hill	1.669	2.515	3.339	4.172	4.980	5.816	6.629
(s.d.)	(0.260)	(0.399)	(0.503)	(0.650)	(0.815)	(0.911)	(1.092)
T. Hill	0.512	0.771	1.023	1.281	1.527	1.785	2.028
(s.d)	(0.073)	(0.111)	(0.143)	(0.187)	(0.228)	(0.262)	(0.307)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.502	0.754	1.003	1.259	1.502	1.752	1.993
(s.d.)	(0.080)	(0.116)	(0.151)	(0.199)	(0.252)	(0.284)	(0.327)
$u = 100$							
Hill	2.704	4.058	5.415	6.769	8.058	9.449	10.808
(s.d.)	(0.440)	(0.656)	(0.842)	(1.109)	(1.264)	(1.591)	(1.772)
T. Hill	0.510	0.767	1.022	1.274	1.522	1.776	2.036
(s.d)	(0.075)	(0.109)	(0.144)	(0.189)	(0.208)	(0.258)	(0.292)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.503	0.756	1.006	1.253	1.497	1.746	2.001
(s.d.)	(0.080)	(0.118)	(0.155)	(0.200)	(0.225)	(0.274)	(0.318)
$u = 500$							
Hill	11.927	17.826	23.907	29.465	35.412	41.314	47.006
(s.d.)	(2.038)	(2.997)	(4.067)	(5.154)	(6.042)	(7.232)	(8.037)
T. Hill	0.511	0.766	1.031	1.269	1.528	1.787	2.038
(s.d)	(0.078)	(0.110)	(0.153)	(0.190)	(0.228)	(0.270)	(0.311)
(Theor. s.d.)	(0.077)	(0.116)	(0.154)	(0.193)	(0.231)	(0.270)	(0.308)
Regression	0.501	0.754	1.019	1.252	1.506	1.761	2.014
(s.d.)	(0.083)	(0.121)	(0.162)	(0.208)	(0.243)	(0.294)	(0.333)

TABLE IV: Pareto Distribution, $k = 10000$, $s = .01(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.503	0.756	1.015	1.266	1.515	1.767	2.027
(s.d.)	(0.052)	(0.075)	(0.103)	(0.124)	(0.157)	(0.173)	(0.208)
T. Hill	0.503	0.756	1.015	1.266	1.515	1.767	2.027
(s.d)	(0.052)	(0.075)	(0.103)	(0.124)	(0.157)	(0.173)	(0.208)
(Theor. s.d.)	(0.051)	(0.076)	(0.102)	(0.127)	(0.152)	(0.178)	(0.203)
Regression	0.468	0.706	0.950	1.181	1.421	1.647	1.902
(s.d.)	(0.070)	(0.099)	(0.130)	(0.168)	(0.204)	(0.235)	(0.273)
$u = 1$							
Hill	0.529	0.790	1.055	1.323	1.589	1.847	2.110
(s.d.)	(0.053)	(0.082)	(0.108)	(0.138)	(0.165)	(0.178)	(0.212)
T. Hill	0.506	0.756	1.010	1.268	1.522	1.769	2.020
(s.d)	(0.051)	(0.078)	(0.103)	(0.132)	(0.158)	(0.171)	(0.202)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.482	0.719	0.965	1.213	1.453	1.691	1.928
(s.d.)	(0.060)	(0.094)	(0.123)	(0.156)	(0.189)	(0.218)	(0.244)
$u = 5$							
Hill	0.595	0.891	1.186	1.483	1.783	2.077	2.386
(s.d.)	(0.063)	(0.089)	(0.125)	(0.153)	(0.187)	(0.221)	(0.251)
T. Hill	0.507	0.758	1.010	1.262	1.516	1.770	2.031
(s.d)	(0.053)	(0.074)	(0.105)	(0.127)	(0.155)	(0.187)	(0.209)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.493	0.739	0.983	1.226	1.473	1.725	1.978
(s.d.)	(0.060)	(0.088)	(0.121)	(0.141)	(0.175)	(0.214)	(0.237)
$u = 10$							
Hill	0.658	0.995	1.325	1.663	1.986	2.318	2.648
(s.d.)	(0.069)	(0.103)	(0.141)	(0.179)	(0.207)	(0.239)	(0.269)
T. Hill	0.502	0.760	1.012	1.268	1.515	1.768	2.021
(s.d)	(0.050)	(0.076)	(0.107)	(0.133)	(0.153)	(0.178)	(0.201)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.492	0.744	0.992	1.240	1.482	1.730	1.985
(s.d.)	(0.057)	(0.084)	(0.121)	(0.148)	(0.170)	(0.199)	(0.232)
$u = 50$							
Hill	1.115	1.673	2.241	2.809	3.355	3.925	4.494
(s.d.)	(0.119)	(0.177)	(0.245)	(0.303)	(0.363)	(0.430)	(0.506)
T. Hill	0.503	0.756	1.010	1.268	1.512	1.768	2.028
(s.d)	(0.051)	(0.076)	(0.102)	(0.130)	(0.153)	(0.181)	(0.212)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.496	0.746	0.996	1.252	1.497	1.746	2.007
(s.d.)	(0.056)	(0.083)	(0.109)	(0.145)	(0.168)	(0.195)	(0.232)
$u = 100$							
Hill	1.637	2.462	3.299	4.130	4.927	5.763	6.601
(s.d.)	(0.187)	(0.266)	(0.368)	(0.453)	(0.563)	(0.662)	(0.745)
T. Hill	0.503	0.755	1.010	1.266	1.513	1.769	2.023
(s.d)	(0.052)	(0.076)	(0.101)	(0.126)	(0.156)	(0.184)	(0.210)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.498	0.748	0.999	1.258	1.500	1.751	2.002
(s.d.)	(0.056)	(0.085)	(0.109)	(0.140)	(0.171)	(0.193)	(0.225)
$u = 500$							
Hill	5.958	8.918	11.835	14.820	17.798	20.816	23.897
(s.d.)	(0.711)	(1.094)	(1.441)	(1.711)	(1.989)	(2.398)	(2.681)
T. Hill	0.506	0.758	1.010	1.262	1.515	1.768	2.024
(s.d)	(0.053)	(0.081)	(0.106)	(0.129)	(0.152)	(0.183)	(0.205)
(Theor. s.d.)	(0.052)	(0.078)	(0.104)	(0.130)	(0.156)	(0.182)	(0.208)
Regression	0.503	0.755	1.002	1.253	1.504	1.757	2.009
(s.d.)	(0.058)	(0.087)	(0.114)	(0.139)	(0.168)	(0.198)	(0.221)
$u = 1000$							
Hill	11.834	17.720	23.820	29.576	35.173	41.469	47.262
(s.d.)	(1.451)	(2.134)	(2.785)	(3.562)	(4.259)	(5.152)	(5.820)
T. Hill	0.506	0.758	1.018	1.265	1.510	1.775	2.029
(s.d)	(0.055)	(0.081)	(0.102)	(0.134)	(0.162)	(0.188)	(0.214)
(Theor. s.d.)	(0.054)	(0.080)	(0.107)	(0.134)	(0.161)	(0.188)	(0.214)
Regression	0.503	0.753	1.011	1.254	1.501	1.761	2.020
(s.d.)	(0.061)	(0.087)	(0.114)	(0.144)	(0.175)	(0.205)	(0.228)

TABLE V: Pareto Distribution, $k = 500$, $s = .025(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.550	0.824	1.085	1.382	1.622	1.892	2.189
(s.d.)	(0.179)	(0.257)	(0.321)	(0.463)	(0.498)	(0.589)	(0.698)
T. Hill	0.550	0.824	1.085	1.382	1.622	1.892	2.189
(s.d)	(0.179)	(0.257)	(0.321)	(0.463)	(0.498)	(0.589)	(0.698)
(Theor. s.d.)	(0.163)	(0.245)	(0.327)	(0.408)	(0.490)	(0.572)	(0.653)
Regression	0.459	0.680	0.889	1.133	1.347	1.547	1.776
(s.d.)	(0.189)	(0.255)	(0.334)	(0.449)	(0.507)	(0.581)	(0.708)
$u = 1$							
Hill	0.672	1.008	1.314	1.665	1.988	2.337	2.663
(s.d.)	(0.205)	(0.319)	(0.405)	(0.546)	(0.620)	(0.756)	(0.860)
T. Hill	0.550	0.825	1.077	1.361	1.629	1.909	2.169
(s.d)	(0.169)	(0.260)	(0.331)	(0.439)	(0.504)	(0.618)	(0.695)
(Theor. s.d.)	(0.163)	(0.245)	(0.327)	(0.408)	(0.490)	(0.572)	(0.653)
Regression	0.479	0.718	0.938	1.190	1.424	1.648	1.860
(s.d.)	(0.172)	(0.255)	(0.325)	(0.429)	(0.498)	(0.608)	(0.693)
$u = 5$							
Hill	1.047	1.577	2.118	2.608	3.211	3.666	4.170
(s.d.)	(0.324)	(0.527)	(0.684)	(0.854)	(1.096)	(1.179)	(1.355)
T. Hill	0.541	0.812	1.089	1.335	1.647	1.883	2.142
(s.d)	(0.165)	(0.261)	(0.332)	(0.415)	(0.528)	(0.575)	(0.661)
(Theor. s.d.)	(0.163)	(0.245)	(0.327)	(0.408)	(0.490)	(0.572)	(0.653)
Regression	0.495	0.740	0.990	1.210	1.508	1.710	1.939
(s.d.)	(0.166)	(0.258)	(0.317)	(0.400)	(0.511)	(0.568)	(0.639)
$u = 10$							
Hill	1.516	2.244	3.016	3.832	4.480	5.245	6.078
(s.d.)	(0.492)	(0.766)	(1.016)	(1.346)	(1.360)	(1.704)	(2.105)
T. Hill	0.546	0.812	1.091	1.378	1.620	1.903	2.188
(s.d)	(0.166)	(0.268)	(0.346)	(0.440)	(0.467)	(0.594)	(0.693)
(Theor. s.d.)	(0.163)	(0.245)	(0.327)	(0.408)	(0.490)	(0.572)	(0.653)
Regression	0.505	0.753	1.008	1.274	1.499	1.756	2.025
(s.d.)	(0.162)	(0.266)	(0.334)	(0.427)	(0.463)	(0.571)	(0.667)
$u = 50$							
Hill	5.404	8.089	10.567	13.450	15.936	18.738	21.492
(s.d.)	(1.870)	(3.049)	(3.824)	(4.919)	(6.003)	(6.853)	(7.526)
T. Hill	0.550	0.824	1.079	1.373	1.627	1.907	2.195
(s.d)	(0.175)	(0.278)	(0.344)	(0.443)	(0.572)	(0.639)	(0.707)
(Theor. s.d.)	(0.172)	(0.259)	(0.345)	(0.431)	(0.517)	(0.604)	(0.690)
Regression	0.517	0.777	1.018	1.295	1.536	1.805	2.068
(s.d.)	(0.169)	(0.272)	(0.339)	(0.429)	(0.575)	(0.630)	(0.694)

TABLE VI: Pareto Distribution, $k = 1000$, $s = .025(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.515	0.790	1.047	1.298	1.562	1.835	2.102
(s.d.)	(0.108)	(0.159)	(0.222)	(0.268)	(0.325)	(0.370)	(0.436)
T. Hill	0.515	0.790	1.047	1.298	1.562	1.835	2.102
(s.d)	(0.108)	(0.159)	(0.222)	(0.268)	(0.325)	(0.370)	(0.436)
(Theor. s.d.)	(0.106)	(0.159)	(0.212)	(0.265)	(0.318)	(0.372)	(0.425)
Regression	0.447	0.684	0.906	1.118	1.350	1.595	1.817
(s.d.)	(0.123)	(0.184)	(0.247)	(0.314)	(0.377)	(0.441)	(0.505)
$u = 1$							
Hill	0.589	0.882	1.178	1.479	1.775	2.062	2.366
(s.d.)	(0.127)	(0.184)	(0.242)	(0.315)	(0.371)	(0.446)	(0.500)
T. Hill	0.520	0.777	1.039	1.306	1.567	1.818	2.088
(s.d)	(0.111)	(0.161)	(0.210)	(0.276)	(0.325)	(0.389)	(0.437)
(Theor. s.d.)	(0.109)	(0.163)	(0.217)	(0.272)	(0.326)	(0.380)	(0.434)
Regression	0.472	0.701	0.942	1.189	1.416	1.642	1.884
(s.d.)	(0.119)	(0.172)	(0.232)	(0.296)	(0.342)	(0.408)	(0.468)
$u = 5$							
Hill	0.791	1.207	1.608	2.006	2.429	2.804	3.213
(s.d.)	(0.177)	(0.264)	(0.349)	(0.427)	(0.553)	(0.611)	(0.709)
T. Hill	0.514	0.782	1.041	1.300	1.572	1.822	2.084
(s.d)	(0.112)	(0.166)	(0.216)	(0.265)	(0.341)	(0.389)	(0.440)
(Theor. s.d.)	(0.109)	(0.163)	(0.217)	(0.272)	(0.326)	(0.380)	(0.434)
Regression	0.486	0.732	0.970	1.221	1.468	1.714	1.963
(s.d.)	(0.120)	(0.171)	(0.222)	(0.272)	(0.348)	(0.408)	(0.450)
$u = 10$							
Hill	1.034	1.558	2.066	2.634	3.186	3.660	4.166
(s.d.)	(0.234)	(0.336)	(0.452)	(0.585)	(0.760)	(0.846)	(0.962)
T. Hill	0.518	0.782	1.038	1.321	1.594	1.833	2.083
(s.d)	(0.108)	(0.159)	(0.214)	(0.280)	(0.358)	(0.401)	(0.454)
(Theor. s.d.)	(0.109)	(0.163)	(0.217)	(0.272)	(0.326)	(0.380)	(0.434)
Regression	0.493	0.744	0.988	1.258	1.506	1.744	1.973
(s.d.)	(0.111)	(0.162)	(0.222)	(0.289)	(0.359)	(0.414)	(0.459)
$u = 50$							
Hill	2.871	4.314	5.760	7.223	8.592	10.077	11.582
(s.d.)	(0.733)	(1.078)	(1.344)	(1.787)	(2.103)	(2.614)	(2.845)
T. Hill	0.525	0.785	1.051	1.311	1.561	1.835	2.104
(s.d)	(0.121)	(0.176)	(0.226)	(0.293)	(0.343)	(0.422)	(0.463)
(Theor. s.d.)	(0.111)	(0.167)	(0.222)	(0.278)	(0.334)	(0.389)	(0.445)
Regression	0.506	0.755	1.008	1.261	1.514	1.776	2.035
(s.d.)	(0.123)	(0.181)	(0.233)	(0.302)	(0.365)	(0.428)	(0.479)
$u = 100$							
Hill	5.294	7.903	10.707	13.341	15.785	18.424	21.112
(s.d.)	(1.290)	(2.016)	(2.751)	(3.497)	(3.938)	(4.711)	(5.053)
T. Hill	0.524	0.788	1.058	1.323	1.564	1.821	2.093
(s.d)	(0.112)	(0.181)	(0.237)	(0.306)	(0.340)	(0.422)	(0.469)
(Theor. s.d.)	(0.114)	(0.171)	(0.228)	(0.285)	(0.342)	(0.399)	(0.456)
Regression	0.509	0.766	1.029	1.282	1.523	1.769	2.027
(s.d.)	(0.115)	(0.183)	(0.246)	(0.327)	(0.353)	(0.432)	(0.481)

TABLE VII: Pareto Distribution, $k = 5000$, $s = .025(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.503	0.755	1.007	1.264	1.515	1.770	2.017
(s.d.)	(0.046)	(0.069)	(0.090)	(0.113)	(0.138)	(0.162)	(0.184)
T. Hill	0.503	0.755	1.007	1.264	1.515	1.770	2.017
(s.d)	(0.046)	(0.069)	(0.090)	(0.113)	(0.138)	(0.162)	(0.184)
(Theor. s.d.)	(0.045)	(0.068)	(0.091)	(0.113)	(0.136)	(0.158)	(0.181)
Regression	0.474	0.709	0.950	1.192	1.430	1.666	1.899
(s.d.)	(0.062)	(0.090)	(0.116)	(0.152)	(0.181)	(0.212)	(0.247)
$u = 1$							
Hill	0.522	0.785	1.042	1.304	1.564	1.834	2.097
(s.d.)	(0.048)	(0.071)	(0.092)	(0.124)	(0.148)	(0.161)	(0.189)
T. Hill	0.503	0.757	1.005	1.258	1.509	1.769	2.022
(s.d)	(0.046)	(0.069)	(0.089)	(0.119)	(0.142)	(0.155)	(0.182)
(Theor. s.d.)	(0.045)	(0.068)	(0.091)	(0.114)	(0.136)	(0.159)	(0.182)
Regression	0.484	0.729	0.965	1.213	1.454	1.700	1.940
(s.d.)	(0.056)	(0.085)	(0.110)	(0.144)	(0.180)	(0.197)	(0.226)
$u = 5$							
Hill	0.578	0.867	1.150	1.446	1.731	2.032	2.310
(s.d.)	(0.054)	(0.080)	(0.107)	(0.134)	(0.161)	(0.182)	(0.216)
T. Hill	0.504	0.756	1.003	1.261	1.510	1.773	2.015
(s.d)	(0.047)	(0.069)	(0.092)	(0.115)	(0.138)	(0.157)	(0.186)
(Theor. s.d.)	(0.045)	(0.068)	(0.091)	(0.114)	(0.136)	(0.159)	(0.182)
Regression	0.493	0.739	0.982	1.229	1.473	1.734	1.968
(s.d.)	(0.054)	(0.080)	(0.110)	(0.129)	(0.163)	(0.187)	(0.208)
$u = 10$							
Hill	0.636	0.951	1.268	1.589	1.906	2.222	2.547
(s.d.)	(0.058)	(0.084)	(0.116)	(0.144)	(0.173)	(0.210)	(0.242)
T. Hill	0.505	0.755	1.008	1.263	1.512	1.765	2.024
(s.d)	(0.045)	(0.066)	(0.090)	(0.113)	(0.135)	(0.162)	(0.189)
(Theor. s.d.)	(0.045)	(0.068)	(0.091)	(0.114)	(0.136)	(0.159)	(0.182)
Regression	0.498	0.740	0.991	1.243	1.481	1.732	1.987
(s.d.)	(0.053)	(0.076)	(0.102)	(0.131)	(0.153)	(0.186)	(0.214)
$u = 50$							
Hill	1.012	1.519	2.018	2.530	3.030	3.541	4.079
(s.d.)	(0.093)	(0.142)	(0.195)	(0.242)	(0.291)	(0.338)	(0.403)
T. Hill	0.505	0.756	1.004	1.257	1.508	1.760	2.027
(s.d)	(0.043)	(0.067)	(0.090)	(0.111)	(0.136)	(0.159)	(0.186)
(Theor. s.d.)	(0.046)	(0.068)	(0.091)	(0.114)	(0.137)	(0.160)	(0.183)
Regression	0.500	0.748	0.994	1.247	1.489	1.741	2.001
(s.d.)	(0.047)	(0.073)	(0.098)	(0.122)	(0.148)	(0.178)	(0.203)
$u = 100$							
Hill	1.456	2.169	2.889	3.612	4.366	5.053	5.791
(s.d.)	(0.148)	(0.214)	(0.284)	(0.343)	(0.441)	(0.484)	(0.574)
T. Hill	0.506	0.754	1.006	1.259	1.518	1.759	2.018
(s.d)	(0.046)	(0.068)	(0.091)	(0.110)	(0.140)	(0.153)	(0.185)
(Theor. s.d.)	(0.046)	(0.069)	(0.092)	(0.115)	(0.137)	(0.160)	(0.183)
Regression	0.503	0.748	0.999	1.249	1.503	1.745	2.002
(s.d.)	(0.050)	(0.074)	(0.099)	(0.120)	(0.149)	(0.165)	(0.201)
$u = 500$							
Hill	5.130	7.678	10.344	12.944	15.346	17.990	20.603
(s.d.)	(0.549)	(0.802)	(1.123)	(1.408)	(1.633)	(1.912)	(2.195)
T. Hill	0.504	0.755	1.014	1.268	1.509	1.767	2.020
(s.d)	(0.049)	(0.070)	(0.097)	(0.120)	(0.144)	(0.167)	(0.188)
(Theor. s.d.)	(0.048)	(0.072)	(0.096)	(0.120)	(0.144)	(0.168)	(0.192)
Regression	0.502	0.749	1.011	1.258	1.500	1.757	2.010
(s.d.)	(0.052)	(0.076)	(0.107)	(0.131)	(0.155)	(0.183)	(0.203)

TABLE VIII: Pareto Distribution, $k = 10000$, $s = .025(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.502	0.753	1.007	1.251	1.509	1.762	2.009
(s.d.)	(0.032)	(0.047)	(0.062)	(0.082)	(0.091)	(0.114)	(0.128)
T. Hill	0.502	0.753	1.007	1.251	1.509	1.762	2.009
(s.d)	(0.032)	(0.047)	(0.062)	(0.082)	(0.091)	(0.114)	(0.128)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.095)	(0.111)	(0.127)
Regression	0.485	0.725	0.969	1.204	1.449	1.700	1.921
(s.d.)	(0.043)	(0.066)	(0.085)	(0.113)	(0.127)	(0.147)	(0.179)
$u = 1$							
Hill	0.511	0.769	1.026	1.278	1.533	1.791	2.058
(s.d.)	(0.033)	(0.049)	(0.065)	(0.081)	(0.101)	(0.111)	(0.129)
T. Hill	0.500	0.753	1.005	1.251	1.502	1.754	2.016
(s.d)	(0.032)	(0.048)	(0.064)	(0.079)	(0.099)	(0.108)	(0.126)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.488	0.732	0.980	1.220	1.462	1.712	1.967
(s.d.)	(0.041)	(0.059)	(0.083)	(0.098)	(0.127)	(0.140)	(0.159)
$u = 5$							
Hill	0.541	0.816	1.085	1.361	1.633	1.898	2.180
(s.d.)	(0.035)	(0.055)	(0.070)	(0.086)	(0.108)	(0.120)	(0.136)
T. Hill	0.500	0.753	1.001	1.256	1.507	1.752	2.013
(s.d)	(0.032)	(0.051)	(0.064)	(0.078)	(0.099)	(0.111)	(0.125)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.492	0.740	0.986	1.236	1.484	1.724	1.984
(s.d.)	(0.038)	(0.061)	(0.076)	(0.094)	(0.117)	(0.140)	(0.154)
$u = 10$							
Hill	0.577	0.863	1.148	1.441	1.726	2.024	2.302
(s.d.)	(0.037)	(0.053)	(0.076)	(0.094)	(0.113)	(0.135)	(0.149)
T. Hill	0.502	0.751	1.000	1.255	1.504	1.762	2.005
(s.d)	(0.032)	(0.045)	(0.066)	(0.081)	(0.096)	(0.117)	(0.128)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.496	0.745	0.990	1.240	1.485	1.738	1.977
(s.d.)	(0.038)	(0.055)	(0.078)	(0.094)	(0.114)	(0.135)	(0.154)
$u = 50$							
Hill	0.780	1.174	1.558	1.955	2.348	2.744	3.139
(s.d.)	(0.050)	(0.079)	(0.104)	(0.137)	(0.152)	(0.180)	(0.208)
T. Hill	0.501	0.753	1.001	1.255	1.506	1.761	2.012
(s.d)	(0.031)	(0.049)	(0.064)	(0.084)	(0.095)	(0.109)	(0.126)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.498	0.748	0.995	1.249	1.497	1.751	1.997
(s.d.)	(0.034)	(0.055)	(0.074)	(0.094)	(0.106)	(0.125)	(0.139)
$u = 100$							
Hill	1.011	1.511	2.029	2.522	3.030	3.535	4.048
(s.d.)	(0.069)	(0.101)	(0.141)	(0.174)	(0.210)	(0.238)	(0.275)
T. Hill	0.503	0.753	1.008	1.254	1.506	1.759	2.015
(s.d)	(0.032)	(0.048)	(0.066)	(0.081)	(0.098)	(0.112)	(0.128)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.501	0.749	1.003	1.248	1.498	1.750	2.007
(s.d.)	(0.036)	(0.053)	(0.073)	(0.090)	(0.108)	(0.123)	(0.140)
$u = 500$							
Hill	2.757	4.127	5.518	6.884	8.281	9.678	11.078
(s.d.)	(0.202)	(0.305)	(0.408)	(0.508)	(0.590)	(0.724)	(0.808)
T. Hill	0.501	0.751	1.003	1.253	1.509	1.758	2.013
(s.d)	(0.033)	(0.049)	(0.065)	(0.082)	(0.098)	(0.116)	(0.129)
(Theor. s.d.)	(0.033)	(0.049)	(0.065)	(0.082)	(0.098)	(0.114)	(0.131)
Regression	0.500	0.748	1.000	1.249	1.505	1.754	2.010
(s.d.)	(0.036)	(0.054)	(0.072)	(0.089)	(0.107)	(0.126)	(0.142)
$u = 1000$							
Hill	5.133	7.684	10.211	12.782	15.280	17.898	20.324
(s.d.)	(0.375)	(0.588)	(0.799)	(0.971)	(1.157)	(1.394)	(1.587)
T. Hill	0.505	0.756	1.004	1.258	1.504	1.759	2.000
(s.d)	(0.033)	(0.052)	(0.069)	(0.086)	(0.099)	(0.118)	(0.139)
(Theor. s.d.)	(0.034)	(0.050)	(0.067)	(0.084)	(0.101)	(0.117)	(0.134)
Regression	0.503	0.754	1.003	1.255	1.498	1.756	1.992
(s.d.)	(0.036)	(0.056)	(0.076)	(0.094)	(0.108)	(0.128)	(0.149)

TABLE IX: Pareto Distribution, $k = 500$, $s = .05(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.519	0.782	1.041	1.305	1.560	1.824	2.103
(s.d.)	(0.107)	(0.168)	(0.210)	(0.268)	(0.315)	(0.384)	(0.427)
T. Hill	0.519	0.782	1.041	1.305	1.560	1.824	2.103
(s.d)	(0.107)	(0.168)	(0.210)	(0.268)	(0.315)	(0.384)	(0.427)
(Theor. s.d.)	(0.106)	(0.159)	(0.212)	(0.265)	(0.318)	(0.372)	(0.425)
Regression	0.451	0.680	0.905	1.127	1.342	1.588	1.812
(s.d.)	(0.123)	(0.186)	(0.252)	(0.309)	(0.374)	(0.459)	(0.505)
$u = 1$							
Hill	0.588	0.878	1.185	1.484	1.785	2.033	2.368
(s.d.)	(0.129)	(0.180)	(0.244)	(0.318)	(0.377)	(0.422)	(0.494)
T. Hill	0.518	0.775	1.044	1.309	1.576	1.794	2.089
(s.d)	(0.112)	(0.159)	(0.214)	(0.279)	(0.331)	(0.370)	(0.432)
(Theor. s.d.)	(0.109)	(0.163)	(0.217)	(0.272)	(0.326)	(0.380)	(0.434)
Regression	0.467	0.697	0.945	1.189	1.425	1.619	1.887
(s.d.)	(0.118)	(0.177)	(0.235)	(0.303)	(0.359)	(0.390)	(0.460)
$u = 5$							
Hill	0.812	1.201	1.612	2.003	2.441	2.789	3.239
(s.d.)	(0.181)	(0.267)	(0.362)	(0.450)	(0.559)	(0.597)	(0.720)
T. Hill	0.527	0.777	1.046	1.296	1.580	1.811	2.097
(s.d)	(0.113)	(0.167)	(0.226)	(0.284)	(0.351)	(0.368)	(0.446)
(Theor. s.d.)	(0.109)	(0.163)	(0.217)	(0.272)	(0.326)	(0.380)	(0.434)
Regression	0.496	0.728	0.982	1.214	1.474	1.709	1.957
(s.d.)	(0.118)	(0.173)	(0.230)	(0.293)	(0.360)	(0.392)	(0.457)
$u = 10$							
Hill	1.052	1.545	2.078	2.587	3.129	3.592	4.190
(s.d.)	(0.249)	(0.330)	(0.453)	(0.565)	(0.721)	(0.809)	(0.950)
T. Hill	0.527	0.777	1.048	1.303	1.568	1.806	2.100
(s.d)	(0.119)	(0.160)	(0.216)	(0.265)	(0.343)	(0.382)	(0.452)
(Theor. s.d.)	(0.109)	(0.163)	(0.217)	(0.272)	(0.326)	(0.380)	(0.434)
Regression	0.500	0.737	0.995	1.241	1.486	1.726	1.998
(s.d.)	(0.124)	(0.166)	(0.225)	(0.277)	(0.352)	(0.398)	(0.454)
$u = 50$							
Hill	2.955	4.442	5.998	7.405	9.058	10.326	11.785
(s.d.)	(0.740)	(1.059)	(1.464)	(1.901)	(2.209)	(2.607)	(3.123)
T. Hill	0.523	0.787	1.052	1.305	1.595	1.828	2.079
(s.d)	(0.119)	(0.171)	(0.227)	(0.298)	(0.362)	(0.422)	(0.488)
(Theor. s.d.)	(0.114)	(0.171)	(0.228)	(0.285)	(0.342)	(0.399)	(0.456)
Regression	0.506	0.757	1.015	1.257	1.541	1.761	2.010
(s.d.)	(0.123)	(0.176)	(0.237)	(0.304)	(0.374)	(0.427)	(0.500)

TABLE X: Pareto Distribution, $k = 1000$, $s = .05(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.511	0.765	1.019	1.282	1.532	1.800	2.035
(s.d.)	(0.076)	(0.114)	(0.150)	(0.183)	(0.212)	(0.267)	(0.313)
T. Hill	0.511	0.765	1.019	1.282	1.532	1.800	2.035
(s.d)	(0.076)	(0.114)	(0.150)	(0.183)	(0.212)	(0.267)	(0.313)
(Theor. s.d.)	(0.073)	(0.109)	(0.146)	(0.182)	(0.219)	(0.255)	(0.291)
Regression	0.462	0.697	0.919	1.160	1.379	1.633	1.847
(s.d.)	(0.092)	(0.135)	(0.184)	(0.224)	(0.268)	(0.329)	(0.381)
$u = 1$							
Hill	0.547	0.827	1.093	1.378	1.643	1.931	2.199
(s.d.)	(0.080)	(0.122)	(0.162)	(0.200)	(0.241)	(0.287)	(0.319)
T. Hill	0.508	0.768	1.015	1.279	1.526	1.793	2.043
(s.d)	(0.074)	(0.113)	(0.149)	(0.184)	(0.223)	(0.265)	(0.295)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.474	0.719	0.944	1.192	1.428	1.671	1.910
(s.d.)	(0.084)	(0.132)	(0.172)	(0.213)	(0.253)	(0.306)	(0.340)
$u = 5$							
Hill	0.668	1.001	1.338	1.681	2.000	2.338	2.676
(s.d.)	(0.105)	(0.145)	(0.201)	(0.247)	(0.300)	(0.369)	(0.413)
T. Hill	0.511	0.766	1.022	1.287	1.528	1.790	2.048
(s.d)	(0.079)	(0.110)	(0.151)	(0.184)	(0.226)	(0.273)	(0.311)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.491	0.735	0.981	1.234	1.464	1.716	1.954
(s.d.)	(0.088)	(0.123)	(0.166)	(0.202)	(0.251)	(0.297)	(0.334)
$u = 10$							
Hill	0.787	1.190	1.575	1.979	2.374	2.794	3.152
(s.d.)	(0.117)	(0.179)	(0.233)	(0.306)	(0.376)	(0.408)	(0.471)
T. Hill	0.508	0.768	1.015	1.275	1.530	1.796	2.034
(s.d)	(0.073)	(0.112)	(0.144)	(0.186)	(0.231)	(0.250)	(0.292)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.493	0.744	0.983	1.238	1.480	1.733	1.969
(s.d.)	(0.081)	(0.125)	(0.156)	(0.209)	(0.249)	(0.269)	(0.314)
$u = 50$							
Hill	1.702	2.553	3.433	4.264	5.124	6.013	6.871
(s.d.)	(0.278)	(0.393)	(0.587)	(0.694)	(0.820)	(1.008)	(1.097)
T. Hill	0.509	0.761	1.021	1.275	1.533	1.805	2.048
(s.d)	(0.077)	(0.108)	(0.155)	(0.187)	(0.223)	(0.278)	(0.298)
(Theor. s.d.)	(0.075)	(0.113)	(0.151)	(0.188)	(0.226)	(0.264)	(0.301)
Regression	0.498	0.747	0.999	1.249	1.508	1.772	2.005
(s.d.)	(0.082)	(0.116)	(0.164)	(0.196)	(0.240)	(0.291)	(0.323)
$u = 100$							
Hill	2.871	4.352	5.788	7.232	8.643	10.177	11.576
(s.d.)	(0.491)	(0.754)	(0.947)	(1.240)	(1.480)	(1.717)	(1.970)
T. Hill	0.509	0.769	1.022	1.275	1.531	1.788	2.047
(s.d)	(0.078)	(0.120)	(0.149)	(0.191)	(0.232)	(0.276)	(0.310)
(Theor. s.d.)	(0.077)	(0.116)	(0.154)	(0.193)	(0.231)	(0.270)	(0.308)
Regression	0.501	0.755	1.008	1.253	1.508	1.758	2.006
(s.d.)	(0.084)	(0.128)	(0.161)	(0.201)	(0.250)	(0.297)	(0.332)

TABLE XI: Pareto Distribution, $k = 5000$, $s = .05(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.503	0.752	1.001	1.257	1.506	1.757	2.008
(s.d.)	(0.032)	(0.048)	(0.062)	(0.080)	(0.093)	(0.112)	(0.129)
T. Hill	0.503	0.752	1.001	1.257	1.506	1.757	2.008
(s.d)	(0.032)	(0.048)	(0.062)	(0.080)	(0.093)	(0.112)	(0.129)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.095)	(0.111)	(0.127)
Regression	0.483	0.723	0.961	1.204	1.452	1.691	1.933
(s.d.)	(0.043)	(0.067)	(0.090)	(0.107)	(0.128)	(0.151)	(0.174)
$u = 1$							
Hill	0.513	0.770	1.023	1.280	1.537	1.794	2.057
(s.d.)	(0.032)	(0.047)	(0.066)	(0.086)	(0.096)	(0.114)	(0.133)
T. Hill	0.502	0.754	1.002	1.253	1.505	1.758	2.014
(s.d)	(0.031)	(0.046)	(0.065)	(0.084)	(0.094)	(0.111)	(0.130)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.490	0.733	0.975	1.222	1.463	1.714	1.960
(s.d.)	(0.040)	(0.059)	(0.082)	(0.106)	(0.122)	(0.143)	(0.159)
$u = 5$							
Hill	0.544	0.815	1.085	1.360	1.633	1.899	2.171
(s.d.)	(0.035)	(0.051)	(0.069)	(0.090)	(0.104)	(0.123)	(0.136)
T. Hill	0.502	0.752	1.002	1.255	1.507	1.753	2.004
(s.d)	(0.032)	(0.047)	(0.064)	(0.082)	(0.096)	(0.112)	(0.124)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.494	0.739	0.987	1.237	1.482	1.723	1.975
(s.d.)	(0.038)	(0.057)	(0.076)	(0.096)	(0.120)	(0.134)	(0.157)
$u = 10$							
Hill	0.578	0.864	1.151	1.438	1.724	2.016	2.303
(s.d.)	(0.037)	(0.055)	(0.074)	(0.094)	(0.110)	(0.128)	(0.149)
T. Hill	0.503	0.752	1.003	1.253	1.502	1.756	2.007
(s.d)	(0.032)	(0.047)	(0.063)	(0.081)	(0.095)	(0.110)	(0.128)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.498	0.744	0.993	1.243	1.482	1.734	1.983
(s.d.)	(0.038)	(0.056)	(0.075)	(0.095)	(0.112)	(0.129)	(0.151)
$u = 50$							
Hill	0.784	1.175	1.568	1.958	2.340	2.737	3.129
(s.d.)	(0.054)	(0.079)	(0.103)	(0.131)	(0.154)	(0.189)	(0.205)
T. Hill	0.503	0.752	1.005	1.256	1.501	1.754	2.005
(s.d)	(0.033)	(0.049)	(0.063)	(0.079)	(0.093)	(0.117)	(0.127)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.500	0.746	1.000	1.247	1.493	1.746	1.992
(s.d.)	(0.037)	(0.054)	(0.072)	(0.088)	(0.104)	(0.131)	(0.140)
$u = 100$							
Hill	1.013	1.516	2.026	2.529	3.041	3.552	4.051
(s.d.)	(0.069)	(0.103)	(0.133)	(0.172)	(0.206)	(0.249)	(0.266)
T. Hill	0.502	0.752	1.005	1.253	1.509	1.763	2.009
(s.d)	(0.032)	(0.048)	(0.061)	(0.082)	(0.097)	(0.117)	(0.125)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.129)
Regression	0.499	0.750	0.998	1.245	1.501	1.754	1.997
(s.d.)	(0.035)	(0.054)	(0.068)	(0.095)	(0.109)	(0.129)	(0.142)
$u = 500$							
Hill	2.880	4.302	5.734	7.211	8.627	10.053	11.476
(s.d.)	(0.216)	(0.332)	(0.444)	(0.538)	(0.634)	(0.711)	(0.851)
T. Hill	0.502	0.753	1.003	1.260	1.510	1.756	2.004
(s.d)	(0.033)	(0.052)	(0.070)	(0.085)	(0.101)	(0.112)	(0.132)
(Theor. s.d.)	(0.034)	(0.050)	(0.067)	(0.084)	(0.101)	(0.117)	(0.134)
Regression	0.500	0.751	1.000	1.255	1.505	1.751	1.998
(s.d.)	(0.036)	(0.056)	(0.076)	(0.092)	(0.111)	(0.125)	(0.145)

TABLE XII: Pareto Distribution, $k = 10000$, $s = .05(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.501	0.751	1.003	1.256	1.503	1.755	2.001
(s.d.)	(0.022)	(0.035)	(0.045)	(0.053)	(0.070)	(0.080)	(0.092)
T. Hill	0.501	0.751	1.003	1.256	1.503	1.755	2.001
(s.d)	(0.022)	(0.035)	(0.045)	(0.053)	(0.070)	(0.080)	(0.092)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.078)	(0.090)
Regression	0.488	0.733	0.982	1.225	1.463	1.712	1.954
(s.d.)	(0.031)	(0.047)	(0.062)	(0.079)	(0.095)	(0.113)	(0.123)
$u = 1$							
Hill	0.507	0.760	1.015	1.269	1.518	1.775	2.027
(s.d.)	(0.022)	(0.034)	(0.044)	(0.057)	(0.066)	(0.079)	(0.094)
T. Hill	0.501	0.751	1.003	1.254	1.500	1.755	2.004
(s.d)	(0.022)	(0.033)	(0.043)	(0.056)	(0.065)	(0.078)	(0.092)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.079)	(0.090)
Regression	0.493	0.737	0.986	1.235	1.476	1.724	1.973
(s.d.)	(0.028)	(0.043)	(0.057)	(0.074)	(0.087)	(0.104)	(0.120)
$u = 5$							
Hill	0.525	0.787	1.050	1.311	1.575	1.840	2.095
(s.d.)	(0.024)	(0.036)	(0.047)	(0.059)	(0.071)	(0.083)	(0.091)
T. Hill	0.501	0.751	1.003	1.252	1.504	1.757	2.000
(s.d)	(0.023)	(0.034)	(0.045)	(0.056)	(0.068)	(0.079)	(0.087)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.079)	(0.090)
Regression	0.496	0.744	0.991	1.240	1.488	1.739	1.976
(s.d.)	(0.028)	(0.042)	(0.055)	(0.071)	(0.084)	(0.098)	(0.110)
$u = 10$							
Hill	0.544	0.816	1.087	1.359	1.628	1.902	2.174
(s.d.)	(0.025)	(0.037)	(0.050)	(0.063)	(0.074)	(0.083)	(0.100)
T. Hill	0.501	0.752	1.003	1.253	1.501	1.754	2.005
(s.d)	(0.023)	(0.034)	(0.046)	(0.057)	(0.067)	(0.076)	(0.092)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.079)	(0.090)
Regression	0.498	0.747	0.995	1.242	1.489	1.742	1.991
(s.d.)	(0.027)	(0.040)	(0.056)	(0.068)	(0.083)	(0.094)	(0.112)
$u = 50$							
Hill	0.660	0.987	1.317	1.646	1.977	2.308	2.638
(s.d.)	(0.031)	(0.046)	(0.061)	(0.073)	(0.089)	(0.104)	(0.121)
T. Hill	0.501	0.751	1.002	1.250	1.502	1.755	2.005
(s.d)	(0.022)	(0.033)	(0.045)	(0.054)	(0.067)	(0.076)	(0.088)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.079)	(0.090)
Regression	0.499	0.748	0.997	1.244	1.495	1.749	1.997
(s.d.)	(0.025)	(0.038)	(0.053)	(0.062)	(0.077)	(0.087)	(0.101)
$u = 100$							
Hill	0.782	1.171	1.562	1.951	2.349	2.732	3.127
(s.d.)	(0.036)	(0.054)	(0.071)	(0.090)	(0.111)	(0.129)	(0.146)
T. Hill	0.501	0.750	1.001	1.250	1.503	1.751	2.003
(s.d)	(0.022)	(0.033)	(0.044)	(0.055)	(0.068)	(0.080)	(0.089)
(Theor. s.d.)	(0.023)	(0.034)	(0.045)	(0.056)	(0.068)	(0.079)	(0.090)
Regression	0.500	0.746	0.999	1.246	1.499	1.748	1.996
(s.d.)	(0.026)	(0.036)	(0.050)	(0.063)	(0.076)	(0.091)	(0.102)
$u = 500$							
Hill	1.684	2.521	3.368	4.204	5.055	5.887	6.726
(s.d.)	(0.087)	(0.131)	(0.169)	(0.213)	(0.254)	(0.299)	(0.339)
T. Hill	0.502	0.750	1.002	1.252	1.504	1.753	2.000
(s.d)	(0.023)	(0.036)	(0.046)	(0.059)	(0.069)	(0.081)	(0.092)
(Theor. s.d.)	(0.023)	(0.035)	(0.046)	(0.058)	(0.069)	(0.081)	(0.092)
Regression	0.501	0.748	0.999	1.249	1.502	1.752	1.995
(s.d.)	(0.025)	(0.039)	(0.051)	(0.064)	(0.077)	(0.088)	(0.102)
$u = 1000$							
Hill	2.870	4.303	5.730	7.166	8.622	10.060	11.446
(s.d.)	(0.155)	(0.233)	(0.302)	(0.375)	(0.468)	(0.530)	(0.616)
T. Hill	0.501	0.752	1.000	1.250	1.503	1.756	2.001
(s.d)	(0.024)	(0.037)	(0.047)	(0.058)	(0.074)	(0.083)	(0.097)
(Theor. s.d.)	(0.024)	(0.035)	(0.047)	(0.059)	(0.071)	(0.083)	(0.095)
Regression	0.499	0.752	0.998	1.248	1.500	1.756	1.997
(s.d.)	(0.026)	(0.039)	(0.051)	(0.064)	(0.081)	(0.091)	(0.105)

TABLE XIII: Pareto Distribution, $k = 500$, $s = .10(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.510	0.766	1.023	1.271	1.542	1.799	2.033
(s.d.)	(0.070)	(0.107)	(0.146)	(0.176)	(0.224)	(0.263)	(0.289)
T. Hill	0.510	0.766	1.023	1.271	1.542	1.799	2.033
(s.d)	(0.070)	(0.107)	(0.146)	(0.176)	(0.224)	(0.263)	(0.289)
(Theor. s.d.)	(0.073)	(0.109)	(0.146)	(0.182)	(0.219)	(0.255)	(0.291)
Regression	0.460	0.692	0.925	1.141	1.388	1.639	1.843
(s.d.)	(0.088)	(0.133)	(0.179)	(0.222)	(0.281)	(0.324)	(0.364)
$u = 1$							
Hill	0.550	0.824	1.101	1.368	1.649	1.920	2.181
(s.d.)	(0.080)	(0.115)	(0.162)	(0.205)	(0.239)	(0.274)	(0.332)
T. Hill	0.510	0.765	1.022	1.271	1.530	1.782	2.026
(s.d)	(0.074)	(0.105)	(0.149)	(0.190)	(0.221)	(0.253)	(0.305)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.475	0.716	0.952	1.183	1.423	1.665	1.895
(s.d.)	(0.084)	(0.121)	(0.169)	(0.215)	(0.247)	(0.288)	(0.344)
$u = 5$							
Hill	0.661	1.000	1.343	1.654	2.025	2.329	2.657
(s.d.)	(0.098)	(0.149)	(0.197)	(0.248)	(0.308)	(0.342)	(0.372)
T. Hill	0.506	0.765	1.027	1.268	1.547	1.784	2.035
(s.d)	(0.074)	(0.111)	(0.147)	(0.187)	(0.232)	(0.256)	(0.274)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.488	0.733	0.981	1.218	1.479	1.720	1.953
(s.d.)	(0.083)	(0.119)	(0.160)	(0.207)	(0.256)	(0.280)	(0.304)
$u = 10$							
Hill	0.794	1.197	1.570	1.981	2.367	2.764	3.155
(s.d.)	(0.124)	(0.187)	(0.224)	(0.300)	(0.359)	(0.416)	(0.450)
T. Hill	0.512	0.773	1.012	1.275	1.530	1.780	2.031
(s.d)	(0.077)	(0.118)	(0.138)	(0.187)	(0.223)	(0.255)	(0.280)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.498	0.749	0.982	1.234	1.480	1.720	1.965
(s.d.)	(0.085)	(0.126)	(0.153)	(0.207)	(0.243)	(0.280)	(0.317)
$u = 50$							
Hill	1.752	2.644	3.530	4.397	5.270	6.127	6.985
(s.d.)	(0.283)	(0.422)	(0.618)	(0.744)	(0.906)	(1.011)	(1.146)
T. Hill	0.509	0.766	1.026	1.281	1.537	1.790	2.035
(s.d)	(0.076)	(0.113)	(0.161)	(0.196)	(0.242)	(0.272)	(0.302)
(Theor. s.d.)	(0.077)	(0.116)	(0.154)	(0.193)	(0.231)	(0.270)	(0.308)
Regression	0.499	0.749	1.005	1.258	1.516	1.755	1.996
(s.d.)	(0.083)	(0.121)	(0.168)	(0.211)	(0.262)	(0.295)	(0.318)

TABLE XIV: Pareto Distribution, $k = 1000$, $s = .10(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.506	0.759	1.009	1.260	1.523	1.768	2.012
(s.d.)	(0.052)	(0.081)	(0.103)	(0.126)	(0.153)	(0.179)	(0.201)
T. Hill	0.506	0.759	1.009	1.260	1.523	1.768	2.012
(s.d)	(0.052)	(0.081)	(0.103)	(0.126)	(0.153)	(0.179)	(0.201)
(Theor. s.d.)	(0.051)	(0.076)	(0.102)	(0.127)	(0.152)	(0.178)	(0.203)
Regression	0.474	0.708	0.942	1.175	1.414	1.653	1.870
(s.d.)	(0.067)	(0.106)	(0.135)	(0.165)	(0.213)	(0.231)	(0.263)
$u = 1$							
Hill	0.524	0.794	1.053	1.323	1.586	1.835	2.101
(s.d.)	(0.052)	(0.083)	(0.113)	(0.137)	(0.166)	(0.192)	(0.218)
T. Hill	0.502	0.760	1.009	1.266	1.518	1.757	2.012
(s.d)	(0.050)	(0.079)	(0.108)	(0.131)	(0.158)	(0.184)	(0.208)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.479	0.725	0.967	1.203	1.449	1.682	1.914
(s.d.)	(0.060)	(0.097)	(0.129)	(0.155)	(0.185)	(0.222)	(0.249)
$u = 5$							
Hill	0.593	0.892	1.185	1.483	1.774	2.076	2.386
(s.d.)	(0.062)	(0.093)	(0.126)	(0.151)	(0.186)	(0.210)	(0.248)
T. Hill	0.505	0.759	1.009	1.262	1.510	1.767	2.029
(s.d)	(0.052)	(0.078)	(0.105)	(0.125)	(0.157)	(0.175)	(0.209)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.490	0.739	0.985	1.227	1.471	1.719	1.969
(s.d.)	(0.059)	(0.089)	(0.118)	(0.138)	(0.178)	(0.204)	(0.243)
$u = 10$							
Hill	0.660	0.999	1.329	1.655	1.993	2.316	2.655
(s.d.)	(0.070)	(0.101)	(0.143)	(0.177)	(0.211)	(0.232)	(0.279)
T. Hill	0.503	0.761	1.012	1.263	1.522	1.766	2.027
(s.d)	(0.052)	(0.076)	(0.105)	(0.132)	(0.157)	(0.172)	(0.209)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.492	0.744	0.985	1.231	1.488	1.721	1.987
(s.d.)	(0.057)	(0.084)	(0.118)	(0.149)	(0.172)	(0.193)	(0.235)
$u = 50$							
Hill	1.147	1.711	2.284	2.848	3.449	3.995	4.542
(s.d.)	(0.124)	(0.188)	(0.261)	(0.319)	(0.394)	(0.444)	(0.489)
T. Hill	0.509	0.756	1.012	1.260	1.522	1.767	2.010
(s.d)	(0.051)	(0.077)	(0.108)	(0.128)	(0.162)	(0.181)	(0.206)
(Theor. s.d.)	(0.052)	(0.078)	(0.104)	(0.130)	(0.156)	(0.182)	(0.208)
Regression	0.504	0.745	1.001	1.244	1.501	1.747	1.984
(s.d.)	(0.057)	(0.082)	(0.118)	(0.140)	(0.175)	(0.198)	(0.229)
$u = 100$							
Hill	1.759	2.620	3.510	4.369	5.264	6.140	6.976
(s.d.)	(0.202)	(0.304)	(0.406)	(0.495)	(0.614)	(0.724)	(0.801)
T. Hill	0.507	0.757	1.012	1.262	1.517	1.771	2.011
(s.d)	(0.053)	(0.081)	(0.105)	(0.133)	(0.163)	(0.188)	(0.212)
(Theor. s.d.)	(0.054)	(0.080)	(0.107)	(0.134)	(0.161)	(0.188)	(0.214)
Regression	0.501	0.749	1.003	1.252	1.504	1.757	1.985
(s.d.)	(0.057)	(0.089)	(0.114)	(0.146)	(0.179)	(0.203)	(0.229)

TABLE XV: Pareto Distribution, $k = 5000$, $s = .10(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.500	0.752	1.002	1.252	1.505	1.756	2.010
(s.d.)	(0.022)	(0.034)	(0.044)	(0.056)	(0.069)	(0.079)	(0.089)
T. Hill	0.500	0.752	1.002	1.252	1.505	1.756	2.010
(s.d)	(0.022)	(0.034)	(0.044)	(0.056)	(0.069)	(0.079)	(0.089)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.078)	(0.090)
Regression	0.488	0.733	0.976	1.224	1.472	1.714	1.961
(s.d.)	(0.031)	(0.046)	(0.063)	(0.078)	(0.096)	(0.109)	(0.126)
$u = 1$							
Hill	0.507	0.760	1.012	1.268	1.519	1.774	2.028
(s.d.)	(0.023)	(0.034)	(0.044)	(0.057)	(0.067)	(0.081)	(0.093)
T. Hill	0.501	0.751	1.000	1.253	1.502	1.754	2.005
(s.d)	(0.023)	(0.034)	(0.044)	(0.056)	(0.066)	(0.080)	(0.092)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.079)	(0.090)
Regression	0.493	0.738	0.983	1.234	1.476	1.726	1.967
(s.d.)	(0.030)	(0.045)	(0.059)	(0.074)	(0.088)	(0.105)	(0.121)
$u = 5$							
Hill	0.524	0.788	1.049	1.311	1.575	1.839	2.099
(s.d.)	(0.023)	(0.036)	(0.047)	(0.059)	(0.072)	(0.081)	(0.094)
T. Hill	0.501	0.752	1.001	1.252	1.504	1.756	2.004
(s.d)	(0.022)	(0.034)	(0.045)	(0.056)	(0.068)	(0.077)	(0.089)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.079)	(0.090)
Regression	0.496	0.743	0.990	1.240	1.487	1.740	1.984
(s.d.)	(0.028)	(0.043)	(0.056)	(0.069)	(0.085)	(0.097)	(0.113)
$u = 10$							
Hill	0.545	0.815	1.086	1.359	1.630	1.898	2.176
(s.d.)	(0.025)	(0.036)	(0.049)	(0.061)	(0.075)	(0.082)	(0.101)
T. Hill	0.503	0.751	1.002	1.253	1.503	1.750	2.006
(s.d)	(0.023)	(0.033)	(0.045)	(0.056)	(0.069)	(0.074)	(0.092)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.079)	(0.090)
Regression	0.499	0.743	0.996	1.244	1.491	1.737	1.992
(s.d.)	(0.027)	(0.040)	(0.056)	(0.070)	(0.083)	(0.093)	(0.114)
$u = 50$							
Hill	0.661	0.989	1.320	1.655	1.981	2.306	2.634
(s.d.)	(0.030)	(0.046)	(0.061)	(0.077)	(0.092)	(0.106)	(0.122)
T. Hill	0.502	0.751	1.003	1.256	1.505	1.751	2.001
(s.d)	(0.022)	(0.034)	(0.045)	(0.056)	(0.068)	(0.078)	(0.091)
(Theor. s.d.)	(0.023)	(0.034)	(0.045)	(0.056)	(0.068)	(0.079)	(0.090)
Regression	0.500	0.747	0.997	1.250	1.501	1.744	1.992
(s.d.)	(0.026)	(0.039)	(0.050)	(0.064)	(0.081)	(0.092)	(0.107)
$u = 100$							
Hill	0.787	1.177	1.573	1.958	2.356	2.748	3.147
(s.d.)	(0.038)	(0.057)	(0.073)	(0.090)	(0.111)	(0.132)	(0.155)
T. Hill	0.502	0.752	1.003	1.251	1.504	1.753	2.008
(s.d)	(0.023)	(0.035)	(0.044)	(0.056)	(0.068)	(0.080)	(0.094)
(Theor. s.d.)	(0.023)	(0.034)	(0.045)	(0.057)	(0.068)	(0.079)	(0.091)
Regression	0.501	0.749	1.001	1.247	1.499	1.747	2.000
(s.d.)	(0.026)	(0.038)	(0.050)	(0.064)	(0.077)	(0.088)	(0.105)
$u = 500$							
Hill	1.744	2.617	3.488	4.359	5.244	6.112	6.959
(s.d.)	(0.093)	(0.132)	(0.183)	(0.228)	(0.278)	(0.312)	(0.356)
T. Hill	0.501	0.752	1.002	1.252	1.504	1.757	1.999
(s.d)	(0.024)	(0.034)	(0.049)	(0.060)	(0.072)	(0.083)	(0.093)
(Theor. s.d.)	(0.024)	(0.035)	(0.047)	(0.059)	(0.071)	(0.083)	(0.095)
Regression	0.500	0.750	1.000	1.250	1.501	1.756	1.997
(s.d.)	(0.026)	(0.037)	(0.054)	(0.064)	(0.080)	(0.090)	(0.101)

TABLE XVI: Pareto Distribution, $k = 10000$, $s = .10(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.500	0.752	1.002	1.252	1.501	1.752	2.004
(s.d.)	(0.016)	(0.024)	(0.031)	(0.039)	(0.047)	(0.056)	(0.064)
T. Hill	0.500	0.752	1.002	1.252	1.501	1.752	2.004
(s.d)	(0.016)	(0.024)	(0.031)	(0.039)	(0.047)	(0.056)	(0.064)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.055)	(0.063)
Regression	0.493	0.741	0.987	1.233	1.478	1.729	1.972
(s.d.)	(0.022)	(0.033)	(0.043)	(0.056)	(0.067)	(0.074)	(0.087)
$u = 1$							
Hill	0.503	0.756	1.008	1.259	1.511	1.762	2.018
(s.d.)	(0.016)	(0.024)	(0.032)	(0.037)	(0.049)	(0.057)	(0.066)
T. Hill	0.500	0.751	1.002	1.251	1.501	1.750	2.005
(s.d)	(0.016)	(0.023)	(0.032)	(0.037)	(0.049)	(0.056)	(0.066)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.055)	(0.063)
Regression	0.494	0.741	0.992	1.238	1.484	1.731	1.986
(s.d.)	(0.022)	(0.033)	(0.043)	(0.051)	(0.065)	(0.075)	(0.085)
$u = 5$							
Hill	0.513	0.771	1.026	1.286	1.541	1.796	2.056
(s.d.)	(0.017)	(0.025)	(0.031)	(0.040)	(0.048)	(0.056)	(0.067)
T. Hill	0.500	0.751	0.999	1.253	1.501	1.749	2.003
(s.d)	(0.016)	(0.024)	(0.030)	(0.039)	(0.046)	(0.054)	(0.065)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.055)	(0.063)
Regression	0.497	0.745	0.992	1.246	1.491	1.736	1.990
(s.d.)	(0.020)	(0.032)	(0.040)	(0.049)	(0.061)	(0.073)	(0.083)
$u = 10$							
Hill	0.524	0.787	1.048	1.310	1.574	1.836	2.094
(s.d.)	(0.016)	(0.025)	(0.034)	(0.042)	(0.049)	(0.058)	(0.070)
T. Hill	0.500	0.751	1.000	1.250	1.502	1.752	1.998
(s.d)	(0.016)	(0.023)	(0.033)	(0.040)	(0.047)	(0.055)	(0.066)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.055)	(0.063)
Regression	0.497	0.748	0.995	1.243	1.496	1.741	1.987
(s.d.)	(0.020)	(0.030)	(0.041)	(0.050)	(0.059)	(0.070)	(0.081)
$u = 50$							
Hill	0.590	0.886	1.181	1.477	1.770	2.068	2.363
(s.d.)	(0.019)	(0.029)	(0.039)	(0.047)	(0.058)	(0.067)	(0.078)
T. Hill	0.500	0.750	1.001	1.251	1.501	1.753	2.003
(s.d)	(0.016)	(0.024)	(0.032)	(0.039)	(0.048)	(0.056)	(0.065)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.056)	(0.063)
Regression	0.498	0.748	0.999	1.247	1.497	1.750	1.998
(s.d.)	(0.019)	(0.028)	(0.038)	(0.046)	(0.056)	(0.067)	(0.076)
$u = 100$							
Hill	0.660	0.988	1.318	1.650	1.978	2.310	2.639
(s.d.)	(0.023)	(0.033)	(0.043)	(0.053)	(0.064)	(0.078)	(0.088)
T. Hill	0.501	0.750	1.001	1.252	1.502	1.754	2.004
(s.d)	(0.017)	(0.024)	(0.032)	(0.039)	(0.048)	(0.057)	(0.065)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.056)	(0.064)
Regression	0.499	0.749	0.999	1.250	1.497	1.751	2.000
(s.d.)	(0.019)	(0.028)	(0.037)	(0.046)	(0.055)	(0.063)	(0.075)
$u = 500$							
Hill	1.140	1.708	2.280	2.839	3.416	3.985	4.551
(s.d.)	(0.040)	(0.059)	(0.076)	(0.097)	(0.118)	(0.144)	(0.156)
T. Hill	0.501	0.751	1.003	1.249	1.503	1.752	2.002
(s.d)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.059)	(0.064)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.041)	(0.049)	(0.057)	(0.065)
Regression	0.500	0.750	1.002	1.248	1.501	1.749	1.999
(s.d.)	(0.018)	(0.026)	(0.036)	(0.044)	(0.053)	(0.065)	(0.071)
$u = 1000$							
Hill	1.746	2.616	3.483	4.367	5.234	6.107	6.982
(s.d.)	(0.066)	(0.098)	(0.124)	(0.160)	(0.188)	(0.221)	(0.257)
T. Hill	0.501	0.750	1.000	1.253	1.502	1.751	2.004
(s.d)	(0.017)	(0.025)	(0.033)	(0.042)	(0.049)	(0.057)	(0.068)
(Theor. s.d.)	(0.017)	(0.025)	(0.033)	(0.042)	(0.050)	(0.058)	(0.067)
Regression	0.500	0.748	1.000	1.252	1.502	1.749	2.001
(s.d.)	(0.018)	(0.027)	(0.036)	(0.046)	(0.054)	(0.064)	(0.073)

TABLE XVII: Pareto Distribution, $k = 500$, $s = .15(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.509	0.758	1.013	1.266	1.513	1.764	2.026
(s.d.)	(0.057)	(0.091)	(0.118)	(0.145)	(0.179)	(0.204)	(0.244)
T. Hill	0.509	0.758	1.013	1.266	1.513	1.764	2.026
(s.d)	(0.057)	(0.091)	(0.118)	(0.145)	(0.179)	(0.204)	(0.244)
(Theor. s.d.)	(0.059)	(0.088)	(0.118)	(0.147)	(0.177)	(0.206)	(0.236)
Regression	0.472	0.694	0.937	1.164	1.393	1.624	1.870
(s.d.)	(0.076)	(0.117)	(0.157)	(0.182)	(0.227)	(0.260)	(0.309)
$u = 1$							
Hill	0.539	0.808	1.061	1.340	1.602	1.869	2.150
(s.d.)	(0.066)	(0.094)	(0.122)	(0.160)	(0.179)	(0.218)	(0.256)
T. Hill	0.510	0.766	1.006	1.270	1.517	1.770	2.037
(s.d)	(0.063)	(0.089)	(0.114)	(0.151)	(0.169)	(0.206)	(0.242)
(Theor. s.d.)	(0.059)	(0.089)	(0.119)	(0.148)	(0.178)	(0.208)	(0.237)
Regression	0.483	0.728	0.954	1.204	1.432	1.676	1.932
(s.d.)	(0.073)	(0.104)	(0.132)	(0.180)	(0.206)	(0.247)	(0.287)
$u = 5$							
Hill	0.618	0.920	1.240	1.540	1.861	2.162	2.491
(s.d.)	(0.074)	(0.108)	(0.150)	(0.181)	(0.232)	(0.255)	(0.306)
T. Hill	0.506	0.754	1.015	1.263	1.523	1.771	2.039
(s.d)	(0.059)	(0.087)	(0.121)	(0.146)	(0.187)	(0.203)	(0.245)
(Theor. s.d.)	(0.059)	(0.089)	(0.119)	(0.148)	(0.178)	(0.208)	(0.237)
Regression	0.491	0.730	0.980	1.226	1.472	1.713	1.969
(s.d.)	(0.066)	(0.101)	(0.138)	(0.169)	(0.203)	(0.228)	(0.277)
$u = 10$							
Hill	0.709	1.062	1.414	1.762	2.125	2.487	2.829
(s.d.)	(0.088)	(0.129)	(0.173)	(0.213)	(0.258)	(0.315)	(0.343)
T. Hill	0.508	0.760	1.010	1.260	1.523	1.779	2.021
(s.d)	(0.060)	(0.089)	(0.119)	(0.148)	(0.179)	(0.215)	(0.236)
(Theor. s.d.)	(0.060)	(0.090)	(0.119)	(0.149)	(0.179)	(0.209)	(0.239)
Regression	0.495	0.740	0.982	1.228	1.484	1.734	1.965
(s.d.)	(0.065)	(0.097)	(0.130)	(0.162)	(0.194)	(0.236)	(0.265)
$u = 50$							
Hill	1.373	2.067	2.738	3.443	4.129	4.827	5.556
(s.d.)	(0.189)	(0.284)	(0.365)	(0.471)	(0.564)	(0.654)	(0.757)
T. Hill	0.506	0.762	1.011	1.270	1.524	1.780	2.048
(s.d)	(0.063)	(0.096)	(0.127)	(0.161)	(0.188)	(0.225)	(0.252)
(Theor. s.d.)	(0.062)	(0.094)	(0.125)	(0.156)	(0.187)	(0.219)	(0.250)
Regression	0.498	0.750	0.997	1.251	1.498	1.755	2.020
(s.d.)	(0.068)	(0.105)	(0.137)	(0.173)	(0.201)	(0.247)	(0.273)

TABLE XVIII: Pareto Distribution, $k = 1000$, $s = .15(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.504	0.756	1.006	1.262	1.512	1.762	2.007
(s.d.)	(0.042)	(0.062)	(0.081)	(0.104)	(0.123)	(0.142)	(0.170)
T. Hill	0.504	0.756	1.006	1.262	1.512	1.762	2.007
(s.d)	(0.042)	(0.062)	(0.081)	(0.104)	(0.123)	(0.142)	(0.170)
(Theor. s.d.)	(0.041)	(0.062)	(0.082)	(0.103)	(0.124)	(0.144)	(0.165)
Regression	0.478	0.714	0.956	1.196	1.437	1.671	1.911
(s.d.)	(0.056)	(0.086)	(0.116)	(0.136)	(0.171)	(0.193)	(0.223)
$u = 1$							
Hill	0.520	0.779	1.037	1.299	1.563	1.822	2.087
(s.d.)	(0.042)	(0.064)	(0.085)	(0.111)	(0.131)	(0.143)	(0.176)
T. Hill	0.504	0.755	1.005	1.259	1.515	1.765	2.022
(s.d)	(0.041)	(0.062)	(0.083)	(0.107)	(0.127)	(0.138)	(0.171)
(Theor. s.d.)	(0.041)	(0.062)	(0.083)	(0.103)	(0.124)	(0.145)	(0.166)
Regression	0.486	0.728	0.967	1.217	1.461	1.695	1.950
(s.d.)	(0.052)	(0.075)	(0.105)	(0.129)	(0.162)	(0.176)	(0.213)
$u = 5$							
Hill	0.567	0.851	1.136	1.422	1.701	1.982	2.273
(s.d.)	(0.047)	(0.071)	(0.096)	(0.118)	(0.138)	(0.165)	(0.190)
T. Hill	0.503	0.756	1.009	1.262	1.511	1.759	2.019
(s.d)	(0.041)	(0.062)	(0.085)	(0.104)	(0.122)	(0.144)	(0.166)
(Theor. s.d.)	(0.041)	(0.062)	(0.083)	(0.103)	(0.124)	(0.145)	(0.166)
Regression	0.491	0.741	0.984	1.233	1.481	1.721	1.978
(s.d.)	(0.048)	(0.073)	(0.100)	(0.123)	(0.143)	(0.168)	(0.188)
$u = 10$							
Hill	0.615	0.922	1.235	1.547	1.847	2.158	2.463
(s.d.)	(0.053)	(0.078)	(0.102)	(0.134)	(0.164)	(0.189)	(0.204)
T. Hill	0.502	0.753	1.009	1.263	1.507	1.763	2.013
(s.d)	(0.042)	(0.062)	(0.082)	(0.107)	(0.130)	(0.150)	(0.164)
(Theor. s.d.)	(0.042)	(0.062)	(0.083)	(0.104)	(0.125)	(0.145)	(0.166)
Regression	0.492	0.741	0.992	1.241	1.483	1.735	1.983
(s.d.)	(0.047)	(0.071)	(0.092)	(0.122)	(0.146)	(0.167)	(0.192)
$u = 50$							
Hill	0.954	1.430	1.916	2.385	2.861	3.341	3.813
(s.d.)	(0.086)	(0.134)	(0.172)	(0.218)	(0.252)	(0.301)	(0.340)
T. Hill	0.503	0.756	1.013	1.259	1.514	1.762	2.014
(s.d)	(0.043)	(0.066)	(0.086)	(0.111)	(0.124)	(0.149)	(0.168)
(Theor. s.d.)	(0.042)	(0.064)	(0.085)	(0.106)	(0.127)	(0.148)	(0.170)
Regression	0.498	0.750	1.003	1.246	1.499	1.742	1.997
(s.d.)	(0.047)	(0.071)	(0.097)	(0.118)	(0.140)	(0.162)	(0.185)
$u = 100$							
Hill	1.365	2.053	2.737	3.421	4.096	4.792	5.486
(s.d.)	(0.130)	(0.193)	(0.247)	(0.324)	(0.377)	(0.466)	(0.515)
T. Hill	0.503	0.757	1.008	1.262	1.512	1.766	2.024
(s.d)	(0.044)	(0.065)	(0.084)	(0.110)	(0.127)	(0.157)	(0.175)
(Theor. s.d.)	(0.044)	(0.065)	(0.087)	(0.109)	(0.131)	(0.152)	(0.174)
Regression	0.499	0.751	1.000	1.254	1.503	1.755	2.009
(s.d.)	(0.048)	(0.068)	(0.092)	(0.120)	(0.138)	(0.171)	(0.188)

TABLE XIX: Pareto Distribution, $k = 5000$, $s = .15(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.500	0.751	1.001	1.251	1.501	1.753	2.003
(s.d.)	(0.018)	(0.028)	(0.036)	(0.046)	(0.055)	(0.063)	(0.072)
T. Hill	0.500	0.751	1.001	1.251	1.501	1.753	2.003
(s.d)	(0.018)	(0.028)	(0.036)	(0.046)	(0.055)	(0.063)	(0.072)
(Theor. s.d.)	(0.018)	(0.027)	(0.037)	(0.046)	(0.055)	(0.064)	(0.073)
Regression	0.491	0.735	0.981	1.225	1.473	1.722	1.968
(s.d.)	(0.025)	(0.039)	(0.051)	(0.066)	(0.077)	(0.090)	(0.100)
$u = 1$							
Hill	0.505	0.758	1.010	1.262	1.513	1.765	2.021
(s.d.)	(0.018)	(0.028)	(0.037)	(0.046)	(0.056)	(0.064)	(0.075)
T. Hill	0.501	0.752	1.002	1.252	1.501	1.750	2.005
(s.d)	(0.018)	(0.028)	(0.036)	(0.046)	(0.055)	(0.063)	(0.074)
(Theor. s.d.)	(0.018)	(0.027)	(0.037)	(0.046)	(0.055)	(0.064)	(0.073)
Regression	0.496	0.741	0.989	1.235	1.480	1.728	1.983
(s.d.)	(0.024)	(0.036)	(0.049)	(0.062)	(0.075)	(0.085)	(0.095)
$u = 5$							
Hill	0.518	0.777	1.034	1.294	1.554	1.809	2.070
(s.d.)	(0.019)	(0.028)	(0.037)	(0.049)	(0.057)	(0.066)	(0.077)
T. Hill	0.501	0.752	1.000	1.252	1.503	1.749	2.002
(s.d)	(0.018)	(0.027)	(0.035)	(0.047)	(0.055)	(0.064)	(0.074)
(Theor. s.d.)	(0.018)	(0.027)	(0.037)	(0.046)	(0.055)	(0.064)	(0.073)
Regression	0.497	0.746	0.991	1.240	1.492	1.734	1.986
(s.d.)	(0.023)	(0.035)	(0.046)	(0.060)	(0.071)	(0.082)	(0.094)
$u = 10$							
Hill	0.532	0.798	1.063	1.327	1.594	1.858	2.120
(s.d.)	(0.019)	(0.029)	(0.039)	(0.049)	(0.058)	(0.070)	(0.079)
T. Hill	0.501	0.752	1.002	1.251	1.503	1.751	1.999
(s.d)	(0.018)	(0.027)	(0.037)	(0.046)	(0.055)	(0.065)	(0.074)
(Theor. s.d.)	(0.018)	(0.027)	(0.037)	(0.046)	(0.055)	(0.064)	(0.073)
Regression	0.498	0.747	0.996	1.242	1.495	1.742	1.988
(s.d.)	(0.023)	(0.033)	(0.045)	(0.057)	(0.070)	(0.081)	(0.091)
$u = 50$							
Hill	0.615	0.921	1.228	1.541	1.843	2.155	2.456
(s.d.)	(0.023)	(0.033)	(0.045)	(0.059)	(0.069)	(0.086)	(0.090)
T. Hill	0.501	0.750	1.000	1.256	1.501	1.755	2.000
(s.d)	(0.018)	(0.026)	(0.037)	(0.047)	(0.055)	(0.069)	(0.072)
(Theor. s.d.)	(0.018)	(0.028)	(0.037)	(0.046)	(0.055)	(0.064)	(0.074)
Regression	0.499	0.748	0.997	1.253	1.496	1.749	1.994
(s.d.)	(0.022)	(0.031)	(0.044)	(0.053)	(0.064)	(0.080)	(0.085)
$u = 100$							
Hill	0.703	1.056	1.405	1.760	2.106	2.463	2.807
(s.d.)	(0.029)	(0.040)	(0.053)	(0.066)	(0.082)	(0.092)	(0.111)
T. Hill	0.500	0.751	1.000	1.253	1.499	1.753	1.999
(s.d)	(0.019)	(0.027)	(0.036)	(0.045)	(0.056)	(0.062)	(0.076)
(Theor. s.d.)	(0.018)	(0.028)	(0.037)	(0.046)	(0.055)	(0.065)	(0.074)
Regression	0.499	0.750	0.998	1.249	1.495	1.749	1.993
(s.d.)	(0.021)	(0.031)	(0.043)	(0.051)	(0.063)	(0.071)	(0.084)
$u = 500$							
Hill	1.363	2.044	2.725	3.401	4.087	4.775	5.451
(s.d.)	(0.059)	(0.089)	(0.115)	(0.146)	(0.172)	(0.205)	(0.236)
T. Hill	0.501	0.751	1.001	1.251	1.501	1.755	2.004
(s.d)	(0.020)	(0.030)	(0.039)	(0.049)	(0.058)	(0.069)	(0.080)
(Theor. s.d.)	(0.019)	(0.029)	(0.039)	(0.048)	(0.058)	(0.068)	(0.077)
Regression	0.500	0.750	1.001	1.249	1.499	1.751	1.999
(s.d.)	(0.022)	(0.032)	(0.044)	(0.053)	(0.063)	(0.075)	(0.087)

TABLE XX: Pareto Distribution, $k = 10000$, $s = .15(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.500	0.750	1.002	1.252	1.501	1.751	2.001
(s.d.)	(0.013)	(0.019)	(0.026)	(0.033)	(0.040)	(0.046)	(0.051)
T. Hill	0.500	0.750	1.002	1.252	1.501	1.751	2.001
(s.d)	(0.013)	(0.019)	(0.026)	(0.033)	(0.040)	(0.046)	(0.051)
(Theor. s.d.)	(0.013)	(0.019)	(0.026)	(0.032)	(0.039)	(0.045)	(0.052)
Regression	0.494	0.742	0.993	1.239	1.483	1.733	1.978
(s.d.)	(0.019)	(0.028)	(0.036)	(0.046)	(0.055)	(0.065)	(0.073)
$u = 1$							
Hill	0.503	0.753	1.005	1.257	1.509	1.760	2.009
(s.d.)	(0.013)	(0.020)	(0.026)	(0.033)	(0.040)	(0.046)	(0.052)
T. Hill	0.500	0.749	1.001	1.252	1.502	1.752	2.000
(s.d)	(0.013)	(0.020)	(0.026)	(0.032)	(0.039)	(0.046)	(0.052)
(Theor. s.d.)	(0.013)	(0.019)	(0.026)	(0.032)	(0.039)	(0.045)	(0.052)
Regression	0.497	0.744	0.993	1.242	1.490	1.739	1.985
(s.d.)	(0.017)	(0.027)	(0.035)	(0.044)	(0.053)	(0.064)	(0.072)
$u = 5$							
Hill	0.510	0.765	1.020	1.273	1.530	1.785	2.038
(s.d.)	(0.013)	(0.020)	(0.026)	(0.033)	(0.040)	(0.048)	(0.053)
T. Hill	0.501	0.751	1.001	1.249	1.502	1.751	2.000
(s.d)	(0.013)	(0.020)	(0.026)	(0.032)	(0.039)	(0.047)	(0.052)
(Theor. s.d.)	(0.013)	(0.019)	(0.026)	(0.032)	(0.039)	(0.045)	(0.052)
Regression	0.498	0.747	0.997	1.242	1.494	1.743	1.991
(s.d.)	(0.017)	(0.026)	(0.034)	(0.041)	(0.048)	(0.060)	(0.069)
$u = 10$							
Hill	0.518	0.776	1.035	1.295	1.550	1.811	2.069
(s.d.)	(0.013)	(0.020)	(0.028)	(0.035)	(0.040)	(0.045)	(0.054)
T. Hill	0.500	0.751	1.001	1.252	1.499	1.751	2.000
(s.d)	(0.013)	(0.019)	(0.027)	(0.034)	(0.039)	(0.043)	(0.052)
(Theor. s.d.)	(0.013)	(0.019)	(0.026)	(0.032)	(0.039)	(0.045)	(0.052)
Regression	0.498	0.748	0.997	1.246	1.492	1.745	1.992
(s.d.)	(0.017)	(0.025)	(0.034)	(0.042)	(0.048)	(0.057)	(0.066)
$u = 50$							
Hill	0.565	0.848	1.131	1.410	1.697	1.978	2.261
(s.d.)	(0.014)	(0.022)	(0.029)	(0.036)	(0.045)	(0.051)	(0.059)
T. Hill	0.500	0.751	1.001	1.248	1.502	1.751	2.002
(s.d)	(0.013)	(0.019)	(0.025)	(0.031)	(0.040)	(0.045)	(0.051)
(Theor. s.d.)	(0.013)	(0.019)	(0.026)	(0.032)	(0.039)	(0.045)	(0.052)
Regression	0.499	0.749	0.999	1.245	1.497	1.746	1.999
(s.d.)	(0.016)	(0.023)	(0.032)	(0.039)	(0.049)	(0.055)	(0.061)
$u = 100$							
Hill	0.615	0.920	1.230	1.534	1.842	2.149	2.458
(s.d.)	(0.016)	(0.024)	(0.033)	(0.040)	(0.048)	(0.059)	(0.064)
T. Hill	0.500	0.749	1.001	1.250	1.499	1.749	2.001
(s.d)	(0.013)	(0.019)	(0.026)	(0.032)	(0.039)	(0.047)	(0.050)
(Theor. s.d.)	(0.013)	(0.019)	(0.026)	(0.032)	(0.039)	(0.045)	(0.052)
Regression	0.500	0.748	1.000	1.249	1.496	1.744	1.999
(s.d.)	(0.016)	(0.023)	(0.031)	(0.038)	(0.045)	(0.055)	(0.059)
$u = 500$							
Hill	0.951	1.426	1.897	2.372	2.848	3.320	3.794
(s.d.)	(0.026)	(0.040)	(0.053)	(0.067)	(0.082)	(0.094)	(0.109)
T. Hill	0.501	0.752	1.000	1.251	1.502	1.750	2.001
(s.d)	(0.013)	(0.020)	(0.026)	(0.033)	(0.041)	(0.046)	(0.054)
(Theor. s.d.)	(0.013)	(0.020)	(0.027)	(0.033)	(0.040)	(0.046)	(0.053)
Regression	0.500	0.751	0.999	1.250	1.500	1.749	1.999
(s.d.)	(0.014)	(0.022)	(0.030)	(0.036)	(0.045)	(0.051)	(0.059)
$u = 1000$							
Hill	1.361	2.043	2.729	3.401	4.084	4.770	5.456
(s.d.)	(0.040)	(0.061)	(0.078)	(0.102)	(0.121)	(0.141)	(0.159)
T. Hill	0.500	0.750	1.002	1.249	1.500	1.753	2.004
(s.d)	(0.014)	(0.020)	(0.026)	(0.034)	(0.040)	(0.048)	(0.053)
(Theor. s.d.)	(0.014)	(0.020)	(0.027)	(0.034)	(0.041)	(0.048)	(0.054)
Regression	0.499	0.749	1.001	1.248	1.498	1.751	2.001
(s.d.)	(0.015)	(0.023)	(0.029)	(0.038)	(0.044)	(0.053)	(0.059)

TABLE XXI: Pareto Distribution, $k = 500$, $s = .20(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.504	0.755	1.014	1.260	1.521	1.759	2.005
(s.d.)	(0.050)	(0.078)	(0.104)	(0.126)	(0.152)	(0.183)	(0.209)
T. Hill	0.504	0.755	1.014	1.260	1.521	1.759	2.005
(s.d)	(0.050)	(0.078)	(0.104)	(0.126)	(0.152)	(0.183)	(0.209)
(Theor. s.d.)	(0.051)	(0.076)	(0.102)	(0.127)	(0.152)	(0.178)	(0.203)
Regression	0.468	0.707	0.947	1.176	1.413	1.642	1.876
(s.d.)	(0.067)	(0.099)	(0.132)	(0.178)	(0.198)	(0.233)	(0.262)
$u = 1$							
Hill	0.528	0.789	1.050	1.319	1.583	1.849	2.114
(s.d.)	(0.055)	(0.080)	(0.109)	(0.128)	(0.156)	(0.189)	(0.213)
T. Hill	0.506	0.755	1.006	1.263	1.516	1.771	2.024
(s.d)	(0.053)	(0.076)	(0.104)	(0.122)	(0.149)	(0.181)	(0.203)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.483	0.719	0.965	1.211	1.446	1.693	1.936
(s.d.)	(0.063)	(0.091)	(0.126)	(0.150)	(0.186)	(0.220)	(0.244)
$u = 5$							
Hill	0.592	0.889	1.183	1.482	1.784	2.079	2.362
(s.d.)	(0.065)	(0.094)	(0.117)	(0.158)	(0.177)	(0.209)	(0.248)
T. Hill	0.504	0.757	1.007	1.261	1.518	1.769	2.010
(s.d)	(0.055)	(0.079)	(0.097)	(0.133)	(0.149)	(0.176)	(0.209)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.491	0.739	0.982	1.226	1.475	1.718	1.946
(s.d.)	(0.063)	(0.091)	(0.111)	(0.153)	(0.173)	(0.201)	(0.234)
$u = 10$							
Hill	0.668	0.995	1.325	1.663	1.979	2.320	2.672
(s.d.)	(0.069)	(0.105)	(0.135)	(0.173)	(0.210)	(0.234)	(0.269)
T. Hill	0.508	0.758	1.007	1.265	1.507	1.766	2.035
(s.d)	(0.051)	(0.077)	(0.100)	(0.128)	(0.154)	(0.175)	(0.198)
(Theor. s.d.)	(0.051)	(0.077)	(0.103)	(0.128)	(0.154)	(0.179)	(0.205)
Regression	0.497	0.743	0.990	1.238	1.475	1.731	1.990
(s.d.)	(0.059)	(0.086)	(0.116)	(0.147)	(0.171)	(0.202)	(0.221)
$u = 50$							
Hill	1.176	1.756	2.359	2.939	3.531	4.100	4.738
(s.d.)	(0.137)	(0.197)	(0.277)	(0.333)	(0.395)	(0.474)	(0.543)
T. Hill	0.506	0.756	1.017	1.265	1.519	1.767	2.041
(s.d)	(0.055)	(0.078)	(0.111)	(0.133)	(0.160)	(0.190)	(0.217)
(Theor. s.d.)	(0.054)	(0.080)	(0.107)	(0.134)	(0.161)	(0.188)	(0.214)
Regression	0.500	0.745	1.003	1.245	1.497	1.745	2.015
(s.d.)	(0.060)	(0.083)	(0.120)	(0.142)	(0.174)	(0.203)	(0.239)

TABLE XXII: Pareto Distribution, $k = 1000$, $s = .20(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.505	0.752	1.009	1.264	1.504	1.761	2.010
(s.d.)	(0.037)	(0.052)	(0.069)	(0.089)	(0.106)	(0.126)	(0.140)
T. Hill	0.505	0.752	1.009	1.264	1.504	1.761	2.010
(s.d)	(0.037)	(0.052)	(0.069)	(0.089)	(0.106)	(0.126)	(0.140)
(Theor. s.d.)	(0.036)	(0.053)	(0.071)	(0.089)	(0.107)	(0.125)	(0.142)
Regression	0.483	0.720	0.966	1.211	1.448	1.682	1.915
(s.d.)	(0.049)	(0.071)	(0.096)	(0.122)	(0.150)	(0.171)	(0.183)
$u = 1$							
Hill	0.514	0.774	1.032	1.287	1.544	1.805	2.065
(s.d.)	(0.037)	(0.056)	(0.073)	(0.090)	(0.109)	(0.127)	(0.149)
T. Hill	0.502	0.755	1.007	1.256	1.507	1.761	2.015
(s.d)	(0.036)	(0.055)	(0.071)	(0.087)	(0.106)	(0.124)	(0.146)
(Theor. s.d.)	(0.036)	(0.054)	(0.071)	(0.089)	(0.107)	(0.125)	(0.143)
Regression	0.488	0.736	0.980	1.218	1.467	1.709	1.962
(s.d.)	(0.046)	(0.071)	(0.089)	(0.109)	(0.136)	(0.156)	(0.185)
$u = 5$							
Hill	0.553	0.827	1.104	1.382	1.657	1.931	2.210
(s.d.)	(0.039)	(0.057)	(0.078)	(0.098)	(0.121)	(0.134)	(0.165)
T. Hill	0.503	0.752	1.003	1.256	1.507	1.756	2.010
(s.d)	(0.035)	(0.051)	(0.071)	(0.089)	(0.109)	(0.121)	(0.149)
(Theor. s.d.)	(0.036)	(0.054)	(0.071)	(0.089)	(0.107)	(0.125)	(0.143)
Regression	0.494	0.738	0.985	1.234	1.481	1.724	1.978
(s.d.)	(0.043)	(0.062)	(0.087)	(0.107)	(0.130)	(0.151)	(0.172)
$u = 10$							
Hill	0.594	0.884	1.182	1.479	1.780	2.080	2.369
(s.d.)	(0.043)	(0.062)	(0.086)	(0.107)	(0.129)	(0.156)	(0.166)
T. Hill	0.504	0.751	1.003	1.256	1.512	1.765	2.012
(s.d)	(0.036)	(0.052)	(0.071)	(0.089)	(0.108)	(0.129)	(0.139)
(Theor. s.d.)	(0.036)	(0.054)	(0.072)	(0.090)	(0.107)	(0.125)	(0.143)
Regression	0.496	0.741	0.989	1.236	1.488	1.739	1.984
(s.d.)	(0.042)	(0.061)	(0.085)	(0.102)	(0.124)	(0.147)	(0.162)
$u = 50$							
Hill	0.852	1.278	1.703	2.128	2.577	2.988	3.396
(s.d.)	(0.065)	(0.099)	(0.133)	(0.168)	(0.197)	(0.220)	(0.257)
T. Hill	0.501	0.751	1.002	1.253	1.517	1.759	2.000
(s.d)	(0.036)	(0.055)	(0.073)	(0.096)	(0.111)	(0.124)	(0.146)
(Theor. s.d.)	(0.037)	(0.055)	(0.073)	(0.091)	(0.110)	(0.128)	(0.146)
Regression	0.497	0.745	0.993	1.242	1.504	1.745	1.988
(s.d.)	(0.040)	(0.061)	(0.081)	(0.104)	(0.124)	(0.140)	(0.164)
$u = 100$							
Hill	1.169	1.758	2.343	2.932	3.507	4.091	4.696
(s.d.)	(0.096)	(0.140)	(0.192)	(0.235)	(0.279)	(0.340)	(0.380)
T. Hill	0.502	0.756	1.005	1.258	1.506	1.759	2.015
(s.d)	(0.038)	(0.057)	(0.076)	(0.094)	(0.113)	(0.134)	(0.150)
(Theor. s.d.)	(0.038)	(0.056)	(0.075)	(0.094)	(0.113)	(0.132)	(0.150)
Regression	0.498	0.751	0.997	1.251	1.498	1.745	1.997
(s.d.)	(0.041)	(0.063)	(0.082)	(0.105)	(0.126)	(0.145)	(0.166)

TABLE XXIII: Pareto Distribution, $k = 5000$, $s = .20(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.500	0.752	0.999	1.253	1.500	1.753	2.007
(s.d.)	(0.016)	(0.023)	(0.031)	(0.041)	(0.048)	(0.055)	(0.064)
T. Hill	0.500	0.752	0.999	1.253	1.500	1.753	2.007
(s.d)	(0.016)	(0.023)	(0.031)	(0.041)	(0.048)	(0.055)	(0.064)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.055)	(0.063)
Regression	0.494	0.740	0.984	1.236	1.474	1.726	1.977
(s.d.)	(0.022)	(0.033)	(0.045)	(0.057)	(0.068)	(0.078)	(0.089)
$u = 1$							
Hill	0.504	0.756	1.008	1.261	1.513	1.765	2.012
(s.d.)	(0.016)	(0.025)	(0.032)	(0.039)	(0.047)	(0.056)	(0.064)
T. Hill	0.500	0.752	1.002	1.253	1.503	1.753	1.999
(s.d)	(0.016)	(0.025)	(0.031)	(0.038)	(0.047)	(0.056)	(0.063)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.055)	(0.063)
Regression	0.495	0.744	0.991	1.240	1.489	1.737	1.979
(s.d.)	(0.021)	(0.033)	(0.041)	(0.054)	(0.063)	(0.074)	(0.085)
$u = 5$							
Hill	0.514	0.772	1.028	1.285	1.538	1.799	2.052
(s.d.)	(0.016)	(0.024)	(0.033)	(0.041)	(0.049)	(0.058)	(0.063)
T. Hill	0.500	0.752	1.002	1.252	1.498	1.752	1.999
(s.d)	(0.016)	(0.023)	(0.032)	(0.039)	(0.048)	(0.057)	(0.061)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.055)	(0.063)
Regression	0.497	0.746	0.997	1.245	1.487	1.744	1.986
(s.d.)	(0.020)	(0.030)	(0.042)	(0.052)	(0.062)	(0.070)	(0.079)
$u = 10$							
Hill	0.524	0.788	1.048	1.311	1.572	1.839	2.095
(s.d.)	(0.017)	(0.025)	(0.032)	(0.042)	(0.049)	(0.058)	(0.066)
T. Hill	0.500	0.752	1.000	1.251	1.500	1.755	1.999
(s.d)	(0.016)	(0.024)	(0.030)	(0.040)	(0.047)	(0.055)	(0.063)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.055)	(0.063)
Regression	0.498	0.747	0.996	1.244	1.494	1.747	1.990
(s.d.)	(0.020)	(0.030)	(0.040)	(0.049)	(0.058)	(0.068)	(0.083)
$u = 50$							
Hill	0.591	0.887	1.180	1.479	1.774	2.070	2.361
(s.d.)	(0.019)	(0.028)	(0.037)	(0.048)	(0.057)	(0.068)	(0.075)
T. Hill	0.501	0.752	1.000	1.253	1.503	1.754	2.001
(s.d)	(0.016)	(0.023)	(0.031)	(0.040)	(0.048)	(0.058)	(0.063)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.056)	(0.064)
Regression	0.499	0.750	0.997	1.249	1.499	1.749	1.994
(s.d.)	(0.019)	(0.026)	(0.038)	(0.047)	(0.058)	(0.069)	(0.076)
$u = 100$							
Hill	0.660	0.993	1.320	1.652	1.980	2.312	2.639
(s.d.)	(0.021)	(0.033)	(0.043)	(0.055)	(0.063)	(0.076)	(0.085)
T. Hill	0.500	0.752	1.001	1.252	1.501	1.751	2.000
(s.d)	(0.016)	(0.024)	(0.032)	(0.040)	(0.046)	(0.056)	(0.063)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.056)	(0.064)
Regression	0.499	0.750	0.998	1.250	1.497	1.747	1.999
(s.d.)	(0.018)	(0.028)	(0.037)	(0.045)	(0.053)	(0.063)	(0.072)
$u = 500$							
Hill	1.169	1.754	2.338	2.921	3.511	4.092	4.667
(s.d.)	(0.041)	(0.064)	(0.083)	(0.106)	(0.125)	(0.151)	(0.168)
T. Hill	0.501	0.751	1.001	1.251	1.503	1.753	1.999
(s.d)	(0.016)	(0.025)	(0.033)	(0.042)	(0.049)	(0.060)	(0.067)
(Theor. s.d.)	(0.017)	(0.025)	(0.033)	(0.042)	(0.050)	(0.058)	(0.067)
Regression	0.501	0.750	1.000	1.249	1.501	1.751	1.997
(s.d.)	(0.018)	(0.028)	(0.037)	(0.047)	(0.055)	(0.066)	(0.074)

TABLE XXIV: Pareto Distribution, $k = 10000$, $s = .20(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.500	0.750	0.999	1.251	1.500	1.750	2.001
(s.d.)	(0.011)	(0.017)	(0.023)	(0.028)	(0.034)	(0.041)	(0.044)
T. Hill	0.500	0.750	0.999	1.251	1.500	1.750	2.001
(s.d)	(0.011)	(0.017)	(0.023)	(0.028)	(0.034)	(0.041)	(0.044)
(Theor. s.d.)	(0.011)	(0.017)	(0.022)	(0.028)	(0.034)	(0.039)	(0.045)
Regression	0.495	0.743	0.989	1.241	1.487	1.731	1.983
(s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.047)	(0.058)	(0.063)
$u = 1$							
Hill	0.502	0.752	1.003	1.254	1.507	1.758	2.007
(s.d.)	(0.011)	(0.016)	(0.023)	(0.028)	(0.035)	(0.040)	(0.045)
T. Hill	0.501	0.749	0.999	1.249	1.501	1.752	1.999
(s.d)	(0.011)	(0.016)	(0.023)	(0.028)	(0.035)	(0.040)	(0.045)
(Theor. s.d.)	(0.011)	(0.017)	(0.022)	(0.028)	(0.034)	(0.039)	(0.045)
Regression	0.498	0.745	0.993	1.241	1.491	1.739	1.987
(s.d.)	(0.015)	(0.023)	(0.031)	(0.039)	(0.046)	(0.054)	(0.061)
$u = 5$							
Hill	0.507	0.762	1.015	1.268	1.524	1.778	2.031
(s.d.)	(0.011)	(0.018)	(0.024)	(0.029)	(0.033)	(0.039)	(0.045)
T. Hill	0.500	0.751	1.000	1.249	1.501	1.752	2.001
(s.d)	(0.011)	(0.018)	(0.023)	(0.029)	(0.033)	(0.039)	(0.044)
(Theor. s.d.)	(0.011)	(0.017)	(0.022)	(0.028)	(0.034)	(0.039)	(0.045)
Regression	0.497	0.748	0.997	1.243	1.494	1.746	1.995
(s.d.)	(0.015)	(0.023)	(0.029)	(0.038)	(0.043)	(0.053)	(0.060)
$u = 10$							
Hill	0.513	0.770	1.029	1.283	1.541	1.799	2.058
(s.d.)	(0.011)	(0.017)	(0.022)	(0.029)	(0.033)	(0.041)	(0.046)
T. Hill	0.500	0.750	1.002	1.250	1.501	1.752	2.004
(s.d)	(0.011)	(0.017)	(0.022)	(0.028)	(0.033)	(0.040)	(0.044)
(Theor. s.d.)	(0.011)	(0.017)	(0.022)	(0.028)	(0.034)	(0.039)	(0.045)
Regression	0.498	0.747	0.998	1.246	1.495	1.746	1.997
(s.d.)	(0.014)	(0.022)	(0.029)	(0.036)	(0.043)	(0.052)	(0.057)
$u = 50$							
Hill	0.552	0.826	1.102	1.379	1.655	1.930	2.208
(s.d.)	(0.012)	(0.018)	(0.025)	(0.030)	(0.038)	(0.044)	(0.050)
T. Hill	0.500	0.750	0.999	1.251	1.501	1.751	2.002
(s.d)	(0.011)	(0.016)	(0.022)	(0.027)	(0.034)	(0.040)	(0.045)
(Theor. s.d.)	(0.011)	(0.017)	(0.022)	(0.028)	(0.034)	(0.039)	(0.045)
Regression	0.500	0.749	0.997	1.250	1.499	1.748	1.997
(s.d.)	(0.014)	(0.020)	(0.027)	(0.033)	(0.041)	(0.049)	(0.056)
$u = 100$							
Hill	0.591	0.886	1.181	1.476	1.774	2.069	2.362
(s.d.)	(0.014)	(0.020)	(0.028)	(0.034)	(0.039)	(0.047)	(0.054)
T. Hill	0.501	0.750	1.000	1.250	1.503	1.752	2.001
(s.d)	(0.011)	(0.017)	(0.023)	(0.028)	(0.032)	(0.040)	(0.045)
(Theor. s.d.)	(0.011)	(0.017)	(0.022)	(0.028)	(0.034)	(0.039)	(0.045)
Regression	0.500	0.750	0.999	1.248	1.501	1.749	1.999
(s.d.)	(0.013)	(0.020)	(0.027)	(0.034)	(0.039)	(0.046)	(0.053)
$u = 500$							
Hill	0.852	1.278	1.707	2.131	2.556	2.983	3.406
(s.d.)	(0.020)	(0.030)	(0.042)	(0.051)	(0.061)	(0.073)	(0.081)
T. Hill	0.500	0.750	1.003	1.252	1.501	1.752	2.000
(s.d)	(0.011)	(0.017)	(0.024)	(0.029)	(0.034)	(0.041)	(0.046)
(Theor. s.d.)	(0.011)	(0.017)	(0.023)	(0.029)	(0.034)	(0.040)	(0.046)
Regression	0.500	0.750	1.002	1.251	1.498	1.752	1.999
(s.d.)	(0.013)	(0.019)	(0.026)	(0.032)	(0.038)	(0.046)	(0.051)
$u = 1000$							
Hill	1.168	1.753	2.336	2.922	3.511	4.089	4.675
(s.d.)	(0.030)	(0.044)	(0.056)	(0.074)	(0.085)	(0.106)	(0.121)
T. Hill	0.500	0.750	1.001	1.251	1.504	1.751	2.001
(s.d)	(0.012)	(0.017)	(0.023)	(0.029)	(0.034)	(0.042)	(0.048)
(Theor. s.d.)	(0.012)	(0.018)	(0.024)	(0.029)	(0.035)	(0.041)	(0.047)
Regression	0.500	0.750	1.001	1.250	1.503	1.750	2.001
(s.d.)	(0.013)	(0.019)	(0.025)	(0.032)	(0.037)	(0.047)	(0.053)

TABLE XXV: Stable Distribution, $k = 500$, $s = .010(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.620	0.941	1.283	1.452	1.930	2.369	10.521
(s.d.)	(0.349)	(0.554)	(0.716)	(0.798)	(1.220)	(1.564)	(5.336)
T. Hill	0.620	0.941	1.283	1.452	1.930	2.369	10.521
(s.d)	(0.349)	(0.554)	(0.716)	(0.798)	(1.220)	(1.564)	(5.336)
(Theor. s.d.)	(0.300)	(0.450)	(0.600)	(0.750)	(0.900)	(1.050)	(1.200)
Regression	0.495	0.752	1.013	1.137	1.546	1.851	8.923
(s.d.)	(0.388)	(0.561)	(0.729)	(0.864)	(1.321)	(1.653)	(5.983)
$u = 1$							
Hill	0.991	1.593	2.042	2.444	3.042	3.734	15.406
(s.d.)	(0.674)	(1.221)	(1.525)	(1.846)	(2.107)	(2.558)	(10.080)
T. Hill	0.670	1.076	1.380	1.655	2.053	2.512	10.459
(s.d)	(0.453)	(0.819)	(1.021)	(1.242)	(1.409)	(1.721)	(6.699)
(Theor. s.d.)	(0.361)	(0.541)	(0.722)	(0.902)	(1.083)	(1.263)	(1.443)
Regression	0.531	0.854	1.097	1.309	1.633	1.975	8.408
(s.d.)	(0.366)	(0.678)	(0.828)	(1.030)	(1.172)	(1.422)	(5.518)
$u = 5$							
Hill	2.194	3.386	4.594	4.934	6.396	9.369	28.056
(s.d.)	(1.583)	(2.863)	(3.800)	(3.390)	(4.140)	(7.153)	(18.033)
T. Hill	0.683	1.044	1.413	1.535	1.969	2.898	8.730
(s.d)	(0.467)	(0.844)	(1.118)	(1.011)	(1.213)	(2.155)	(5.249)
(Theor. s.d.)	(0.361)	(0.541)	(0.722)	(0.902)	(1.083)	(1.263)	(1.443)
Regression	0.582	0.887	1.199	1.307	1.658	2.468	7.453
(s.d.)	(0.402)	(0.727)	(0.977)	(0.881)	(1.014)	(1.883)	(4.551)
$u = 10$							
Hill	3.470	5.634	7.635	7.447	10.424	18.187	40.150
(s.d.)	(2.387)	(3.901)	(5.419)	(5.882)	(8.287)	(14.122)	(27.080)
T. Hill	0.657	1.057	1.432	1.403	1.942	3.381	7.575
(s.d)	(0.430)	(0.696)	(0.985)	(1.148)	(1.497)	(2.424)	(4.786)
(Theor. s.d.)	(0.361)	(0.541)	(0.722)	(0.902)	(1.083)	(1.263)	(1.443)
Regression	0.575	0.922	1.245	1.228	1.686	2.935	6.616
(s.d.)	(0.378)	(0.630)	(0.862)	(1.092)	(1.314)	(2.064)	(4.205)
$u = 50$							
Hill	14.891	25.338	29.238	136.403	84.042	84.267	95.188
(s.d.)	(10.734)	(16.460)	(22.002)	(107.061)	(66.066)	(61.479)	(68.909)
T. Hill	0.668	1.143	1.328	6.130	3.840	3.863	4.301
(s.d)	(0.445)	(0.740)	(0.954)	(4.366)	(2.959)	(2.632)	(2.921)
(Theor. s.d.)	(0.361)	(0.541)	(0.722)	(0.902)	(1.083)	(1.263)	(1.443)
Regression	0.600	1.024	1.195	5.474	3.474	3.449	3.847
(s.d.)	(0.407)	(0.647)	(0.871)	(3.927)	(2.741)	(2.327)	(2.642)

TABLE XXVI: Stable Distribution, $k = 1000$, $s = .010(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.559	0.853	1.146	1.336	1.642	2.119	9.760
(s.d.)	(0.192)	(0.305)	(0.433)	(0.465)	(0.588)	(0.762)	(3.165)
T. Hill	0.559	0.853	1.146	1.336	1.642	2.119	9.760
(s.d)	(0.192)	(0.305)	(0.433)	(0.465)	(0.588)	(0.762)	(3.165)
(Theor. s.d.)	(0.183)	(0.275)	(0.367)	(0.458)	(0.550)	(0.642)	(0.733)
Regression	0.450	0.690	0.925	1.103	1.331	1.687	8.619
(s.d.)	(0.197)	(0.311)	(0.440)	(0.470)	(0.602)	(0.746)	(3.349)
$u = 1$							
Hill	0.694	1.076	1.480	1.738	2.141	2.747	11.981
(s.d.)	(0.237)	(0.391)	(0.604)	(0.649)	(0.874)	(1.074)	(4.283)
T. Hill	0.544	0.841	1.158	1.362	1.677	2.144	9.510
(s.d)	(0.183)	(0.300)	(0.468)	(0.504)	(0.680)	(0.836)	(3.326)
(Theor. s.d.)	(0.196)	(0.295)	(0.393)	(0.491)	(0.589)	(0.687)	(0.786)
Regression	0.462	0.706	0.969	1.158	1.410	1.792	8.364
(s.d.)	(0.178)	(0.276)	(0.419)	(0.483)	(0.622)	(0.780)	(3.073)
$u = 5$							
Hill	1.260	1.885	2.525	2.923	3.705	4.789	18.382
(s.d.)	(0.551)	(0.772)	(0.997)	(1.228)	(1.486)	(2.027)	(6.908)
T. Hill	0.578	0.862	1.150	1.339	1.695	2.183	8.556
(s.d)	(0.240)	(0.333)	(0.435)	(0.531)	(0.656)	(0.895)	(3.090)
(Theor. s.d.)	(0.196)	(0.295)	(0.393)	(0.491)	(0.589)	(0.687)	(0.786)
Regression	0.516	0.773	1.028	1.197	1.520	1.946	7.767
(s.d.)	(0.222)	(0.312)	(0.407)	(0.484)	(0.624)	(0.844)	(2.959)
$u = 10$							
Hill	1.860	2.879	3.878	4.228	5.441	7.776	24.572
(s.d.)	(0.730)	(1.145)	(1.565)	(1.673)	(2.078)	(3.143)	(9.638)
T. Hill	0.569	0.880	1.192	1.301	1.663	2.375	7.638
(s.d)	(0.204)	(0.326)	(0.466)	(0.479)	(0.600)	(0.897)	(2.759)
(Theor. s.d.)	(0.196)	(0.295)	(0.393)	(0.491)	(0.589)	(0.687)	(0.786)
Regression	0.515	0.797	1.085	1.189	1.516	2.162	7.018
(s.d.)	(0.191)	(0.307)	(0.440)	(0.452)	(0.574)	(0.846)	(2.687)
$u = 50$							
Hill	6.473	10.671	13.424	23.956	26.578	40.111	58.363
(s.d.)	(2.585)	(4.434)	(5.695)	(16.008)	(12.168)	(16.132)	(26.110)
T. Hill	0.562	0.919	1.157	1.975	2.253	3.483	5.047
(s.d)	(0.207)	(0.345)	(0.436)	(1.275)	(0.962)	(1.266)	(2.067)
(Theor. s.d.)	(0.196)	(0.295)	(0.393)	(0.491)	(0.589)	(0.687)	(0.786)
Regression	0.524	0.857	1.075	1.835	2.099	3.261	4.712
(s.d.)	(0.204)	(0.332)	(0.420)	(1.209)	(0.913)	(1.238)	(1.950)
$u = 100$							
Hill	12.002	21.308	24.099	110.178	71.258	68.980	80.745
(s.d.)	(4.814)	(8.868)	(9.737)	(44.121)	(28.879)	(29.936)	(33.009)
T. Hill	0.552	0.973	1.103	5.048	3.246	3.182	3.714
(s.d)	(0.191)	(0.352)	(0.411)	(1.836)	(1.199)	(1.200)	(1.385)
(Theor. s.d.)	(0.196)	(0.295)	(0.393)	(0.491)	(0.589)	(0.687)	(0.786)
Regression	0.526	0.919	1.048	4.810	3.086	3.019	3.525
(s.d.)	(0.210)	(0.375)	(0.454)	(1.965)	(1.339)	(1.247)	(1.525)

TABLE XXVII: Stable Distribution, $k = 5000$, $s = .010(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.510	0.765	1.039	1.231	1.534	1.897	9.163
(s.d.)	(0.071)	(0.116)	(0.145)	(0.166)	(0.223)	(0.288)	(1.156)
T. Hill	0.510	0.765	1.039	1.231	1.534	1.897	9.163
(s.d)	(0.071)	(0.116)	(0.145)	(0.166)	(0.223)	(0.288)	(1.156)
(Theor. s.d.)	(0.073)	(0.109)	(0.146)	(0.182)	(0.219)	(0.255)	(0.291)
Regression	0.463	0.687	0.934	1.127	1.399	1.657	9.478
(s.d.)	(0.089)	(0.146)	(0.183)	(0.208)	(0.280)	(0.344)	(1.346)
$u = 1$							
Hill	0.550	0.834	1.119	1.336	1.647	2.055	9.608
(s.d.)	(0.080)	(0.121)	(0.168)	(0.190)	(0.245)	(0.311)	(1.255)
T. Hill	0.510	0.774	1.038	1.242	1.529	1.905	9.016
(s.d)	(0.074)	(0.112)	(0.155)	(0.177)	(0.227)	(0.289)	(1.152)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.475	0.718	0.963	1.176	1.427	1.753	9.262
(s.d.)	(0.083)	(0.130)	(0.176)	(0.203)	(0.255)	(0.344)	(1.240)
$u = 5$							
Hill	0.666	1.015	1.361	1.602	1.996	2.518	11.058
(s.d.)	(0.099)	(0.146)	(0.209)	(0.231)	(0.308)	(0.376)	(1.465)
T. Hill	0.510	0.776	1.041	1.229	1.529	1.918	8.655
(s.d)	(0.075)	(0.108)	(0.158)	(0.170)	(0.229)	(0.280)	(1.097)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.489	0.744	0.995	1.185	1.472	1.813	8.724
(s.d.)	(0.082)	(0.122)	(0.174)	(0.183)	(0.246)	(0.310)	(1.249)
$u = 10$							
Hill	0.790	1.205	1.611	1.887	2.340	3.007	12.649
(s.d.)	(0.119)	(0.179)	(0.242)	(0.275)	(0.354)	(0.469)	(1.836)
T. Hill	0.510	0.776	1.041	1.222	1.511	1.928	8.375
(s.d)	(0.075)	(0.110)	(0.151)	(0.171)	(0.220)	(0.294)	(1.138)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.493	0.750	1.005	1.191	1.462	1.843	8.405
(s.d.)	(0.081)	(0.118)	(0.160)	(0.191)	(0.237)	(0.309)	(1.220)
$u = 50$							
Hill	1.666	2.580	3.455	3.801	4.903	7.065	22.018
(s.d.)	(0.266)	(0.413)	(0.560)	(0.591)	(0.749)	(1.200)	(3.357)
T. Hill	0.513	0.790	1.059	1.176	1.508	2.128	6.910
(s.d)	(0.074)	(0.117)	(0.161)	(0.166)	(0.210)	(0.332)	(0.938)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.504	0.773	1.039	1.152	1.482	2.075	6.825
(s.d.)	(0.077)	(0.125)	(0.171)	(0.177)	(0.229)	(0.357)	(1.019)
u = 100							
Hill	2.728	4.256	5.616	5.819	7.758	13.654	30.826
(s.d.)	(0.437)	(0.713)	(0.882)	(0.937)	(1.279)	(2.481)	(4.941)
T. Hill	0.515	0.801	1.058	1.101	1.468	2.511	5.912
(s.d)	(0.073)	(0.121)	(0.151)	(0.157)	(0.215)	(0.420)	(0.825)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.508	0.789	1.042	1.083	1.445	2.446	5.829
(s.d.)	(0.079)	(0.129)	(0.165)	(0.167)	(0.233)	(0.444)	(0.897)
$u = 500$							
Hill	11.779	20.795	23.441	109.585	69.511	67.955	77.984
(s.d.)	(2.062)	(3.473)	(3.993)	(19.186)	(11.819)	(11.951)	(13.805)
T. Hill	0.506	0.897	1.013	4.699	3.002	2.936	3.374
(s.d)	(0.076)	(0.133)	(0.150)	(0.744)	(0.454)	(0.445)	(0.514)
(Theor. s.d.)	(0.077)	(0.116)	(0.154)	(0.193)	(0.231)	(0.270)	(0.308)
Regression	0.500	0.884	0.996	4.631	2.960	2.898	3.345
(s.d.)	(0.081)	(0.141)	(0.163)	(0.794)	(0.484)	(0.469)	(0.555)

TABLE XXVIII: Stable Distribution, $k = 10000$, $s = .010(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.507	0.767	1.027	1.228	1.507	1.872	9.079
(s.d.)	(0.052)	(0.079)	(0.107)	(0.125)	(0.149)	(0.191)	(0.801)
T. Hill	0.507	0.767	1.027	1.228	1.507	1.872	9.079
(s.d)	(0.052)	(0.079)	(0.107)	(0.125)	(0.149)	(0.191)	(0.801)
(Theor. s.d.)	(0.051)	(0.076)	(0.102)	(0.127)	(0.152)	(0.178)	(0.203)
Regression	0.474	0.717	0.957	1.168	1.421	1.705	9.817
(s.d.)	(0.067)	(0.104)	(0.139)	(0.170)	(0.196)	(0.249)	(0.937)
$u = 1$							
Hill	0.528	0.800	1.070	1.285	1.574	1.952	9.350
(s.d.)	(0.054)	(0.079)	(0.108)	(0.129)	(0.158)	(0.204)	(0.843)
T. Hill	0.506	0.766	1.024	1.231	1.507	1.866	9.024
(s.d)	(0.052)	(0.075)	(0.103)	(0.123)	(0.151)	(0.194)	(0.805)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.485	0.729	0.974	1.190	1.434	1.735	9.664
(s.d.)	(0.063)	(0.091)	(0.124)	(0.150)	(0.186)	(0.238)	(0.984)
$u = 5$							
Hill	0.593	0.896	1.215	1.433	1.777	2.213	10.201
(s.d.)	(0.062)	(0.091)	(0.127)	(0.148)	(0.178)	(0.246)	(0.944)
T. Hill	0.505	0.762	1.033	1.222	1.514	1.876	8.852
(s.d)	(0.052)	(0.077)	(0.107)	(0.125)	(0.149)	(0.204)	(0.788)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.491	0.739	1.002	1.200	1.476	1.788	9.259
(s.d.)	(0.058)	(0.087)	(0.123)	(0.143)	(0.173)	(0.224)	(0.919)
$u = 10$							
Hill	0.663	1.007	1.350	1.602	1.979	2.495	11.068
(s.d.)	(0.071)	(0.104)	(0.144)	(0.166)	(0.204)	(0.276)	(1.089)
T. Hill	0.506	0.767	1.028	1.225	1.508	1.892	8.643
(s.d)	(0.053)	(0.078)	(0.108)	(0.125)	(0.151)	(0.203)	(0.800)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.495	0.747	1.002	1.204	1.478	1.823	8.882
(s.d.)	(0.059)	(0.086)	(0.122)	(0.137)	(0.170)	(0.224)	(0.898)
$u = 50$							
Hill	1.121	1.716	2.304	2.630	3.347	4.430	16.296
(s.d.)	(0.121)	(0.185)	(0.246)	(0.276)	(0.365)	(0.493)	(1.691)
T. Hill	0.506	0.772	1.038	1.193	1.510	1.977	7.555
(s.d)	(0.052)	(0.079)	(0.104)	(0.117)	(0.152)	(0.209)	(0.706)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.501	0.762	1.026	1.183	1.492	1.939	7.576
(s.d.)	(0.056)	(0.088)	(0.113)	(0.127)	(0.161)	(0.229)	(0.787)
$u = 100$							
Hill	1.651	2.541	3.415	3.753	4.859	6.908	21.634
(s.d.)	(0.189)	(0.289)	(0.382)	(0.409)	(0.542)	(0.790)	(2.348)
T. Hill	0.507	0.779	1.044	1.160	1.498	2.087	6.794
(s.d)	(0.054)	(0.081)	(0.109)	(0.114)	(0.155)	(0.225)	(0.652)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.502	0.771	1.033	1.152	1.486	2.047	6.768
(s.d.)	(0.058)	(0.092)	(0.119)	(0.123)	(0.169)	(0.241)	(0.716)
$u = 500$							
Hill	5.957	9.792	12.346	20.794	24.130	37.028	52.250
(s.d.)	(0.682)	(1.197)	(1.421)	(4.836)	(3.196)	(4.254)	(5.926)
T. Hill	0.506	0.831	1.051	1.565	1.998	3.151	4.491
(s.d)	(0.051)	(0.090)	(0.108)	(0.364)	(0.252)	(0.322)	(0.457)
(Theor. s.d.)	(0.052)	(0.078)	(0.104)	(0.130)	(0.156)	(0.182)	(0.208)
Regression	0.502	0.824	1.045	1.536	1.985	3.138	4.468
(s.d.)	(0.057)	(0.095)	(0.119)	(0.373)	(0.268)	(0.355)	(0.505)
$u = 1000$							
Hill	11.824	20.690	23.336	109.042	68.365	68.668	78.325
(s.d.)	(1.407)	(2.598)	(2.756)	(13.395)	(8.417)	(8.223)	(9.777)
T. Hill	0.504	0.885	0.999	4.656	2.922	2.951	3.371
(s.d)	(0.052)	(0.096)	(0.104)	(0.507)	(0.312)	(0.308)	(0.370)
(Theor. s.d.)	(0.054)	(0.080)	(0.107)	(0.134)	(0.161)	(0.188)	(0.214)
Regression	0.501	0.880	0.993	4.619	2.897	2.927	3.355
(s.d.)	(0.056)	(0.105)	(0.112)	(0.546)	(0.341)	(0.337)	(0.392)

TABLE XXIX: Stable Distribution, $k = 500$, $s = .025(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.549	0.833	1.106	1.242	1.585	2.183	7.773
(s.d.)	(0.176)	(0.271)	(0.351)	(0.371)	(0.483)	(0.731)	(2.047)
T. Hill	0.549	0.833	1.106	1.242	1.585	2.183	7.773
(s.d)	(0.176)	(0.271)	(0.351)	(0.371)	(0.483)	(0.731)	(2.047)
(Theor. s.d.)	(0.163)	(0.245)	(0.327)	(0.408)	(0.490)	(0.572)	(0.653)
Regression	0.452	0.681	0.905	1.048	1.318	1.678	7.143
(s.d.)	(0.182)	(0.269)	(0.353)	(0.396)	(0.509)	(0.707)	(2.081)
$u = 1$							
Hill	0.660	1.030	1.374	1.521	1.985	2.851	8.785
(s.d.)	(0.213)	(0.338)	(0.441)	(0.470)	(0.609)	(0.973)	(2.660)
T. Hill	0.539	0.839	1.122	1.248	1.625	2.298	7.299
(s.d)	(0.172)	(0.273)	(0.357)	(0.380)	(0.503)	(0.774)	(2.129)
(Theor. s.d.)	(0.163)	(0.245)	(0.327)	(0.408)	(0.490)	(0.572)	(0.653)
Regression	0.466	0.715	0.969	1.100	1.418	1.888	6.749
(s.d.)	(0.168)	(0.262)	(0.355)	(0.382)	(0.507)	(0.723)	(2.058)
$u = 5$							
Hill	1.061	1.665	2.208	2.265	3.113	5.275	12.017
(s.d.)	(0.374)	(0.546)	(0.724)	(0.707)	(1.010)	(2.068)	(3.549)
T. Hill	0.546	0.853	1.142	1.175	1.597	2.633	6.357
(s.d)	(0.183)	(0.268)	(0.366)	(0.341)	(0.492)	(0.999)	(1.763)
(Theor. s.d.)	(0.163)	(0.245)	(0.327)	(0.408)	(0.490)	(0.572)	(0.653)
Regression	0.496	0.775	1.047	1.080	1.449	2.330	5.929
(s.d.)	(0.174)	(0.261)	(0.357)	(0.338)	(0.483)	(0.961)	(1.721)
$u = 10$							
Hill	1.511	2.401	3.163	3.177	4.540	8.585	15.695
(s.d.)	(0.491)	(0.804)	(1.105)	(1.139)	(1.558)	(3.228)	(5.289)
T. Hill	0.543	0.863	1.140	1.149	1.623	3.006	5.796
(s.d)	(0.167)	(0.270)	(0.376)	(0.363)	(0.506)	(1.077)	(1.773)
(Theor. s.d.)	(0.163)	(0.245)	(0.327)	(0.408)	(0.490)	(0.572)	(0.653)
Regression	0.501	0.796	1.059	1.054	1.493	2.749	5.442
(s.d.)	(0.166)	(0.264)	(0.375)	(0.342)	(0.493)	(1.017)	(1.718)
$u = 50$							
Hill	5.346	9.322	10.407	50.823	31.170	30.396	33.422
(s.d.)	(1.937)	(3.348)	(3.943)	(20.878)	(11.059)	(10.728)	(11.671)
T. Hill	0.546	0.952	1.073	5.146	3.175	3.110	3.480
(s.d)	(0.182)	(0.306)	(0.341)	(1.877)	(1.007)	(1.005)	(1.109)
(Theor. s.d.)	(0.172)	(0.259)	(0.345)	(0.431)	(0.517)	(0.604)	(0.690)
Regression	0.517	0.895	1.016	4.856	3.013	2.934	3.290
(s.d.)	(0.182)	(0.301)	(0.340)	(1.861)	(1.000)	(1.008)	(1.139)

TABLE XXX: Stable Distribution, $k = 1000$, $s = .025(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.520	0.813	1.066	1.193	1.537	2.152	7.445
(s.d.)	(0.111)	(0.172)	(0.219)	(0.231)	(0.315)	(0.504)	(1.372)
T. Hill	0.520	0.813	1.066	1.193	1.537	2.152	7.445
(s.d)	(0.111)	(0.172)	(0.219)	(0.231)	(0.315)	(0.504)	(1.372)
(Theor. s.d.)	(0.106)	(0.159)	(0.212)	(0.265)	(0.318)	(0.372)	(0.425)
Regression	0.452	0.700	0.915	1.063	1.336	1.737	7.502
(s.d.)	(0.128)	(0.196)	(0.255)	(0.277)	(0.367)	(0.545)	(1.511)
$u = 1$							
Hill	0.589	0.911	1.225	1.375	1.746	2.468	8.074
(s.d.)	(0.127)	(0.198)	(0.269)	(0.293)	(0.372)	(0.588)	(1.543)
T. Hill	0.520	0.804	1.080	1.217	1.541	2.159	7.234
(s.d)	(0.111)	(0.175)	(0.236)	(0.256)	(0.327)	(0.516)	(1.342)
(Theor. s.d.)	(0.109)	(0.163)	(0.217)	(0.272)	(0.326)	(0.380)	(0.434)
Regression	0.469	0.725	0.968	1.130	1.404	1.846	7.147
(s.d.)	(0.118)	(0.193)	(0.249)	(0.275)	(0.365)	(0.532)	(1.379)
$u = 5$							
Hill	0.807	1.248	1.662	1.792	2.359	3.627	10.080
(s.d.)	(0.178)	(0.279)	(0.382)	(0.383)	(0.516)	(0.895)	(1.981)
T. Hill	0.524	0.807	1.078	1.173	1.530	2.287	6.728
(s.d)	(0.111)	(0.175)	(0.239)	(0.241)	(0.319)	(0.536)	(1.245)
(Theor. s.d.)	(0.109)	(0.163)	(0.217)	(0.272)	(0.326)	(0.380)	(0.434)
Regression	0.493	0.754	1.012	1.114	1.443	2.047	6.572
(s.d.)	(0.116)	(0.184)	(0.248)	(0.250)	(0.323)	(0.524)	(1.324)
$u = 10$							
Hill	1.029	1.645	2.192	2.235	3.028	5.066	12.065
(s.d.)	(0.233)	(0.381)	(0.495)	(0.512)	(0.706)	(1.332)	(2.596)
T. Hill	0.516	0.820	1.096	1.134	1.523	2.456	6.250
(s.d)	(0.112)	(0.179)	(0.231)	(0.240)	(0.326)	(0.632)	(1.224)
(Theor. s.d.)	(0.109)	(0.163)	(0.217)	(0.272)	(0.326)	(0.380)	(0.434)
Regression	0.491	0.776	1.038	1.092	1.452	2.274	6.089
(s.d.)	(0.114)	(0.186)	(0.240)	(0.250)	(0.337)	(0.641)	(1.269)
$u = 50$							
Hill	2.845	4.846	5.973	15.154	13.460	17.565	23.601
(s.d.)	(0.677)	(1.158)	(1.550)	(6.585)	(3.834)	(4.255)	(5.706)
T. Hill	0.519	0.879	1.088	2.401	2.343	3.216	4.373
(s.d)	(0.111)	(0.190)	(0.255)	(1.134)	(0.654)	(0.718)	(0.935)
(Theor. s.d.)	(0.111)	(0.167)	(0.222)	(0.278)	(0.334)	(0.389)	(0.445)
Regression	0.505	0.846	1.052	2.285	2.261	3.110	4.229
(s.d.)	(0.118)	(0.192)	(0.261)	(1.152)	(0.665)	(0.751)	(0.967)
$u = 100$							
Hill	5.251	9.393	10.389	48.740	30.344	29.452	33.685
(s.d.)	(1.304)	(2.335)	(2.482)	(12.699)	(7.025)	(7.173)	(8.592)
T. Hill	0.520	0.929	1.032	4.807	3.024	2.943	3.384
(s.d)	(0.116)	(0.206)	(0.219)	(1.123)	(0.631)	(0.629)	(0.782)
(Theor. s.d.)	(0.114)	(0.171)	(0.228)	(0.285)	(0.342)	(0.399)	(0.456)
Regression	0.506	0.902	1.002	4.654	2.947	2.860	3.288
(s.d.)	(0.122)	(0.218)	(0.228)	(1.138)	(0.673)	(0.648)	(0.816)

TABLE XXXI: Stable Distribution, $k = 5000$, $s = .025(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.504	0.775	1.034	1.175	1.487	2.040	7.314
(s.d.)	(0.045)	(0.069)	(0.095)	(0.106)	(0.135)	(0.193)	(0.582)
T. Hill	0.504	0.775	1.034	1.175	1.487	2.040	7.314
(s.d)	(0.045)	(0.069)	(0.095)	(0.106)	(0.135)	(0.193)	(0.582)
(Theor. s.d.)	(0.045)	(0.068)	(0.091)	(0.113)	(0.136)	(0.158)	(0.181)
Regression	0.475	0.723	0.964	1.145	1.414	1.779	8.182
(s.d.)	(0.059)	(0.089)	(0.125)	(0.146)	(0.175)	(0.241)	(0.687)
$u = 1$							
Hill	0.526	0.802	1.079	1.216	1.550	2.131	7.506
(s.d.)	(0.049)	(0.073)	(0.098)	(0.107)	(0.139)	(0.222)	(0.600)
T. Hill	0.508	0.773	1.040	1.175	1.495	2.048	7.297
(s.d)	(0.047)	(0.071)	(0.095)	(0.103)	(0.134)	(0.214)	(0.577)
(Theor. s.d.)	(0.045)	(0.068)	(0.091)	(0.114)	(0.136)	(0.159)	(0.182)
Regression	0.488	0.736	0.995	1.163	1.447	1.828	8.053
(s.d.)	(0.057)	(0.087)	(0.116)	(0.129)	(0.168)	(0.246)	(0.651)
$u = 5$							
Hill	0.580	0.887	1.188	1.330	1.711	2.393	8.001
(s.d.)	(0.053)	(0.080)	(0.109)	(0.114)	(0.152)	(0.238)	(0.620)
T. Hill	0.506	0.773	1.035	1.167	1.494	2.067	7.128
(s.d)	(0.045)	(0.069)	(0.093)	(0.098)	(0.131)	(0.204)	(0.538)
(Theor. s.d.)	(0.045)	(0.068)	(0.091)	(0.114)	(0.136)	(0.159)	(0.182)
Regression	0.496	0.752	1.004	1.171	1.465	1.906	7.630
(s.d.)	(0.052)	(0.082)	(0.108)	(0.118)	(0.157)	(0.231)	(0.636)
$u = 10$							
Hill	0.634	0.983	1.307	1.452	1.877	2.682	8.572
(s.d.)	(0.059)	(0.095)	(0.124)	(0.126)	(0.171)	(0.290)	(0.717)
T. Hill	0.504	0.778	1.038	1.161	1.494	2.095	6.990
(s.d)	(0.046)	(0.073)	(0.096)	(0.098)	(0.132)	(0.220)	(0.556)
(Theor. s.d.)	(0.045)	(0.068)	(0.091)	(0.114)	(0.136)	(0.159)	(0.182)
Regression	0.494	0.758	1.013	1.165	1.471	1.952	7.343
(s.d.)	(0.053)	(0.084)	(0.108)	(0.113)	(0.152)	(0.230)	(0.634)
$u = 50$							
Hill	1.008	1.583	2.110	2.184	2.931	4.943	11.838
(s.d.)	(0.100)	(0.152)	(0.202)	(0.196)	(0.281)	(0.540)	(1.099)
T. Hill	0.502	0.785	1.049	1.102	1.468	2.353	6.097
(s.d)	(0.046)	(0.071)	(0.094)	(0.092)	(0.132)	(0.241)	(0.514)
(Theor. s.d.)	(0.046)	(0.068)	(0.091)	(0.114)	(0.137)	(0.160)	(0.183)
Regression	0.496	0.775	1.039	1.106	1.464	2.243	6.145
(s.d.)	(0.050)	(0.080)	(0.103)	(0.102)	(0.146)	(0.256)	(0.562)
$u = 100$							
Hill	1.448	2.312	3.015	2.926	4.241	8.163	15.036
(s.d.)	(0.149)	(0.230)	(0.300)	(0.278)	(0.441)	(0.869)	(1.406)
T. Hill	0.504	0.801	1.050	1.040	1.474	2.729	5.392
(s.d)	(0.047)	(0.073)	(0.098)	(0.088)	(0.137)	(0.287)	(0.454)
(Theor. s.d.)	(0.046)	(0.069)	(0.092)	(0.115)	(0.137)	(0.160)	(0.183)
Regression	0.500	0.794	1.041	1.040	1.450	2.681	5.406
(s.d.)	(0.051)	(0.079)	(0.105)	(0.099)	(0.145)	(0.310)	(0.510)
$u = 500$							
Hill	5.101	9.125	10.025	47.890	29.989	28.648	32.566
(s.d.)	(0.558)	(1.015)	(1.099)	(5.139)	(3.337)	(3.004)	(3.548)
T. Hill	0.500	0.894	0.992	4.686	2.941	2.837	3.255
(s.d)	(0.048)	(0.088)	(0.096)	(0.448)	(0.283)	(0.257)	(0.307)
(Theor. s.d.)	(0.048)	(0.072)	(0.096)	(0.120)	(0.144)	(0.168)	(0.192)
Regression	0.497	0.887	0.984	4.658	2.925	2.821	3.250
(s.d.)	(0.050)	(0.095)	(0.103)	(0.485)	(0.310)	(0.276)	(0.335)

TABLE XXXII: Stable Distribution, $k = 10000$, $s = .025(k-u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.501	0.769	1.031	1.171	1.486	2.030	7.301
(s.d.)	(0.031)	(0.048)	(0.066)	(0.072)	(0.096)	(0.138)	(0.394)
T. Hill	0.501	0.769	1.031	1.171	1.486	2.030	7.301
(s.d)	(0.031)	(0.048)	(0.066)	(0.072)	(0.096)	(0.138)	(0.394)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.095)	(0.111)	(0.127)
Regression	0.482	0.733	0.982	1.168	1.437	1.813	8.377
(s.d.)	(0.043)	(0.065)	(0.090)	(0.099)	(0.129)	(0.178)	(0.482)
$u = 1$							
Hill	0.513	0.789	1.055	1.191	1.524	2.072	7.411
(s.d.)	(0.034)	(0.050)	(0.066)	(0.072)	(0.094)	(0.147)	(0.425)
T. Hill	0.503	0.773	1.033	1.167	1.494	2.025	7.298
(s.d)	(0.033)	(0.049)	(0.065)	(0.070)	(0.092)	(0.144)	(0.415)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.490	0.746	0.998	1.174	1.465	1.835	8.265
(s.d.)	(0.041)	(0.064)	(0.082)	(0.093)	(0.118)	(0.170)	(0.504)
$u = 5$							
Hill	0.544	0.839	1.116	1.263	1.613	2.229	7.672
(s.d.)	(0.035)	(0.055)	(0.073)	(0.079)	(0.103)	(0.156)	(0.445)
T. Hill	0.502	0.774	1.028	1.170	1.489	2.044	7.190
(s.d)	(0.032)	(0.050)	(0.067)	(0.073)	(0.095)	(0.141)	(0.408)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.494	0.756	1.003	1.187	1.475	1.890	7.929
(s.d.)	(0.038)	(0.059)	(0.081)	(0.088)	(0.118)	(0.162)	(0.484)
$u = 10$							
Hill	0.579	0.886	1.186	1.331	1.705	2.390	7.951
(s.d.)	(0.039)	(0.058)	(0.080)	(0.083)	(0.108)	(0.178)	(0.446)
T. Hill	0.504	0.771	1.032	1.165	1.487	2.060	7.078
(s.d)	(0.033)	(0.050)	(0.069)	(0.071)	(0.094)	(0.151)	(0.384)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.498	0.755	1.013	1.179	1.477	1.921	7.667
(s.d.)	(0.039)	(0.057)	(0.081)	(0.086)	(0.113)	(0.166)	(0.460)
$u = 50$							
Hill	0.784	1.217	1.626	1.751	2.286	3.497	9.870
(s.d.)	(0.050)	(0.080)	(0.110)	(0.111)	(0.149)	(0.260)	(0.614)
T. Hill	0.503	0.778	1.041	1.136	1.472	2.180	6.553
(s.d)	(0.031)	(0.049)	(0.067)	(0.069)	(0.091)	(0.156)	(0.379)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.500	0.769	1.031	1.149	1.470	2.062	6.781
(s.d.)	(0.035)	(0.055)	(0.075)	(0.077)	(0.101)	(0.162)	(0.420)
$u = 100$							
Hill	1.012	1.582	2.099	2.184	2.928	4.907	11.727
(s.d.)	(0.069)	(0.108)	(0.137)	(0.142)	(0.194)	(0.383)	(0.732)
T. Hill	0.504	0.784	1.043	1.102	1.464	2.334	6.046
(s.d)	(0.032)	(0.049)	(0.063)	(0.068)	(0.090)	(0.175)	(0.337)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.502	0.778	1.037	1.110	1.461	2.238	6.161
(s.d.)	(0.036)	(0.053)	(0.071)	(0.076)	(0.100)	(0.188)	(0.378)
$u = 500$							
Hill	2.761	4.589	5.714	14.803	13.067	17.207	22.801
(s.d.)	(0.193)	(0.338)	(0.430)	(2.191)	(1.138)	(1.260)	(1.586)
T. Hill	0.503	0.832	1.043	2.178	2.240	3.135	4.246
(s.d)	(0.032)	(0.056)	(0.071)	(0.380)	(0.195)	(0.208)	(0.263)
(Theor. s.d.)	(0.033)	(0.049)	(0.065)	(0.082)	(0.098)	(0.114)	(0.131)
Regression	0.500	0.829	1.040	2.166	2.227	3.126	4.249
(s.d.)	(0.035)	(0.061)	(0.077)	(0.406)	(0.206)	(0.220)	(0.291)
$u = 1000$							
Hill	5.072	9.013	10.021	47.576	29.676	28.658	32.377
(s.d.)	(0.385)	(0.721)	(0.723)	(3.591)	(2.201)	(2.182)	(2.441)
T. Hill	0.499	0.885	0.989	4.656	2.920	2.847	3.231
(s.d)	(0.033)	(0.062)	(0.064)	(0.315)	(0.195)	(0.186)	(0.210)
(Theor. s.d.)	(0.034)	(0.050)	(0.067)	(0.084)	(0.101)	(0.117)	(0.134)
Regression	0.498	0.882	0.987	4.639	2.915	2.845	3.222
(s.d.)	(0.036)	(0.068)	(0.070)	(0.345)	(0.208)	(0.205)	(0.231)

TABLE XXXIII: Stable Distribution, $k = 500$, $s = .050(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.525	0.821	1.071	1.157	1.585	2.544	6.110
(s.d.)	(0.111)	(0.175)	(0.220)	(0.244)	(0.335)	(0.647)	(1.071)
T. Hill	0.525	0.821	1.071	1.157	1.585	2.544	6.110
(s.d)	(0.111)	(0.175)	(0.220)	(0.244)	(0.335)	(0.647)	(1.071)
(Theor. s.d.)	(0.106)	(0.159)	(0.212)	(0.265)	(0.318)	(0.372)	(0.425)
Regression	0.455	0.703	0.926	1.031	1.346	1.920	6.345
(s.d.)	(0.124)	(0.201)	(0.260)	(0.255)	(0.363)	(0.651)	(1.212)
$u = 1$							
Hill	0.590	0.942	1.224	1.299	1.786	2.927	6.708
(s.d.)	(0.125)	(0.207)	(0.271)	(0.284)	(0.390)	(0.741)	(1.275)
T. Hill	0.521	0.829	1.079	1.151	1.568	2.538	6.020
(s.d)	(0.110)	(0.180)	(0.237)	(0.246)	(0.337)	(0.640)	(1.109)
(Theor. s.d.)	(0.109)	(0.163)	(0.217)	(0.272)	(0.326)	(0.380)	(0.434)
Regression	0.475	0.736	0.970	1.058	1.373	2.066	6.011
(s.d.)	(0.116)	(0.183)	(0.250)	(0.251)	(0.338)	(0.632)	(1.144)
$u = 5$							
Hill	0.802	1.275	1.669	1.861	2.600	4.433	8.166
(s.d.)	(0.170)	(0.279)	(0.368)	(0.515)	(0.704)	(1.152)	(1.689)
T. Hill	0.521	0.821	1.077	1.188	1.654	2.781	5.475
(s.d)	(0.107)	(0.173)	(0.230)	(0.290)	(0.412)	(0.744)	(1.053)
(Theor. s.d.)	(0.109)	(0.163)	(0.217)	(0.272)	(0.326)	(0.380)	(0.434)
Regression	0.489	0.767	1.007	1.058	1.480	2.502	5.363
(s.d.)	(0.112)	(0.181)	(0.234)	(0.237)	(0.378)	(0.781)	(1.051)
$u = 10$							
Hill	1.038	1.718	2.176	2.961	3.803	6.155	9.486
(s.d.)	(0.243)	(0.393)	(0.504)	(1.179)	(1.102)	(1.560)	(2.022)
T. Hill	0.522	0.853	1.089	1.353	1.807	3.002	4.934
(s.d)	(0.114)	(0.186)	(0.238)	(0.440)	(0.482)	(0.765)	(0.937)
(Theor. s.d.)	(0.109)	(0.163)	(0.217)	(0.272)	(0.326)	(0.380)	(0.434)
Regression	0.494	0.803	1.036	1.138	1.611	2.817	4.820
(s.d.)	(0.114)	(0.191)	(0.251)	(0.355)	(0.459)	(0.781)	(0.981)
$u = 50$							
Hill	2.972	5.236	5.653	27.158	16.960	16.117	17.451
(s.d.)	(0.744)	(1.209)	(1.356)	(6.956)	(4.490)	(4.135)	(4.207)
T. Hill	0.524	0.915	1.007	4.804	3.002	2.867	3.155
(s.d)	(0.116)	(0.191)	(0.217)	(1.129)	(0.703)	(0.653)	(0.689)
(Theor. s.d.)	(0.114)	(0.171)	(0.228)	(0.285)	(0.342)	(0.399)	(0.456)
Regression	0.507	0.883	0.976	4.654	2.893	2.783	3.063
(s.d.)	(0.120)	(0.199)	(0.225)	(1.156)	(0.721)	(0.666)	(0.723)

TABLE XXXIV: Stable Distribution, $k = 1000$, $s = .050(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.513	0.804	1.068	1.116	1.512	2.440	6.080
(s.d.)	(0.076)	(0.119)	(0.148)	(0.153)	(0.229)	(0.405)	(0.773)
T. Hill	0.513	0.804	1.068	1.116	1.512	2.440	6.080
(s.d)	(0.076)	(0.119)	(0.148)	(0.153)	(0.229)	(0.405)	(0.773)
(Theor. s.d.)	(0.073)	(0.109)	(0.146)	(0.182)	(0.219)	(0.255)	(0.291)
Regression	0.464	0.715	0.960	1.055	1.347	1.895	6.604
(s.d.)	(0.091)	(0.147)	(0.189)	(0.188)	(0.259)	(0.436)	(0.883)
$u = 1$							
Hill	0.546	0.862	1.150	1.195	1.653	2.687	6.348
(s.d.)	(0.080)	(0.124)	(0.173)	(0.170)	(0.258)	(0.453)	(0.824)
T. Hill	0.508	0.799	1.067	1.114	1.533	2.462	5.978
(s.d)	(0.074)	(0.114)	(0.159)	(0.157)	(0.238)	(0.415)	(0.759)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.473	0.735	0.994	1.084	1.403	2.012	6.374
(s.d.)	(0.086)	(0.133)	(0.184)	(0.178)	(0.252)	(0.424)	(0.824)
$u = 5$							
Hill	0.668	1.057	1.395	1.463	2.067	3.530	7.161
(s.d.)	(0.096)	(0.156)	(0.201)	(0.253)	(0.340)	(0.642)	(0.946)
T. Hill	0.511	0.805	1.066	1.125	1.574	2.601	5.660
(s.d)	(0.072)	(0.117)	(0.151)	(0.182)	(0.246)	(0.466)	(0.706)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.491	0.763	1.023	1.075	1.463	2.282	5.853
(s.d.)	(0.081)	(0.127)	(0.171)	(0.171)	(0.243)	(0.473)	(0.796)
$u = 10$							
Hill	0.782	1.279	1.650	1.815	2.540	4.405	8.041
(s.d.)	(0.120)	(0.202)	(0.263)	(0.368)	(0.432)	(0.818)	(1.129)
T. Hill	0.504	0.819	1.063	1.160	1.608	2.721	5.380
(s.d)	(0.074)	(0.125)	(0.163)	(0.204)	(0.255)	(0.508)	(0.701)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.487	0.786	1.030	1.067	1.481	2.507	5.492
(s.d.)	(0.079)	(0.131)	(0.176)	(0.171)	(0.248)	(0.553)	(0.773)
$u = 50$							
Hill	1.697	2.881	3.487	12.026	8.950	10.443	12.872
(s.d.)	(0.281)	(0.470)	(0.560)	(3.456)	(1.730)	(1.691)	(2.051)
T. Hill	0.507	0.853	1.047	3.021	2.516	3.137	3.986
(s.d)	(0.077)	(0.127)	(0.151)	(1.083)	(0.516)	(0.473)	(0.557)
(Theor. s.d.)	(0.075)	(0.113)	(0.151)	(0.188)	(0.226)	(0.264)	(0.301)
Regression	0.498	0.834	1.032	2.927	2.452	3.076	3.962
(s.d.)	(0.081)	(0.135)	(0.162)	(1.137)	(0.536)	(0.494)	(0.601)
$u = 100$							
Hill	2.867	5.217	5.540	26.452	16.631	15.581	17.035
(s.d.)	(0.477)	(0.885)	(0.934)	(4.725)	(2.756)	(2.549)	(2.739)
T. Hill	0.508	0.916	0.987	4.697	2.952	2.801	3.089
(s.d)	(0.078)	(0.135)	(0.149)	(0.749)	(0.442)	(0.399)	(0.440)
(Theor. s.d.)	(0.077)	(0.116)	(0.154)	(0.193)	(0.231)	(0.270)	(0.308)
Regression	0.501	0.900	0.975	4.645	2.908	2.766	3.064
(s.d.)	(0.084)	(0.143)	(0.162)	(0.805)	(0.481)	(0.446)	(0.484)

TABLE XXXV: Stable Distribution, $k = 5000$, $s = .050(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.502	0.784	1.038	1.087	1.494	2.383	5.989
(s.d.)	(0.032)	(0.049)	(0.064)	(0.063)	(0.092)	(0.187)	(0.311)
T. Hill	0.502	0.784	1.038	1.087	1.494	2.383	5.989
(s.d)	(0.032)	(0.049)	(0.064)	(0.063)	(0.092)	(0.187)	(0.311)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.095)	(0.111)	(0.127)
Regression	0.482	0.738	0.991	1.122	1.432	1.932	7.012
(s.d.)	(0.045)	(0.068)	(0.092)	(0.089)	(0.124)	(0.222)	(0.384)
$u = 1$							
Hill	0.514	0.803	1.063	1.111	1.529	2.454	6.089
(s.d.)	(0.033)	(0.052)	(0.070)	(0.066)	(0.101)	(0.187)	(0.330)
T. Hill	0.503	0.786	1.041	1.090	1.498	2.392	6.000
(s.d)	(0.032)	(0.051)	(0.069)	(0.065)	(0.099)	(0.182)	(0.322)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.490	0.750	1.003	1.135	1.453	2.000	6.927
(s.d.)	(0.041)	(0.062)	(0.086)	(0.084)	(0.120)	(0.202)	(0.392)
$u = 5$							
Hill	0.545	0.854	1.122	1.172	1.626	2.671	6.269
(s.d.)	(0.034)	(0.056)	(0.072)	(0.071)	(0.106)	(0.204)	(0.352)
T. Hill	0.503	0.786	1.034	1.087	1.500	2.427	5.888
(s.d)	(0.032)	(0.051)	(0.066)	(0.065)	(0.096)	(0.184)	(0.324)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.495	0.763	1.013	1.123	1.464	2.088	6.621
(s.d.)	(0.038)	(0.061)	(0.081)	(0.081)	(0.112)	(0.199)	(0.392)
$u = 10$							
Hill	0.575	0.911	1.198	1.232	1.723	2.885	6.533
(s.d.)	(0.038)	(0.060)	(0.079)	(0.074)	(0.114)	(0.229)	(0.377)
T. Hill	0.501	0.791	1.041	1.082	1.500	2.454	5.833
(s.d)	(0.033)	(0.051)	(0.068)	(0.063)	(0.098)	(0.193)	(0.324)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.496	0.770	1.024	1.113	1.458	2.157	6.409
(s.d.)	(0.040)	(0.060)	(0.082)	(0.075)	(0.111)	(0.205)	(0.373)
$u = 50$							
Hill	0.781	1.250	1.633	1.751	2.493	4.372	7.912
(s.d.)	(0.053)	(0.088)	(0.107)	(0.143)	(0.195)	(0.345)	(0.497)
T. Hill	0.500	0.797	1.046	1.120	1.569	2.667	5.274
(s.d)	(0.032)	(0.054)	(0.066)	(0.081)	(0.114)	(0.208)	(0.303)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.128)
Regression	0.498	0.786	1.040	1.068	1.482	2.517	5.504
(s.d.)	(0.036)	(0.060)	(0.075)	(0.071)	(0.112)	(0.233)	(0.346)
$u = 100$							
Hill	1.013	1.635	2.103	2.728	3.637	6.119	9.353
(s.d.)	(0.069)	(0.117)	(0.142)	(0.301)	(0.311)	(0.465)	(0.612)
T. Hill	0.502	0.805	1.043	1.256	1.703	2.921	4.833
(s.d)	(0.032)	(0.055)	(0.065)	(0.110)	(0.128)	(0.228)	(0.280)
(Theor. s.d.)	(0.032)	(0.048)	(0.064)	(0.080)	(0.096)	(0.112)	(0.129)
Regression	0.501	0.797	1.041	1.090	1.572	2.877	4.947
(s.d.)	(0.035)	(0.061)	(0.073)	(0.083)	(0.127)	(0.254)	(0.311)
$u = 500$							
Hill	2.854	5.132	5.500	26.491	16.429	15.420	17.043
(s.d.)	(0.221)	(0.388)	(0.412)	(1.950)	(1.196)	(1.138)	(1.260)
T. Hill	0.498	0.892	0.970	4.637	2.889	2.744	3.065
(s.d)	(0.034)	(0.062)	(0.066)	(0.308)	(0.188)	(0.173)	(0.198)
(Theor. s.d.)	(0.034)	(0.050)	(0.067)	(0.084)	(0.101)	(0.117)	(0.134)
Regression	0.496	0.890	0.969	4.632	2.886	2.745	3.070
(s.d.)	(0.037)	(0.066)	(0.074)	(0.341)	(0.202)	(0.195)	(0.216)

TABLE XXXVI: Stable Distribution, $k = 10000$, $s = .050(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.501	0.786	1.041	1.081	1.489	2.384	5.999
(s.d.)	(0.023)	(0.036)	(0.048)	(0.046)	(0.064)	(0.125)	(0.230)
T. Hill	0.501	0.786	1.041	1.081	1.489	2.384	5.999
(s.d)	(0.023)	(0.036)	(0.048)	(0.046)	(0.064)	(0.125)	(0.230)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.078)	(0.090)
Regression	0.489	0.751	1.008	1.131	1.450	1.972	7.135
(s.d.)	(0.032)	(0.048)	(0.067)	(0.065)	(0.092)	(0.152)	(0.274)
$u = 1$							
Hill	0.507	0.793	1.048	1.093	1.508	2.417	6.039
(s.d.)	(0.023)	(0.038)	(0.048)	(0.046)	(0.069)	(0.124)	(0.234)
T. Hill	0.501	0.783	1.036	1.082	1.491	2.382	5.992
(s.d)	(0.023)	(0.037)	(0.047)	(0.045)	(0.068)	(0.122)	(0.232)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.079)	(0.090)
Regression	0.492	0.756	1.007	1.142	1.458	2.000	7.044
(s.d.)	(0.030)	(0.050)	(0.063)	(0.060)	(0.087)	(0.144)	(0.280)
$u = 5$							
Hill	0.525	0.821	1.085	1.130	1.568	2.529	6.165
(s.d.)	(0.024)	(0.038)	(0.049)	(0.048)	(0.072)	(0.132)	(0.243)
T. Hill	0.501	0.782	1.035	1.083	1.497	2.391	5.956
(s.d)	(0.023)	(0.036)	(0.047)	(0.046)	(0.069)	(0.124)	(0.232)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.079)	(0.090)
Regression	0.497	0.761	1.016	1.138	1.472	2.048	6.856
(s.d.)	(0.029)	(0.045)	(0.059)	(0.060)	(0.085)	(0.133)	(0.286)
$u = 10$							
Hill	0.543	0.855	1.125	1.165	1.621	2.648	6.295
(s.d.)	(0.025)	(0.040)	(0.052)	(0.050)	(0.077)	(0.146)	(0.246)
T. Hill	0.501	0.787	1.036	1.081	1.494	2.404	5.910
(s.d)	(0.023)	(0.036)	(0.047)	(0.046)	(0.070)	(0.132)	(0.226)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.079)	(0.090)
Regression	0.497	0.768	1.021	1.131	1.467	2.090	6.693
(s.d.)	(0.028)	(0.044)	(0.056)	(0.057)	(0.082)	(0.142)	(0.257)
$u = 50$							
Hill	0.659	1.047	1.372	1.407	2.010	3.453	7.108
(s.d.)	(0.031)	(0.050)	(0.066)	(0.065)	(0.097)	(0.190)	(0.283)
T. Hill	0.501	0.792	1.041	1.081	1.520	2.523	5.598
(s.d)	(0.023)	(0.037)	(0.049)	(0.047)	(0.070)	(0.137)	(0.209)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.079)	(0.090)
Regression	0.499	0.781	1.034	1.092	1.471	2.301	5.998
(s.d.)	(0.026)	(0.043)	(0.055)	(0.051)	(0.075)	(0.147)	(0.244)
$u = 100$							
Hill	0.782	1.254	1.631	1.735	2.479	4.360	7.946
(s.d.)	(0.037)	(0.062)	(0.078)	(0.100)	(0.136)	(0.235)	(0.330)
T. Hill	0.500	0.798	1.043	1.111	1.561	2.654	5.298
(s.d)	(0.023)	(0.038)	(0.048)	(0.056)	(0.079)	(0.146)	(0.202)
(Theor. s.d.)	(0.023)	(0.034)	(0.045)	(0.056)	(0.068)	(0.079)	(0.090)
Regression	0.498	0.788	1.039	1.065	1.479	2.513	5.545
(s.d.)	(0.025)	(0.042)	(0.055)	(0.050)	(0.077)	(0.169)	(0.234)
$u = 500$							
Hill	1.678	2.860	3.437	11.972	8.913	10.318	12.722
(s.d.)	(0.086)	(0.146)	(0.173)	(1.134)	(0.546)	(0.527)	(0.628)
T. Hill	0.499	0.844	1.028	2.828	2.481	3.091	3.924
(s.d)	(0.023)	(0.040)	(0.047)	(0.373)	(0.162)	(0.142)	(0.171)
(Theor. s.d.)	(0.023)	(0.035)	(0.046)	(0.058)	(0.069)	(0.081)	(0.092)
Regression	0.499	0.840	1.028	2.854	2.471	3.108	3.960
(s.d.)	(0.024)	(0.043)	(0.051)	(0.408)	(0.172)	(0.155)	(0.192)
$u = 1000$							
Hill	2.844	5.157	5.513	26.473	16.447	15.459	16.993
(s.d.)	(0.149)	(0.277)	(0.292)	(1.393)	(0.894)	(0.834)	(0.872)
T. Hill	0.497	0.894	0.972	4.634	2.886	2.749	3.055
(s.d)	(0.023)	(0.043)	(0.045)	(0.218)	(0.137)	(0.128)	(0.137)
(Theor. s.d.)	(0.024)	(0.035)	(0.047)	(0.059)	(0.071)	(0.083)	(0.095)
Regression	0.496	0.892	0.972	4.634	2.892	2.753	3.066
(s.d.)	(0.025)	(0.047)	(0.049)	(0.232)	(0.146)	(0.138)	(0.148)

TABLE XXXVII: Stable Distribution, $k = 500$, $s = .10(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.511	0.832	1.053	1.647	1.908	2.757	4.743
(s.d.)	(0.075)	(0.129)	(0.160)	(0.353)	(0.350)	(0.461)	(0.588)
T. Hill	0.511	0.832	1.053	1.647	1.908	2.757	4.743
(s.d)	(0.075)	(0.129)	(0.160)	(0.353)	(0.350)	(0.461)	(0.588)
(Theor. s.d.)	(0.073)	(0.109)	(0.146)	(0.182)	(0.219)	(0.255)	(0.291)
Regression	0.465	0.726	0.951	1.135	1.444	2.120	5.379
(s.d.)	(0.090)	(0.158)	(0.201)	(0.244)	(0.329)	(0.582)	(0.694)
$u = 1$							
Hill	0.548	0.904	1.134	1.793	2.104	3.038	4.983
(s.d.)	(0.078)	(0.138)	(0.171)	(0.410)	(0.391)	(0.524)	(0.626)
T. Hill	0.509	0.837	1.052	1.631	1.927	2.781	4.703
(s.d)	(0.072)	(0.127)	(0.158)	(0.361)	(0.353)	(0.485)	(0.578)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.476	0.761	0.988	1.161	1.522	2.326	5.157
(s.d.)	(0.083)	(0.148)	(0.184)	(0.235)	(0.304)	(0.570)	(0.638)
$u = 5$							
Hill	0.665	1.118	1.364	2.567	2.799	3.873	5.540
(s.d.)	(0.103)	(0.169)	(0.210)	(0.691)	(0.555)	(0.632)	(0.758)
T. Hill	0.510	0.850	1.048	1.787	2.038	2.898	4.407
(s.d)	(0.076)	(0.125)	(0.157)	(0.426)	(0.383)	(0.489)	(0.557)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.490	0.801	1.015	1.278	1.686	2.724	4.631
(s.d.)	(0.083)	(0.136)	(0.174)	(0.289)	(0.351)	(0.590)	(0.594)
$u = 10$							
Hill	0.789	1.339	1.607	3.765	3.658	4.700	6.083
(s.d.)	(0.116)	(0.210)	(0.244)	(1.111)	(0.754)	(0.783)	(0.839)
T. Hill	0.509	0.856	1.040	2.040	2.182	2.994	4.110
(s.d)	(0.072)	(0.130)	(0.151)	(0.537)	(0.439)	(0.508)	(0.526)
(Theor. s.d.)	(0.074)	(0.110)	(0.147)	(0.184)	(0.221)	(0.258)	(0.295)
Regression	0.492	0.814	1.018	1.498	1.900	2.936	4.240
(s.d.)	(0.080)	(0.139)	(0.168)	(0.450)	(0.463)	(0.569)	(0.574)
$u = 50$							
Hill	1.746	3.261	3.197	15.309	9.415	8.592	9.270
(s.d.)	(0.295)	(0.580)	(0.511)	(2.607)	(1.544)	(1.392)	(1.458)
T. Hill	0.508	0.936	0.948	4.530	2.793	2.580	2.823
(s.d)	(0.077)	(0.149)	(0.135)	(0.713)	(0.424)	(0.368)	(0.386)
(Theor. s.d.)	(0.077)	(0.116)	(0.154)	(0.193)	(0.231)	(0.270)	(0.308)
Regression	0.498	0.914	0.940	4.502	2.768	2.565	2.820
(s.d.)	(0.085)	(0.160)	(0.148)	(0.774)	(0.456)	(0.390)	(0.430)

TABLE XXXVIII: Stable Distribution, $k = 1000$, $s = .10(k-u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.505	0.816	1.039	1.609	1.885	2.701	4.740
(s.d.)	(0.051)	(0.087)	(0.107)	(0.244)	(0.235)	(0.307)	(0.400)
T. Hill	0.505	0.816	1.039	1.609	1.885	2.701	4.740
(s.d)	(0.051)	(0.087)	(0.107)	(0.244)	(0.235)	(0.307)	(0.400)
(Theor. s.d.)	(0.051)	(0.076)	(0.102)	(0.127)	(0.152)	(0.178)	(0.203)
Regression	0.469	0.732	0.971	1.136	1.475	2.105	5.556
(s.d.)	(0.067)	(0.106)	(0.140)	(0.173)	(0.227)	(0.398)	(0.488)
$u = 1$							
Hill	0.526	0.861	1.082	1.682	1.991	2.877	4.883
(s.d.)	(0.053)	(0.091)	(0.111)	(0.258)	(0.254)	(0.337)	(0.417)
T. Hill	0.504	0.823	1.036	1.592	1.892	2.727	4.731
(s.d)	(0.051)	(0.087)	(0.105)	(0.239)	(0.239)	(0.323)	(0.398)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.480	0.763	0.996	1.160	1.519	2.276	5.419
(s.d.)	(0.063)	(0.105)	(0.128)	(0.156)	(0.217)	(0.395)	(0.469)
$u = 5$							
Hill	0.594	0.966	1.228	2.052	2.355	3.382	5.205
(s.d.)	(0.061)	(0.102)	(0.121)	(0.347)	(0.306)	(0.396)	(0.481)
T. Hill	0.506	0.818	1.045	1.663	1.947	2.803	4.566
(s.d)	(0.051)	(0.086)	(0.102)	(0.262)	(0.245)	(0.333)	(0.402)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.493	0.780	1.025	1.213	1.616	2.537	5.027
(s.d.)	(0.059)	(0.102)	(0.122)	(0.175)	(0.227)	(0.402)	(0.455)
$u = 10$							
Hill	0.660	1.104	1.365	2.543	2.780	3.871	5.491
(s.d.)	(0.070)	(0.120)	(0.144)	(0.456)	(0.369)	(0.443)	(0.531)
T. Hill	0.504	0.835	1.043	1.765	2.013	2.870	4.360
(s.d)	(0.052)	(0.089)	(0.106)	(0.280)	(0.254)	(0.337)	(0.397)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.493	0.799	1.031	1.275	1.687	2.736	4.694
(s.d.)	(0.059)	(0.100)	(0.119)	(0.195)	(0.238)	(0.411)	(0.461)
$u = 50$							
Hill	1.133	2.008	2.241	9.531	6.377	6.458	7.437
(s.d.)	(0.131)	(0.233)	(0.232)	(1.565)	(0.816)	(0.721)	(0.791)
T. Hill	0.501	0.876	1.004	3.627	2.691	2.924	3.463
(s.d)	(0.054)	(0.097)	(0.096)	(0.861)	(0.366)	(0.299)	(0.327)
(Theor. s.d.)	(0.052)	(0.078)	(0.104)	(0.130)	(0.156)	(0.182)	(0.208)
Regression	0.495	0.859	1.004	3.601	2.673	2.943	3.531
(s.d.)	(0.058)	(0.104)	(0.106)	(0.949)	(0.396)	(0.328)	(0.368)
$u = 100$							
Hill	1.719	3.228	3.189	15.323	9.440	8.584	9.186
(s.d.)	(0.206)	(0.377)	(0.357)	(1.698)	(1.055)	(0.948)	(1.028)
T. Hill	0.498	0.923	0.939	4.499	2.779	2.568	2.781
(s.d)	(0.053)	(0.102)	(0.096)	(0.465)	(0.286)	(0.250)	(0.265)
(Theor. s.d.)	(0.054)	(0.080)	(0.107)	(0.134)	(0.161)	(0.188)	(0.214)
Regression	0.494	0.909	0.940	4.513	2.781	2.585	2.795
(s.d.)	(0.056)	(0.110)	(0.106)	(0.520)	(0.307)	(0.278)	(0.296)

TABLE XXXIX: Stable Distribution, $k = 5000$, $s = .10(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.499	0.815	1.032	1.570	1.867	2.688	4.718
(s.d.)	(0.022)	(0.037)	(0.048)	(0.105)	(0.104)	(0.147)	(0.177)
T. Hill	0.499	0.815	1.032	1.570	1.867	2.688	4.718
(s.d)	(0.022)	(0.037)	(0.048)	(0.105)	(0.104)	(0.147)	(0.177)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.078)	(0.090)
Regression	0.489	0.766	1.006	1.179	1.526	2.178	5.802
(s.d.)	(0.031)	(0.049)	(0.067)	(0.078)	(0.106)	(0.192)	(0.222)
$u = 1$							
Hill	0.506	0.823	1.044	1.600	1.887	2.727	4.749
(s.d.)	(0.022)	(0.039)	(0.047)	(0.104)	(0.110)	(0.145)	(0.182)
T. Hill	0.500	0.813	1.032	1.577	1.861	2.685	4.714
(s.d)	(0.022)	(0.038)	(0.046)	(0.102)	(0.108)	(0.143)	(0.180)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.079)	(0.090)
Regression	0.492	0.769	1.019	1.192	1.537	2.226	5.733
(s.d.)	(0.029)	(0.049)	(0.063)	(0.072)	(0.106)	(0.183)	(0.216)
$u = 5$							
Hill	0.525	0.856	1.078	1.687	1.985	2.860	4.829
(s.d.)	(0.023)	(0.040)	(0.048)	(0.110)	(0.110)	(0.158)	(0.193)
T. Hill	0.501	0.815	1.029	1.592	1.878	2.696	4.674
(s.d)	(0.022)	(0.038)	(0.046)	(0.102)	(0.103)	(0.150)	(0.185)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.079)	(0.090)
Regression	0.497	0.781	1.020	1.204	1.566	2.310	5.534
(s.d.)	(0.028)	(0.047)	(0.059)	(0.073)	(0.096)	(0.182)	(0.222)
$u = 10$							
Hill	0.543	0.890	1.118	1.774	2.082	3.003	4.929
(s.d.)	(0.025)	(0.042)	(0.052)	(0.121)	(0.120)	(0.166)	(0.201)
T. Hill	0.500	0.817	1.031	1.602	1.891	2.717	4.641
(s.d)	(0.023)	(0.039)	(0.047)	(0.105)	(0.107)	(0.151)	(0.184)
(Theor. s.d.)	(0.022)	(0.034)	(0.045)	(0.056)	(0.067)	(0.079)	(0.090)
Regression	0.497	0.787	1.027	1.207	1.586	2.390	5.391
(s.d.)	(0.027)	(0.045)	(0.057)	(0.072)	(0.098)	(0.183)	(0.215)
$u = 50$							
Hill	0.657	1.095	1.352	2.501	2.755	3.853	5.493
(s.d.)	(0.031)	(0.053)	(0.062)	(0.198)	(0.164)	(0.198)	(0.233)
T. Hill	0.499	0.825	1.028	1.733	1.984	2.833	4.352
(s.d)	(0.023)	(0.039)	(0.047)	(0.120)	(0.112)	(0.150)	(0.173)
(Theor. s.d.)	(0.023)	(0.034)	(0.045)	(0.056)	(0.068)	(0.079)	(0.090)
Regression	0.498	0.805	1.031	1.281	1.696	2.743	4.749
(s.d.)	(0.026)	(0.045)	(0.056)	(0.084)	(0.106)	(0.182)	(0.202)
$u = 100$							
Hill	0.784	1.322	1.598	3.639	3.617	4.665	6.057
(s.d.)	(0.036)	(0.064)	(0.075)	(0.342)	(0.234)	(0.225)	(0.262)
T. Hill	0.500	0.835	1.025	1.939	2.118	2.935	4.065
(s.d)	(0.022)	(0.039)	(0.047)	(0.157)	(0.130)	(0.149)	(0.161)
(Theor. s.d.)	(0.023)	(0.034)	(0.045)	(0.057)	(0.068)	(0.079)	(0.091)
Regression	0.499	0.821	1.032	1.453	1.882	2.972	4.321
(s.d.)	(0.025)	(0.045)	(0.052)	(0.125)	(0.138)	(0.174)	(0.183)
$u = 500$							
Hill	1.724	3.211	3.176	15.297	9.447	8.573	9.207
(s.d.)	(0.089)	(0.175)	(0.164)	(0.749)	(0.482)	(0.438)	(0.465)
T. Hill	0.496	0.912	0.932	4.469	2.769	2.552	2.774
(s.d)	(0.023)	(0.046)	(0.043)	(0.203)	(0.130)	(0.114)	(0.123)
(Theor. s.d.)	(0.024)	(0.035)	(0.047)	(0.059)	(0.071)	(0.083)	(0.095)
Regression	0.496	0.907	0.938	4.516	2.794	2.579	2.810
(s.d.)	(0.025)	(0.050)	(0.046)	(0.230)	(0.140)	(0.126)	(0.138)

TABLE XL: Stable Distribution, $k = 10000$, $s = .10(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.499	0.813	1.030	1.570	1.865	2.687	4.725
(s.d.)	(0.016)	(0.026)	(0.032)	(0.069)	(0.072)	(0.100)	(0.130)
T. Hill	0.499	0.813	1.030	1.570	1.865	2.687	4.725
(s.d)	(0.016)	(0.026)	(0.032)	(0.069)	(0.072)	(0.100)	(0.130)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.055)	(0.063)
Regression	0.493	0.772	1.012	1.189	1.538	2.199	5.856
(s.d.)	(0.023)	(0.036)	(0.048)	(0.054)	(0.075)	(0.140)	(0.152)
$u = 1$							
Hill	0.503	0.817	1.036	1.584	1.883	2.706	4.733
(s.d.)	(0.016)	(0.028)	(0.034)	(0.074)	(0.077)	(0.105)	(0.129)
T. Hill	0.499	0.812	1.029	1.571	1.868	2.683	4.714
(s.d)	(0.016)	(0.027)	(0.034)	(0.073)	(0.076)	(0.104)	(0.128)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.055)	(0.063)
Regression	0.495	0.774	1.017	1.195	1.552	2.220	5.796
(s.d.)	(0.022)	(0.036)	(0.046)	(0.052)	(0.073)	(0.135)	(0.154)
$u = 5$							
Hill	0.512	0.835	1.060	1.630	1.933	2.784	4.779
(s.d.)	(0.017)	(0.028)	(0.034)	(0.076)	(0.076)	(0.105)	(0.134)
T. Hill	0.499	0.812	1.032	1.577	1.873	2.690	4.695
(s.d)	(0.016)	(0.027)	(0.033)	(0.073)	(0.073)	(0.102)	(0.131)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.055)	(0.063)
Regression	0.496	0.779	1.029	1.202	1.564	2.283	5.674
(s.d.)	(0.021)	(0.034)	(0.044)	(0.052)	(0.072)	(0.129)	(0.158)
$u = 10$							
Hill	0.524	0.854	1.080	1.679	1.986	2.856	4.830
(s.d.)	(0.017)	(0.029)	(0.034)	(0.077)	(0.080)	(0.110)	(0.133)
T. Hill	0.500	0.813	1.030	1.584	1.878	2.692	4.674
(s.d)	(0.016)	(0.027)	(0.032)	(0.071)	(0.075)	(0.104)	(0.127)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.055)	(0.063)
Regression	0.498	0.784	1.029	1.201	1.573	2.324	5.567
(s.d.)	(0.020)	(0.034)	(0.042)	(0.051)	(0.070)	(0.131)	(0.152)
$u = 50$							
Hill	0.589	0.970	1.213	2.040	2.332	3.349	5.162
(s.d.)	(0.020)	(0.033)	(0.039)	(0.100)	(0.099)	(0.124)	(0.151)
T. Hill	0.500	0.818	1.029	1.648	1.917	2.758	4.522
(s.d)	(0.016)	(0.027)	(0.032)	(0.075)	(0.079)	(0.103)	(0.126)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.056)	(0.063)
Regression	0.498	0.797	1.032	1.232	1.622	2.552	5.093
(s.d.)	(0.019)	(0.031)	(0.039)	(0.053)	(0.072)	(0.125)	(0.145)
$u = 100$							
Hill	0.658	1.090	1.354	2.488	2.755	3.843	5.488
(s.d.)	(0.020)	(0.037)	(0.045)	(0.138)	(0.120)	(0.140)	(0.162)
T. Hill	0.500	0.821	1.030	1.723	1.984	2.825	4.347
(s.d)	(0.015)	(0.027)	(0.034)	(0.085)	(0.082)	(0.105)	(0.120)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.056)	(0.064)
Regression	0.499	0.804	1.037	1.278	1.699	2.744	4.752
(s.d.)	(0.017)	(0.030)	(0.038)	(0.059)	(0.078)	(0.132)	(0.134)
$u = 500$							
Hill	1.130	1.996	2.239	9.494	6.361	6.465	7.427
(s.d.)	(0.039)	(0.074)	(0.078)	(0.486)	(0.245)	(0.209)	(0.240)
T. Hill	0.498	0.867	0.997	3.486	2.666	2.910	3.443
(s.d)	(0.016)	(0.030)	(0.032)	(0.301)	(0.115)	(0.088)	(0.097)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.041)	(0.049)	(0.057)	(0.065)
Regression	0.497	0.860	1.006	3.589	2.689	2.967	3.543
(s.d.)	(0.018)	(0.033)	(0.035)	(0.329)	(0.125)	(0.100)	(0.111)
$u = 1000$							
Hill	1.719	3.202	3.179	15.302	9.416	8.571	9.186
(s.d.)	(0.060)	(0.123)	(0.112)	(0.533)	(0.333)	(0.306)	(0.332)
T. Hill	0.494	0.909	0.931	4.469	2.758	2.549	2.767
(s.d)	(0.016)	(0.033)	(0.029)	(0.143)	(0.086)	(0.079)	(0.087)
(Theor. s.d.)	(0.017)	(0.025)	(0.033)	(0.042)	(0.050)	(0.058)	(0.067)
Regression	0.494	0.906	0.937	4.520	2.787	2.577	2.804
(s.d.)	(0.018)	(0.036)	(0.032)	(0.157)	(0.097)	(0.088)	(0.097)

TABLE XLI: Stable Distribution, $k = 500$, $s = .15(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.504	0.849	1.020	2.104	2.159	2.719	3.998
(s.d.)	(0.057)	(0.103)	(0.121)	(0.410)	(0.327)	(0.370)	(0.415)
T. Hill	0.504	0.849	1.020	2.104	2.159	2.719	3.998
(s.d)	(0.057)	(0.103)	(0.121)	(0.410)	(0.327)	(0.370)	(0.415)
(Theor. s.d.)	(0.059)	(0.088)	(0.118)	(0.147)	(0.177)	(0.206)	(0.236)
Regression	0.467	0.742	0.957	1.256	1.580	2.230	4.750
(s.d.)	(0.073)	(0.133)	(0.159)	(0.263)	(0.328)	(0.521)	(0.486)
$u = 1$							
Hill	0.531	0.904	1.074	2.273	2.340	2.933	4.102
(s.d.)	(0.062)	(0.113)	(0.128)	(0.452)	(0.345)	(0.381)	(0.436)
T. Hill	0.503	0.853	1.019	2.093	2.185	2.753	3.946
(s.d)	(0.059)	(0.107)	(0.120)	(0.406)	(0.321)	(0.364)	(0.412)
(Theor. s.d.)	(0.059)	(0.089)	(0.119)	(0.148)	(0.178)	(0.208)	(0.237)
Regression	0.477	0.773	0.989	1.307	1.674	2.443	4.567
(s.d.)	(0.069)	(0.124)	(0.147)	(0.257)	(0.305)	(0.499)	(0.459)
$u = 5$							
Hill	0.614	1.056	1.224	3.068	2.873	3.459	4.409
(s.d.)	(0.075)	(0.134)	(0.145)	(0.669)	(0.463)	(0.431)	(0.474)
T. Hill	0.504	0.857	1.008	2.262	2.242	2.803	3.753
(s.d)	(0.060)	(0.107)	(0.118)	(0.468)	(0.358)	(0.365)	(0.383)
(Theor. s.d.)	(0.059)	(0.089)	(0.119)	(0.148)	(0.178)	(0.208)	(0.237)
Regression	0.489	0.803	1.002	1.506	1.867	2.760	4.115
(s.d.)	(0.065)	(0.118)	(0.137)	(0.353)	(0.379)	(0.466)	(0.433)
$u = 10$							
Hill	0.700	1.227	1.399	4.102	3.501	3.927	4.720
(s.d.)	(0.081)	(0.155)	(0.164)	(0.932)	(0.540)	(0.463)	(0.531)
T. Hill	0.501	0.868	1.010	2.465	2.345	2.816	3.551
(s.d)	(0.057)	(0.107)	(0.115)	(0.534)	(0.368)	(0.338)	(0.367)
(Theor. s.d.)	(0.060)	(0.090)	(0.119)	(0.149)	(0.179)	(0.209)	(0.239)
Regression	0.491	0.829	1.009	1.792	2.086	2.863	3.792
(s.d.)	(0.065)	(0.117)	(0.132)	(0.489)	(0.423)	(0.410)	(0.412)
$u = 50$							
Hill	1.362	2.584	2.360	11.111	6.921	6.213	6.525
(s.d.)	(0.181)	(0.354)	(0.312)	(1.490)	(0.950)	(0.803)	(0.832)
T. Hill	0.504	0.940	0.897	4.215	2.627	2.399	2.554
(s.d)	(0.062)	(0.120)	(0.106)	(0.513)	(0.325)	(0.272)	(0.290)
(Theor. s.d.)	(0.062)	(0.094)	(0.125)	(0.156)	(0.187)	(0.219)	(0.250)
Regression	0.497	0.920	0.896	4.247	2.640	2.421	2.582
(s.d.)	(0.065)	(0.129)	(0.113)	(0.534)	(0.341)	(0.296)	(0.319)

TABLE XLII: Stable Distribution, $k = 1000$, $s = .15(k-u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.502	0.843	1.011	2.055	2.131	2.697	3.976
(s.d.)	(0.041)	(0.076)	(0.080)	(0.277)	(0.218)	(0.254)	(0.278)
T. Hill	0.502	0.843	1.011	2.055	2.131	2.697	3.976
(s.d)	(0.041)	(0.076)	(0.080)	(0.277)	(0.218)	(0.254)	(0.278)
(Theor. s.d.)	(0.041)	(0.062)	(0.082)	(0.103)	(0.124)	(0.144)	(0.165)
Regression	0.478	0.761	0.974	1.266	1.593	2.245	4.877
(s.d.)	(0.054)	(0.097)	(0.115)	(0.180)	(0.224)	(0.370)	(0.324)
$u = 1$							
Hill	0.517	0.868	1.039	2.154	2.228	2.831	4.067
(s.d.)	(0.043)	(0.077)	(0.087)	(0.278)	(0.244)	(0.271)	(0.300)
T. Hill	0.501	0.840	1.008	2.055	2.140	2.724	3.981
(s.d)	(0.041)	(0.074)	(0.085)	(0.262)	(0.234)	(0.263)	(0.291)
(Theor. s.d.)	(0.041)	(0.062)	(0.083)	(0.103)	(0.124)	(0.145)	(0.166)
Regression	0.485	0.778	0.995	1.299	1.650	2.396	4.751
(s.d.)	(0.051)	(0.089)	(0.110)	(0.170)	(0.222)	(0.371)	(0.338)
$u = 5$							
Hill	0.564	0.965	1.133	2.549	2.524	3.149	4.214
(s.d.)	(0.046)	(0.082)	(0.095)	(0.375)	(0.265)	(0.287)	(0.308)
T. Hill	0.501	0.851	1.008	2.128	2.174	2.754	3.851
(s.d)	(0.041)	(0.072)	(0.084)	(0.298)	(0.224)	(0.257)	(0.272)
(Theor. s.d.)	(0.041)	(0.062)	(0.083)	(0.103)	(0.124)	(0.145)	(0.166)
Regression	0.493	0.805	1.008	1.395	1.763	2.623	4.416
(s.d.)	(0.049)	(0.085)	(0.101)	(0.200)	(0.223)	(0.348)	(0.313)
$u = 10$							
Hill	0.614	1.049	1.226	3.049	2.842	3.451	4.403
(s.d.)	(0.051)	(0.093)	(0.098)	(0.472)	(0.315)	(0.305)	(0.352)
T. Hill	0.502	0.848	1.005	2.232	2.204	2.779	3.741
(s.d)	(0.041)	(0.073)	(0.080)	(0.319)	(0.240)	(0.259)	(0.286)
(Theor. s.d.)	(0.042)	(0.062)	(0.083)	(0.104)	(0.125)	(0.145)	(0.166)
Regression	0.493	0.809	1.012	1.504	1.847	2.767	4.176
(s.d.)	(0.047)	(0.082)	(0.098)	(0.230)	(0.251)	(0.340)	(0.321)
$u = 50$							
Hill	0.947	1.720	1.805	7.985	5.186	4.969	5.492
(s.d.)	(0.083)	(0.155)	(0.159)	(0.943)	(0.478)	(0.438)	(0.454)
T. Hill	0.501	0.895	0.971	3.769	2.671	2.715	3.084
(s.d)	(0.041)	(0.078)	(0.080)	(0.672)	(0.274)	(0.223)	(0.231)
(Theor. s.d.)	(0.042)	(0.064)	(0.085)	(0.106)	(0.127)	(0.148)	(0.170)
Regression	0.498	0.878	0.985	3.852	2.714	2.798	3.209
(s.d.)	(0.046)	(0.087)	(0.089)	(0.767)	(0.309)	(0.252)	(0.254)
$u = 100$							
Hill	1.340	2.566	2.342	11.127	6.840	6.182	6.467
(s.d.)	(0.122)	(0.245)	(0.206)	(1.020)	(0.610)	(0.539)	(0.591)
T. Hill	0.496	0.932	0.891	4.234	2.610	2.390	2.530
(s.d)	(0.041)	(0.084)	(0.071)	(0.357)	(0.212)	(0.186)	(0.201)
(Theor. s.d.)	(0.044)	(0.065)	(0.087)	(0.109)	(0.131)	(0.152)	(0.174)
Regression	0.492	0.920	0.900	4.311	2.660	2.434	2.583
(s.d.)	(0.045)	(0.093)	(0.081)	(0.396)	(0.230)	(0.210)	(0.216)

TABLE XLIII: Stable Distribution, $k = 5000$, $s = .15(k-u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.498	0.837	1.011	2.014	2.113	2.694	3.974
(s.d.)	(0.018)	(0.033)	(0.037)	(0.120)	(0.099)	(0.110)	(0.128)
T. Hill	0.498	0.837	1.011	2.014	2.113	2.694	3.974
(s.d)	(0.018)	(0.033)	(0.037)	(0.120)	(0.099)	(0.110)	(0.128)
(Theor. s.d.)	(0.018)	(0.027)	(0.037)	(0.046)	(0.055)	(0.064)	(0.073)
Regression	0.491	0.781	1.011	1.293	1.633	2.306	5.041
(s.d.)	(0.025)	(0.044)	(0.053)	(0.082)	(0.108)	(0.171)	(0.155)
$u = 1$							
Hill	0.503	0.845	1.016	2.045	2.136	2.720	3.989
(s.d.)	(0.018)	(0.032)	(0.039)	(0.117)	(0.097)	(0.114)	(0.129)
T. Hill	0.499	0.838	1.008	2.019	2.113	2.691	3.970
(s.d)	(0.018)	(0.032)	(0.038)	(0.115)	(0.096)	(0.113)	(0.128)
(Theor. s.d.)	(0.018)	(0.027)	(0.037)	(0.046)	(0.055)	(0.064)	(0.073)
Regression	0.493	0.786	1.015	1.304	1.654	2.344	4.984
(s.d.)	(0.024)	(0.041)	(0.050)	(0.077)	(0.099)	(0.170)	(0.156)
$u = 5$							
Hill	0.516	0.869	1.041	2.143	2.220	2.822	4.030
(s.d.)	(0.018)	(0.034)	(0.036)	(0.123)	(0.104)	(0.119)	(0.133)
T. Hill	0.499	0.838	1.008	2.037	2.126	2.708	3.943
(s.d)	(0.018)	(0.032)	(0.034)	(0.116)	(0.099)	(0.115)	(0.129)
(Theor. s.d.)	(0.018)	(0.027)	(0.037)	(0.046)	(0.055)	(0.064)	(0.073)
Regression	0.496	0.795	1.022	1.332	1.691	2.452	4.844
(s.d.)	(0.022)	(0.040)	(0.047)	(0.078)	(0.102)	(0.162)	(0.152)
$u = 10$							
Hill	0.529	0.895	1.067	2.234	2.295	2.912	4.085
(s.d.)	(0.019)	(0.036)	(0.040)	(0.133)	(0.109)	(0.120)	(0.132)
T. Hill	0.499	0.840	1.007	2.043	2.128	2.712	3.922
(s.d)	(0.018)	(0.034)	(0.037)	(0.119)	(0.099)	(0.113)	(0.124)
(Theor. s.d.)	(0.018)	(0.027)	(0.037)	(0.046)	(0.055)	(0.064)	(0.073)
Regression	0.497	0.801	1.021	1.342	1.708	2.508	4.724
(s.d.)	(0.023)	(0.041)	(0.049)	(0.077)	(0.097)	(0.158)	(0.150)
$u = 50$							
Hill	0.611	1.055	1.223	2.988	2.853	3.455	4.391
(s.d.)	(0.023)	(0.041)	(0.045)	(0.197)	(0.139)	(0.134)	(0.148)
T. Hill	0.498	0.850	1.001	2.178	2.207	2.768	3.728
(s.d)	(0.018)	(0.033)	(0.036)	(0.130)	(0.104)	(0.111)	(0.120)
(Theor. s.d.)	(0.018)	(0.028)	(0.037)	(0.046)	(0.055)	(0.064)	(0.074)
Regression	0.497	0.823	1.021	1.484	1.874	2.797	4.217
(s.d.)	(0.022)	(0.037)	(0.044)	(0.093)	(0.112)	(0.150)	(0.142)
$u = 100$							
Hill	0.700	1.222	1.383	4.053	3.485	3.951	4.709
(s.d.)	(0.027)	(0.048)	(0.053)	(0.294)	(0.170)	(0.148)	(0.164)
T. Hill	0.499	0.859	0.993	2.395	2.302	2.812	3.532
(s.d)	(0.019)	(0.033)	(0.036)	(0.161)	(0.111)	(0.109)	(0.113)
(Theor. s.d.)	(0.018)	(0.028)	(0.037)	(0.046)	(0.055)	(0.065)	(0.074)
Regression	0.499	0.838	1.015	1.750	2.085	2.933	3.865
(s.d.)	(0.022)	(0.037)	(0.042)	(0.145)	(0.133)	(0.132)	(0.129)
$u = 500$							
Hill	1.337	2.561	2.344	11.114	6.870	6.139	6.482
(s.d.)	(0.058)	(0.109)	(0.092)	(0.450)	(0.280)	(0.253)	(0.257)
T. Hill	0.492	0.927	0.888	4.217	2.612	2.369	2.529
(s.d)	(0.019)	(0.037)	(0.031)	(0.161)	(0.095)	(0.087)	(0.089)
(Theor. s.d.)	(0.019)	(0.029)	(0.039)	(0.048)	(0.058)	(0.068)	(0.077)
Regression	0.493	0.922	0.902	4.314	2.669	2.423	2.590
(s.d.)	(0.021)	(0.041)	(0.035)	(0.176)	(0.106)	(0.096)	(0.098)

TABLE XLIV: Stable Distribution, $k = 10000$, $s = .15(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.498	0.838	1.008	2.012	2.111	2.690	3.975
(s.d.)	(0.013)	(0.023)	(0.026)	(0.084)	(0.068)	(0.079)	(0.090)
T. Hill	0.498	0.838	1.008	2.012	2.111	2.690	3.975
(s.d)	(0.013)	(0.023)	(0.026)	(0.084)	(0.068)	(0.079)	(0.090)
(Theor. s.d.)	(0.013)	(0.019)	(0.026)	(0.032)	(0.039)	(0.045)	(0.052)
Regression	0.495	0.787	1.016	1.300	1.645	2.312	5.075
(s.d.)	(0.018)	(0.032)	(0.038)	(0.058)	(0.071)	(0.129)	(0.107)
$u = 1$							
Hill	0.501	0.842	1.011	2.030	2.124	2.708	3.978
(s.d.)	(0.013)	(0.024)	(0.024)	(0.083)	(0.068)	(0.079)	(0.088)
T. Hill	0.499	0.838	1.007	2.015	2.111	2.692	3.968
(s.d)	(0.013)	(0.024)	(0.024)	(0.082)	(0.068)	(0.078)	(0.088)
(Theor. s.d.)	(0.013)	(0.019)	(0.026)	(0.032)	(0.039)	(0.045)	(0.052)
Regression	0.497	0.790	1.018	1.310	1.657	2.346	5.032
(s.d.)	(0.018)	(0.031)	(0.036)	(0.056)	(0.069)	(0.118)	(0.107)
$u = 5$							
Hill	0.508	0.854	1.027	2.078	2.166	2.762	4.000
(s.d.)	(0.013)	(0.023)	(0.026)	(0.088)	(0.071)	(0.083)	(0.090)
T. Hill	0.499	0.837	1.008	2.019	2.113	2.697	3.954
(s.d)	(0.013)	(0.022)	(0.026)	(0.085)	(0.069)	(0.081)	(0.088)
(Theor. s.d.)	(0.013)	(0.019)	(0.026)	(0.032)	(0.039)	(0.045)	(0.052)
Regression	0.498	0.795	1.023	1.321	1.672	2.405	4.936
(s.d.)	(0.017)	(0.028)	(0.035)	(0.055)	(0.071)	(0.119)	(0.104)
$u = 10$							
Hill	0.516	0.868	1.042	2.127	2.211	2.814	4.026
(s.d.)	(0.013)	(0.024)	(0.026)	(0.087)	(0.073)	(0.087)	(0.089)
T. Hill	0.499	0.837	1.008	2.021	2.116	2.698	3.938
(s.d)	(0.013)	(0.023)	(0.026)	(0.081)	(0.070)	(0.084)	(0.086)
(Theor. s.d.)	(0.013)	(0.019)	(0.026)	(0.032)	(0.039)	(0.045)	(0.052)
Regression	0.498	0.797	1.025	1.327	1.690	2.447	4.850
(s.d.)	(0.016)	(0.029)	(0.034)	(0.052)	(0.069)	(0.118)	(0.104)
$u = 50$							
Hill	0.562	0.959	1.132	2.520	2.514	3.143	4.215
(s.d.)	(0.015)	(0.026)	(0.029)	(0.112)	(0.086)	(0.089)	(0.095)
T. Hill	0.498	0.844	1.004	2.094	2.153	2.733	3.847
(s.d)	(0.013)	(0.023)	(0.026)	(0.087)	(0.072)	(0.079)	(0.085)
(Theor. s.d.)	(0.013)	(0.019)	(0.026)	(0.032)	(0.039)	(0.045)	(0.052)
Regression	0.498	0.813	1.024	1.399	1.772	2.652	4.492
(s.d.)	(0.016)	(0.027)	(0.032)	(0.060)	(0.070)	(0.109)	(0.102)
$u = 100$							
Hill	0.612	1.051	1.224	2.977	2.848	3.453	4.389
(s.d.)	(0.017)	(0.030)	(0.031)	(0.136)	(0.101)	(0.096)	(0.105)
T. Hill	0.498	0.848	1.001	2.172	2.202	2.766	3.726
(s.d)	(0.013)	(0.024)	(0.025)	(0.092)	(0.076)	(0.080)	(0.084)
(Theor. s.d.)	(0.013)	(0.019)	(0.026)	(0.032)	(0.039)	(0.045)	(0.052)
Regression	0.499	0.822	1.023	1.483	1.872	2.805	4.220
(s.d.)	(0.016)	(0.027)	(0.031)	(0.067)	(0.079)	(0.109)	(0.097)
$u = 500$							
Hill	0.937	1.705	1.786	8.025	5.179	4.947	5.476
(s.d.)	(0.027)	(0.052)	(0.049)	(0.299)	(0.158)	(0.136)	(0.142)
T. Hill	0.495	0.885	0.960	3.725	2.660	2.703	3.070
(s.d)	(0.013)	(0.026)	(0.024)	(0.242)	(0.091)	(0.069)	(0.071)
(Theor. s.d.)	(0.013)	(0.020)	(0.027)	(0.033)	(0.040)	(0.046)	(0.053)
Regression	0.496	0.874	0.979	3.916	2.732	2.811	3.224
(s.d.)	(0.015)	(0.028)	(0.027)	(0.260)	(0.100)	(0.077)	(0.076)
$u = 1000$							
Hill	1.336	2.554	2.344	11.083	6.870	6.148	6.474
(s.d.)	(0.039)	(0.078)	(0.067)	(0.314)	(0.200)	(0.179)	(0.184)
T. Hill	0.492	0.926	0.888	4.205	2.611	2.371	2.527
(s.d)	(0.013)	(0.026)	(0.023)	(0.110)	(0.069)	(0.060)	(0.062)
(Theor. s.d.)	(0.014)	(0.020)	(0.027)	(0.034)	(0.041)	(0.048)	(0.054)
Regression	0.493	0.920	0.903	4.308	2.671	2.428	2.592
(s.d.)	(0.015)	(0.028)	(0.026)	(0.123)	(0.075)	(0.066)	(0.070)

TABLE XLV: Stable Distribution, $k = 500$, $s = .20(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.501	0.864	0.979	2.364	2.258	2.594	3.451
(s.d.)	(0.050)	(0.089)	(0.094)	(0.397)	(0.288)	(0.278)	(0.293)
T. Hill	0.501	0.864	0.979	2.364	2.258	2.594	3.451
(s.d)	(0.050)	(0.089)	(0.094)	(0.397)	(0.288)	(0.278)	(0.293)
(Theor. s.d.)	(0.051)	(0.076)	(0.102)	(0.127)	(0.152)	(0.178)	(0.203)
Regression	0.473	0.759	0.960	1.341	1.675	2.281	4.272
(s.d.)	(0.067)	(0.115)	(0.132)	(0.279)	(0.306)	(0.442)	(0.346)
$u = 1$							
Hill	0.523	0.910	1.017	2.544	2.387	2.742	3.540
(s.d.)	(0.052)	(0.102)	(0.099)	(0.435)	(0.300)	(0.278)	(0.327)
T. Hill	0.501	0.869	0.976	2.369	2.258	2.612	3.437
(s.d)	(0.050)	(0.097)	(0.094)	(0.401)	(0.284)	(0.269)	(0.314)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.479	0.789	0.978	1.415	1.767	2.460	4.101
(s.d.)	(0.060)	(0.113)	(0.123)	(0.278)	(0.311)	(0.420)	(0.347)
$u = 5$							
Hill	0.589	1.042	1.130	3.261	2.797	3.119	3.716
(s.d.)	(0.059)	(0.111)	(0.115)	(0.624)	(0.341)	(0.331)	(0.343)
T. Hill	0.502	0.879	0.969	2.513	2.293	2.645	3.285
(s.d)	(0.049)	(0.093)	(0.097)	(0.461)	(0.282)	(0.292)	(0.290)
(Theor. s.d.)	(0.051)	(0.077)	(0.102)	(0.128)	(0.153)	(0.179)	(0.204)
Regression	0.489	0.823	0.988	1.653	1.961	2.702	3.757
(s.d.)	(0.056)	(0.104)	(0.116)	(0.379)	(0.339)	(0.396)	(0.334)
$u = 10$							
Hill	0.660	1.191	1.250	4.182	3.283	3.443	3.912
(s.d.)	(0.068)	(0.136)	(0.126)	(0.745)	(0.397)	(0.347)	(0.378)
T. Hill	0.503	0.893	0.964	2.728	2.380	2.656	3.133
(s.d)	(0.050)	(0.100)	(0.094)	(0.499)	(0.303)	(0.273)	(0.282)
(Theor. s.d.)	(0.051)	(0.077)	(0.103)	(0.128)	(0.154)	(0.179)	(0.205)
Regression	0.496	0.849	0.986	2.029	2.206	2.805	3.479
(s.d.)	(0.056)	(0.112)	(0.110)	(0.555)	(0.398)	(0.338)	(0.320)
$u = 50$							
Hill	1.141	2.239	1.896	8.796	5.419	4.851	5.057
(s.d.)	(0.132)	(0.267)	(0.212)	(1.023)	(0.581)	(0.535)	(0.542)
T. Hill	0.493	0.949	0.849	3.966	2.440	2.216	2.331
(s.d)	(0.053)	(0.108)	(0.084)	(0.416)	(0.235)	(0.217)	(0.221)
(Theor. s.d.)	(0.054)	(0.080)	(0.107)	(0.134)	(0.161)	(0.188)	(0.214)
Regression	0.489	0.930	0.866	4.083	2.512	2.277	2.403
(s.d.)	(0.057)	(0.116)	(0.094)	(0.456)	(0.265)	(0.237)	(0.244)

TABLE XLVI: Stable Distribution, $k = 1000$, $s = .20(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.499	0.861	0.973	2.333	2.231	2.595	3.448
(s.d.)	(0.035)	(0.066)	(0.067)	(0.263)	(0.197)	(0.193)	(0.212)
T. Hill	0.499	0.861	0.973	2.333	2.231	2.595	3.448
(s.d)	(0.035)	(0.066)	(0.067)	(0.263)	(0.197)	(0.193)	(0.212)
(Theor. s.d.)	(0.036)	(0.053)	(0.071)	(0.089)	(0.107)	(0.125)	(0.142)
Regression	0.480	0.772	0.971	1.355	1.689	2.329	4.369
(s.d.)	(0.049)	(0.086)	(0.097)	(0.181)	(0.224)	(0.325)	(0.251)
$u = 1$							
Hill	0.511	0.889	1.000	2.462	2.318	2.685	3.488
(s.d.)	(0.036)	(0.068)	(0.069)	(0.296)	(0.214)	(0.206)	(0.229)
T. Hill	0.499	0.865	0.976	2.362	2.244	2.609	3.434
(s.d)	(0.035)	(0.066)	(0.067)	(0.281)	(0.207)	(0.202)	(0.224)
(Theor. s.d.)	(0.036)	(0.054)	(0.071)	(0.089)	(0.107)	(0.125)	(0.143)
Regression	0.486	0.793	0.987	1.423	1.762	2.454	4.263
(s.d.)	(0.044)	(0.080)	(0.088)	(0.188)	(0.221)	(0.318)	(0.264)
$u = 5$							
Hill	0.548	0.958	1.066	2.813	2.557	2.899	3.596
(s.d.)	(0.039)	(0.074)	(0.075)	(0.339)	(0.227)	(0.228)	(0.226)
T. Hill	0.498	0.865	0.973	2.404	2.263	2.616	3.360
(s.d)	(0.035)	(0.067)	(0.069)	(0.281)	(0.201)	(0.212)	(0.206)
(Theor. s.d.)	(0.036)	(0.054)	(0.071)	(0.089)	(0.107)	(0.125)	(0.143)
Regression	0.492	0.812	1.003	1.521	1.866	2.633	4.007
(s.d.)	(0.042)	(0.078)	(0.087)	(0.209)	(0.226)	(0.308)	(0.243)
$u = 10$							
Hill	0.588	1.042	1.130	3.250	2.810	3.101	3.701
(s.d.)	(0.040)	(0.082)	(0.078)	(0.404)	(0.262)	(0.224)	(0.234)
T. Hill	0.499	0.876	0.967	2.489	2.290	2.622	3.268
(s.d)	(0.034)	(0.068)	(0.066)	(0.298)	(0.212)	(0.196)	(0.197)
(Theor. s.d.)	(0.036)	(0.054)	(0.072)	(0.090)	(0.107)	(0.125)	(0.143)
Regression	0.494	0.831	0.997	1.655	1.983	2.733	3.784
(s.d.)	(0.041)	(0.078)	(0.081)	(0.248)	(0.248)	(0.278)	(0.231)
$u = 50$							
Hill	0.836	1.574	1.522	6.849	4.390	4.087	4.392
(s.d.)	(0.065)	(0.129)	(0.107)	(0.636)	(0.340)	(0.300)	(0.319)
T. Hill	0.494	0.911	0.919	3.720	2.574	2.517	2.765
(s.d)	(0.037)	(0.071)	(0.061)	(0.530)	(0.221)	(0.175)	(0.181)
(Theor. s.d.)	(0.037)	(0.055)	(0.073)	(0.091)	(0.110)	(0.128)	(0.146)
Regression	0.492	0.889	0.946	3.893	2.675	2.651	2.949
(s.d.)	(0.041)	(0.077)	(0.068)	(0.624)	(0.256)	(0.196)	(0.202)
$u = 100$							
Hill	1.148	2.244	1.890	8.791	5.466	4.833	5.061
(s.d.)	(0.091)	(0.189)	(0.139)	(0.676)	(0.413)	(0.364)	(0.398)
T. Hill	0.494	0.947	0.844	3.954	2.462	2.201	2.331
(s.d)	(0.036)	(0.077)	(0.057)	(0.284)	(0.166)	(0.147)	(0.160)
(Theor. s.d.)	(0.038)	(0.056)	(0.075)	(0.094)	(0.113)	(0.132)	(0.150)
Regression	0.493	0.934	0.863	4.090	2.542	2.275	2.414
(s.d.)	(0.040)	(0.084)	(0.066)	(0.316)	(0.184)	(0.170)	(0.176)

TABLE XLVII: Stable Distribution, $k = 5000$, $s = .20(k - u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.497	0.860	0.974	2.320	2.213	2.597	3.445
(s.d.)	(0.015)	(0.031)	(0.030)	(0.119)	(0.090)	(0.088)	(0.097)
T. Hill	0.497	0.860	0.974	2.320	2.213	2.597	3.445
(s.d)	(0.015)	(0.031)	(0.030)	(0.119)	(0.090)	(0.088)	(0.097)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.055)	(0.063)
Regression	0.492	0.796	1.005	1.394	1.720	2.387	4.488
(s.d.)	(0.022)	(0.041)	(0.044)	(0.087)	(0.103)	(0.155)	(0.118)
$u = 1$							
Hill	0.500	0.865	0.979	2.346	2.237	2.615	3.456
(s.d.)	(0.015)	(0.030)	(0.029)	(0.119)	(0.090)	(0.089)	(0.098)
T. Hill	0.496	0.859	0.973	2.320	2.218	2.594	3.444
(s.d)	(0.015)	(0.030)	(0.029)	(0.117)	(0.089)	(0.089)	(0.098)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.055)	(0.063)
Regression	0.494	0.799	1.007	1.408	1.743	2.414	4.443
(s.d.)	(0.022)	(0.039)	(0.042)	(0.081)	(0.101)	(0.145)	(0.117)
$u = 5$							
Hill	0.510	0.887	0.998	2.438	2.307	2.678	3.478
(s.d.)	(0.016)	(0.031)	(0.030)	(0.127)	(0.092)	(0.092)	(0.097)
T. Hill	0.497	0.862	0.973	2.333	2.228	2.595	3.423
(s.d)	(0.016)	(0.030)	(0.029)	(0.120)	(0.088)	(0.090)	(0.095)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.055)	(0.063)
Regression	0.496	0.809	1.011	1.438	1.782	2.495	4.327
(s.d.)	(0.021)	(0.037)	(0.041)	(0.083)	(0.096)	(0.148)	(0.112)
$u = 10$							
Hill	0.521	0.906	1.016	2.524	2.371	2.746	3.508
(s.d.)	(0.017)	(0.031)	(0.031)	(0.136)	(0.096)	(0.094)	(0.104)
T. Hill	0.498	0.861	0.972	2.334	2.229	2.602	3.402
(s.d)	(0.016)	(0.029)	(0.029)	(0.124)	(0.090)	(0.090)	(0.100)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.055)	(0.063)
Regression	0.497	0.813	1.011	1.457	1.807	2.560	4.232
(s.d.)	(0.020)	(0.035)	(0.040)	(0.085)	(0.096)	(0.142)	(0.114)
$u = 50$							
Hill	0.586	1.035	1.128	3.247	2.807	3.113	3.704
(s.d.)	(0.019)	(0.034)	(0.036)	(0.186)	(0.117)	(0.100)	(0.107)
T. Hill	0.497	0.868	0.963	2.473	2.279	2.623	3.268
(s.d)	(0.016)	(0.029)	(0.030)	(0.136)	(0.093)	(0.087)	(0.090)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.056)	(0.064)
Regression	0.497	0.833	1.005	1.661	1.991	2.769	3.814
(s.d.)	(0.019)	(0.034)	(0.037)	(0.111)	(0.111)	(0.124)	(0.105)
$u = 100$							
Hill	0.654	1.172	1.242	4.146	3.285	3.432	3.914
(s.d.)	(0.022)	(0.041)	(0.039)	(0.252)	(0.139)	(0.109)	(0.119)
T. Hill	0.496	0.876	0.953	2.659	2.351	2.631	3.127
(s.d)	(0.016)	(0.030)	(0.030)	(0.159)	(0.103)	(0.086)	(0.088)
(Theor. s.d.)	(0.016)	(0.024)	(0.032)	(0.040)	(0.048)	(0.056)	(0.064)
Regression	0.497	0.848	0.992	1.976	2.214	2.833	3.515
(s.d.)	(0.017)	(0.034)	(0.036)	(0.162)	(0.135)	(0.105)	(0.099)
$u = 500$							
Hill	1.140	2.230	1.885	8.776	5.460	4.846	5.036
(s.d.)	(0.040)	(0.084)	(0.065)	(0.308)	(0.186)	(0.169)	(0.166)
T. Hill	0.490	0.940	0.842	3.938	2.451	2.203	2.314
(s.d)	(0.016)	(0.033)	(0.026)	(0.127)	(0.076)	(0.068)	(0.067)
(Theor. s.d.)	(0.017)	(0.025)	(0.033)	(0.042)	(0.050)	(0.058)	(0.067)
Regression	0.491	0.933	0.867	4.097	2.541	2.284	2.407
(s.d.)	(0.018)	(0.036)	(0.030)	(0.140)	(0.084)	(0.076)	(0.075)

TABLE XLVIII: Stable Distribution, $k = 10000$, $s = .20(k-u)$

α	.50	.75	1.00	1.25	1.50	1.75	2.00
$u = 0$							
Hill	0.497	0.859	0.973	2.316	2.215	2.591	3.443
(s.d.)	(0.011)	(0.020)	(0.021)	(0.083)	(0.063)	(0.065)	(0.069)
T. Hill	0.497	0.859	0.973	2.316	2.215	2.591	3.443
(s.d)	(0.011)	(0.020)	(0.021)	(0.083)	(0.063)	(0.065)	(0.069)
(Theor. s.d.)	(0.011)	(0.017)	(0.022)	(0.028)	(0.034)	(0.039)	(0.045)
Regression	0.495	0.799	1.009	1.398	1.730	2.388	4.505
(s.d.)	(0.016)	(0.028)	(0.031)	(0.059)	(0.072)	(0.114)	(0.085)
$u = 1$							
Hill	0.499	0.865	0.977	2.321	2.226	2.608	3.450
(s.d.)	(0.012)	(0.021)	(0.022)	(0.087)	(0.059)	(0.061)	(0.069)
T. Hill	0.497	0.861	0.974	2.307	2.215	2.596	3.444
(s.d)	(0.011)	(0.021)	(0.021)	(0.087)	(0.059)	(0.061)	(0.069)
(Theor. s.d.)	(0.011)	(0.017)	(0.022)	(0.028)	(0.034)	(0.039)	(0.045)
Regression	0.496	0.804	1.012	1.404	1.741	2.420	4.480
(s.d.)	(0.016)	(0.027)	(0.031)	(0.061)	(0.067)	(0.107)	(0.086)
$u = 5$							
Hill	0.505	0.874	0.985	2.377	2.264	2.642	3.466
(s.d.)	(0.011)	(0.021)	(0.021)	(0.089)	(0.064)	(0.065)	(0.069)
T. Hill	0.497	0.860	0.971	2.318	2.219	2.594	3.437
(s.d)	(0.011)	(0.021)	(0.021)	(0.086)	(0.062)	(0.064)	(0.068)
(Theor. s.d.)	(0.011)	(0.017)	(0.022)	(0.028)	(0.034)	(0.039)	(0.045)
Regression	0.497	0.807	1.011	1.425	1.768	2.465	4.411
(s.d.)	(0.015)	(0.026)	(0.030)	(0.059)	(0.068)	(0.103)	(0.084)
$u = 10$							
Hill	0.509	0.886	0.997	2.434	2.300	2.679	3.481
(s.d.)	(0.011)	(0.021)	(0.022)	(0.088)	(0.067)	(0.065)	(0.071)
T. Hill	0.496	0.861	0.972	2.328	2.220	2.596	3.425
(s.d)	(0.011)	(0.021)	(0.021)	(0.083)	(0.064)	(0.063)	(0.069)
(Theor. s.d.)	(0.011)	(0.017)	(0.022)	(0.028)	(0.034)	(0.039)	(0.045)
Regression	0.497	0.811	1.014	1.441	1.784	2.505	4.343
(s.d.)	(0.014)	(0.025)	(0.030)	(0.056)	(0.069)	(0.102)	(0.084)
$u = 50$							
Hill	0.547	0.959	1.063	2.803	2.545	2.904	3.591
(s.d.)	(0.012)	(0.023)	(0.023)	(0.107)	(0.072)	(0.068)	(0.074)
T. Hill	0.496	0.864	0.968	2.386	2.245	2.611	3.351
(s.d)	(0.011)	(0.021)	(0.020)	(0.087)	(0.063)	(0.063)	(0.067)
(Theor. s.d.)	(0.011)	(0.017)	(0.022)	(0.028)	(0.034)	(0.039)	(0.045)
Regression	0.497	0.825	1.011	1.534	1.884	2.675	4.044
(s.d.)	(0.014)	(0.025)	(0.026)	(0.062)	(0.070)	(0.095)	(0.078)
$u = 100$							
Hill	0.585	1.038	1.129	3.242	2.810	3.114	3.708
(s.d.)	(0.013)	(0.025)	(0.025)	(0.132)	(0.082)	(0.073)	(0.075)
T. Hill	0.496	0.870	0.964	2.468	2.280	2.623	3.271
(s.d)	(0.011)	(0.021)	(0.021)	(0.097)	(0.067)	(0.063)	(0.063)
(Theor. s.d.)	(0.011)	(0.017)	(0.022)	(0.028)	(0.034)	(0.039)	(0.045)
Regression	0.498	0.836	1.006	1.657	1.994	2.771	3.819
(s.d.)	(0.013)	(0.024)	(0.027)	(0.078)	(0.082)	(0.089)	(0.076)
$u = 500$							
Hill	0.838	1.567	1.519	6.910	4.392	4.079	4.399
(s.d.)	(0.020)	(0.041)	(0.036)	(0.202)	(0.107)	(0.090)	(0.100)
T. Hill	0.494	0.905	0.916	3.728	2.565	2.504	2.764
(s.d)	(0.011)	(0.023)	(0.021)	(0.193)	(0.068)	(0.052)	(0.057)
(Theor. s.d.)	(0.011)	(0.017)	(0.023)	(0.029)	(0.034)	(0.040)	(0.046)
Regression	0.495	0.890	0.950	4.005	2.694	2.656	2.965
(s.d.)	(0.013)	(0.025)	(0.023)	(0.209)	(0.077)	(0.061)	(0.065)
$u = 1000$							
Hill	1.140	2.230	1.883	8.764	5.458	4.836	5.037
(s.d.)	(0.028)	(0.058)	(0.045)	(0.222)	(0.133)	(0.113)	(0.122)
T. Hill	0.490	0.940	0.840	3.934	2.450	2.196	2.314
(s.d)	(0.012)	(0.023)	(0.018)	(0.092)	(0.054)	(0.046)	(0.050)
(Theor. s.d.)	(0.012)	(0.018)	(0.024)	(0.029)	(0.035)	(0.041)	(0.047)
Regression	0.491	0.932	0.865	4.094	2.542	2.278	2.409
(s.d.)	(0.013)	(0.025)	(0.020)	(0.100)	(0.059)	(0.053)	(0.055)